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HISTORY OF POWER TRANSMISSION

BY

W. A. WILLIAMS[1]

INTRODUCTION

There has been an over-abundance of vagueness and considerable misconception surrounding the actual origin of the elements of Power Transmission in use today. For instance, it is a popular belief that the flat belt and pulley combination is as old as the ages, and that such means of power transmission were used by the Egyptians, Babylonians and other early civilizations. A careful study of the available literature shows that the toothed gear was the earliest form of mechanical power transmission in use for the transmission of continuous motion, and that pulleys and belts did not come into use for many centuries after the development of gears.

In the pre-Christian era, toothed wheels were in use as early as 330 B.C., according to the writings of the philosopher Aristotle, in which toothed wheels are mentioned for use in windlasses. By 250 B.C. toothed wheels were in use in hydraulic organs and clocks. About this time an advanced form of toothed wheel was the lantern pinion consisting of two disks with bars connecting them at their peripheries; the bars forming the teeth which engaged projecting pins on a gear wheel usually ran on an axis 90° from the axis of the lantern pinion. This construction is illustrated in Fig. 1 on a set of gears used to drive a stone mill in Rome some time prior to 16 B.C.

All of the evidence points to the fact that power was first applied to the grinding of wheat and other grains, and that power was seldom used for any other purpose until more than 1000 years later. Several forms of water-driven mills have been recorded. The earliest form did not have gears, but utilized a water wheel that revolved on a vertical

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axis and connected directly to the mill stone. Figure 1 is taken from the writings of Vitruvius which appeared about 16 B.C. The actual origin of the construction may have been much earlier because Vitruvius was not claiming originality for the constructions shown in his writings but was simply showing what was in use in his day.

Another interesting illustration from Vitruvius but which also appears in the works of Hero of Alexandria about 100 years earlier is shown in Fig. 2, which shows a very ingenious set of worm drives which operated from the axle of a stagecoach or wagon as a means of recording the distance traveled. The device was geared so that a day's journey

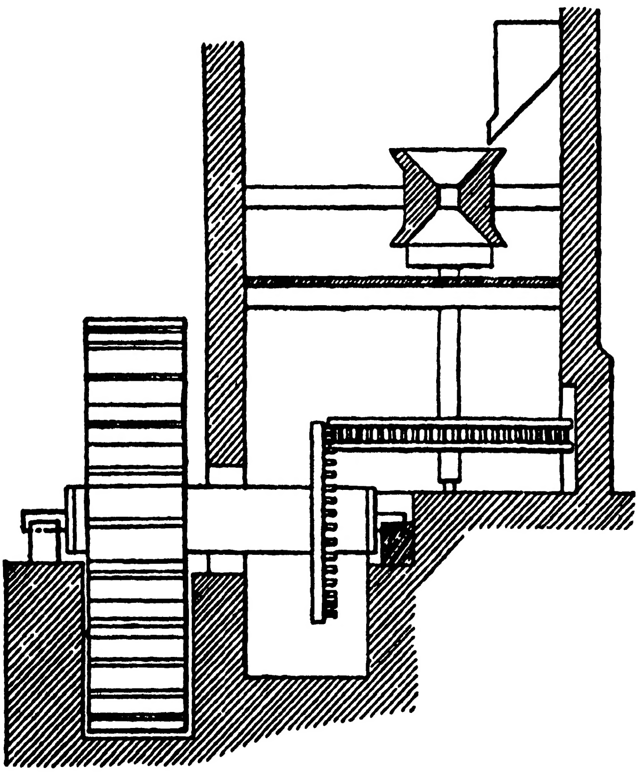

FIG. 1.

usually resulted in one complete revolution of the indicating hand on the dial at the top of the instrument.

The predecessor of the modern bucket elevator was the chain-of-pots well pump which had its origin either in the Far East or in Egypt, and was operated either by animal power through wooden gears similar to those shown in the mill in Fig. 1, or by means of the rope drive shown in Fig. 3. An interesting aspect of the illustration in Fig. 3 is that the rope drive apparently does not provide for continuous motion and yet the chain-of-pots is constructed so that it could rotate continuously. It is a matter of speculation as to whether the illustrator failed to

understand that the drive was intended to provide for continuous motion or whether the art had actually not progressed to that degree when this drawing was made. This picture was taken from Usher, "History of Mechanical Inventions."

There is very little evidence of advancement in mechanical power transmission after the fall of the Roman Empire. About the only major development was the use of windmills for grinding grain following the Crusades. The evidence indicates that the Crusaders brought back the designs of windmills from the East around 1200 A.D., and

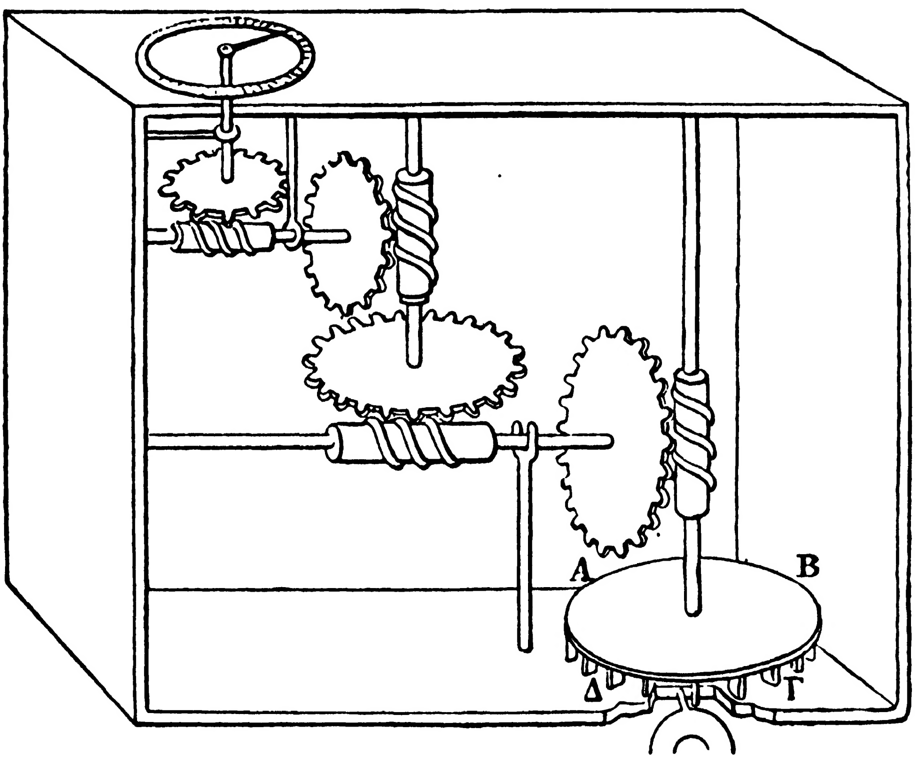

FIG. 2.

that the mills found ready acceptance in areas where water power was not available. Windmills were not used for anything but milling until several hundred years later. Incidentally, the term "millwright" was applied to the early specialists who constructed and maintained both water-powered and wind-powered mills.

The earliest recorded use of continuous power transmission by a cord travelling over two pulleys was on a quilling wheel for winding thread on a bobbin. This development took place around 1200 A.D. The early American and European spinning wheels took their basic construction from these quilling wheels and drill spindles. The earliest evidence of the use of water power for purposes other than

milling is the illustration shown in Fig. 4 of a sawmill erected sometime in the 14th century in Germany. The drawing is interesting because it not only shows the crudeness of their mechanical construction but also the roughness of their draftsmanship. It takes a considerable amount of imagination to figure out how this sawmill worked, but there is sufficient evidence in this illustration to indicate that it did work.

During the Renaissance period, from 1400 A.D. to 1600 A.D., great strides were made in the use of power and mechanical contrivances for the manufacture of implements, clothing and building materials.

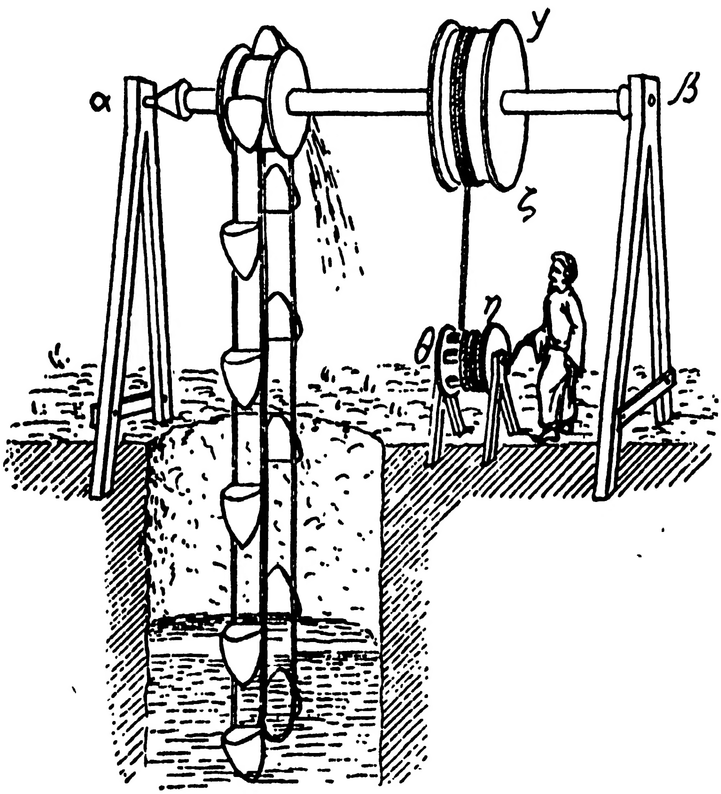

Fig. 3.

The incomparable Leonardo Da Vinci is credited with a long list of inventions among which are the flat belt and crossed belt, spiral gears, linked chain, the flyer for continuous spinning of silk and other fibers, and antifriction bearing. Leonardo was so productive that many of his inventions did not see daylight for from 100 to 200 years after his death when his notes were used by subsequent enterprisers to further their own gains. Many inventions, which were for a long time credited to others, have since been found in the notes left by Leonardo.

In the search for illustrative material on this subject, a veritable gold mine was found in Diderot's *Encyclopédie*, which was published in

1778 in France under the auspices of Catherine the Great of Russia. This encyclopedia describes in detail all of the mechanical arts of the period. It does not designate the date of origin of each of the machines and processes shown, but it is to be assumed that most of them had been in use for some time prior to the publication of the encyclopedia and we can be reasonably sure that many of them were developed during the Renaissance period. This encyclopedia is highly recommended to those interested in the historical aspects of production methods. Another source of equal interest is *Diverse et Artificiose Machine* by Ramelli published in 1588.

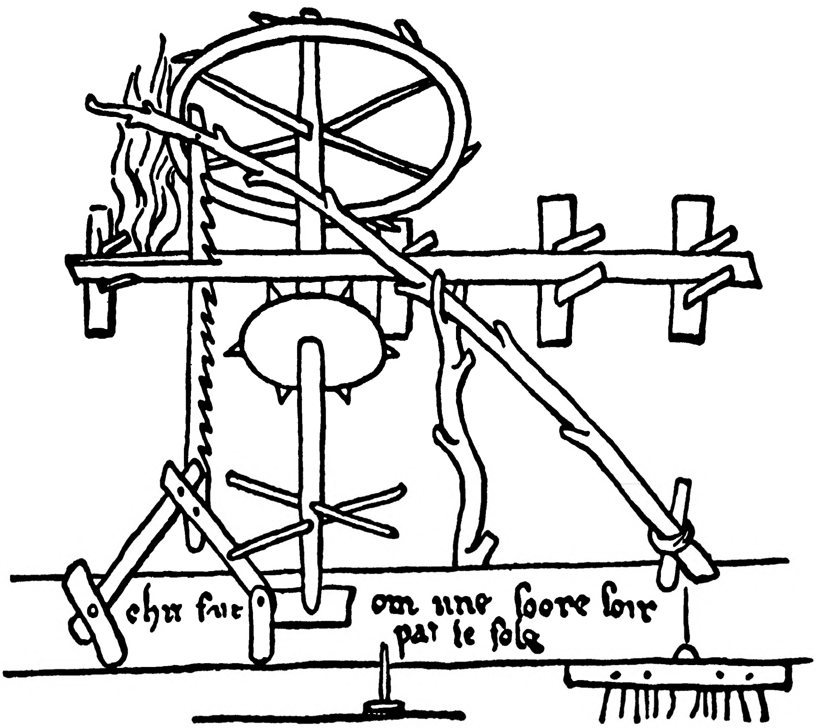

FIG. 4.

Probably the first practical use of the flat belt was on multiple spindle spinning frames, both hand and power operated. On one hand-powered machine illustrated by Diderot ten vertical spindles were arranged in an oval so that one belt could be used without the necessity of idlers to keep the belt in contact with all of the spindles. On a larger machine which was power-driven the spindles were arranged in two straight rows of 24 each, with a single belt driving them all aided by idler pulleys between spindles to increase the arc of contact. All of these spinning machines employed the flyer invented by Leonardo.

That power distribution was a fairly well-known art by the 18th century is clearly indicated by the many illustrations in Diderot's

Encyclopédie which show a wide variety of methods for driving production machinery. Ropes or round belts were used between shafts where the torque was not high, such as in the grinding department of a cutlery, and were usually crossed to increase the arc of contact between belt and pulley. One illustration shows two ropes in parallel, thus anticipating the V-belt drive which is so popular today. Where water power was not available, horses or other animals were hitched in a variety of ways to obtain the necessary power. The most common method is illustrated in Fig. 5, but animals were also hitched to various types of treadmills to rotate mill stones and to provide the necessary

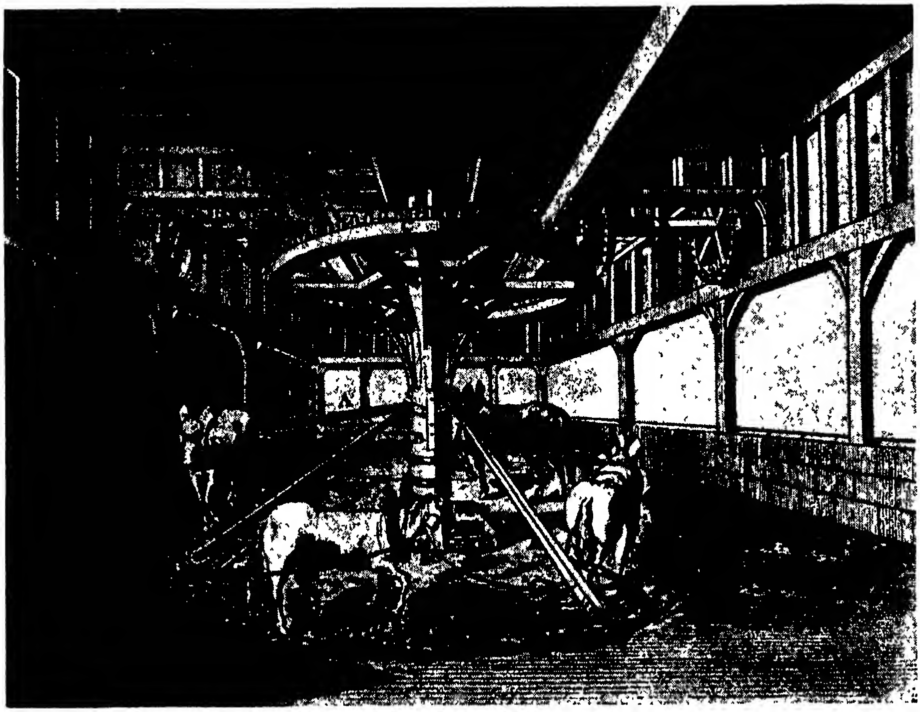

FIG. 5.

power for drilling and boring operations in metal and woodworking shops.

The four-horse capstan shown in Fig. 5 was used to drive two mill stones in a brewery as well as to provide power for lifting grain to the grinding floor. A similar capstan was used in a lead rolling mill to reduce lead billets to sheets. In this machine an ingenious clutch was used to reverse the direction of rotation of the rolls, permitting the horses always to travel in the same direction.

Great ingenuity and inventiveness was demonstrated by the Renaissance engineers in the development of water works for their cities.

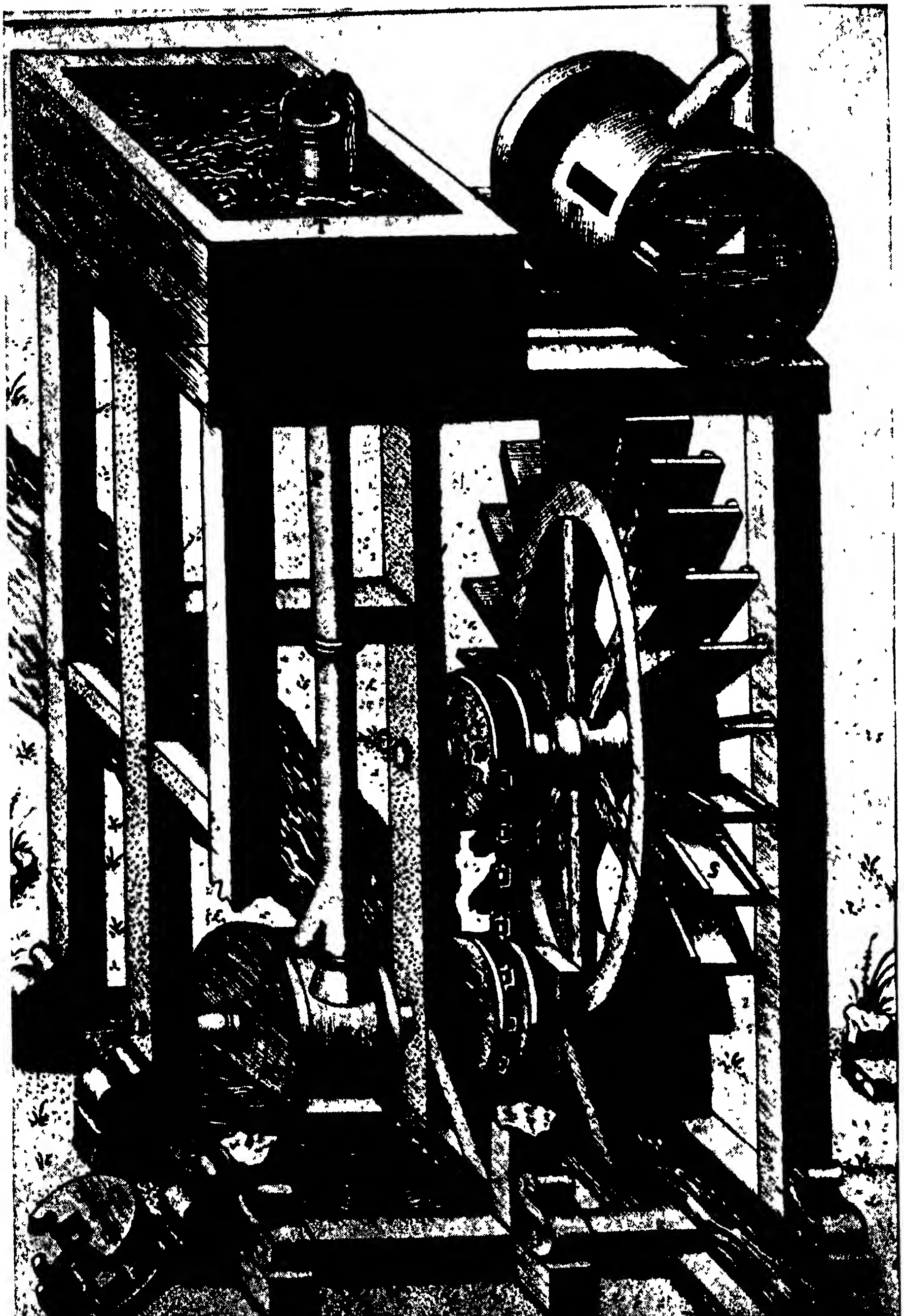

FIG. 6.

Figure 6 shows the details of a rotary pump connected to an undershot water wheel by a chain running on wooden sprockets. This illustration is from *Diverse et Artificiose Machine* by Ramelli, published in 1588, and

is the first practical utilization of a chain drive following its invention by Leonardo da Vinci. The details of the pump are particularly interesting because a well known vacuum pump on the market today utilizes the same construction. Ramelli shows two other rotary pumps, both of which have modern counterparts.

DEVELOPMENT OF STEAM POWER

It did not take long after the introduction of mechanical processes for manufacturers to start looking for ways to take physical labor out of all of their operations. In England, coal was an important commodity, and as the mines began to get deeper the problem of pumping water out of them became serious. Many mines had to be abandoned because of the difficulty of pumping water out of them with the inadequate equipment available. Some mines used mechanical pumps operated through oscillating cables connected to the crank of a water wheel, but obviously a stream had to be readily accessible for this system to be practical. In 1628 the Marquis of Worcester invented a pump which was called the "water commanding engine." This engine lifted and forced water out of the mines by the direct action of steam on the water. Condensation caused water to be drawn into a receiver from the bottom of the mine and direct pressure from the boiler into the receiver forced the water out of it and susbequently out of the mine. The receiver necessarily had to be located near the bottom of the mine and, for convenience, the boilers as well. The relatively high boiler pressures required to lift water out of deep mines caused boiler failures. In the event of a failure of any kind, the whole apparatus was flooded and the mine had to be pumped out by hand before the steam apparatus could be utilized again.

Of greater significance in the development of the steam engine was the Newcomen atmospheric pumping engine. In this engine, steam was admitted to a cylinder at atmospheric pressure and was followed by a spray of water which condensed the steam, thus forming a vacuum. Atmospheric pressure then forced the cylinder downward. The cylinder was connected to a working beam, whose other end was fastened to a pumping rod which could go as deep into a mine as necessary. This engine was strictly a pumping engine and was wholly unsuited to the development of rotary motion. It had a distinct advantage in that the boiler could be outside of the mine and that the boiler did not have to withstand high pressure. By 1772 engines up to 75-in. bore with 96-in. stroke had been built and were rated up to 75 hp. While these engines were somewhat wasteful of fuel they continued to be used as late as 1830, because fuel was so cheap and because the pumps were so reliable.

The development by Watt in 1765 of a condenser separate from the steam cylinder, resulted in higher efficiency for the Newcomen engine.

Later his invention of the double acting cylinder, utilizing steam pressure to produce work in both directions, greatly increased the efficiency of the steam engine and made it possible for the development of rotative engines. In introducing the rotative engine, Watt had to circumvent a previous patent, covering the use of the crank for the development of rotative power, issued to Wasbrough in 1779. Wasbrough failed to make practical use of his development because his work was confined to Newcomen engines. The final development of the rotative engine permitted the use of power in plants away from water and, coming as it did, at the time when mechanization of textile processes was being achieved, it is not surprising that within a year or two after the development of a steam engine prime mover, it was used in a textile mill. During the next 100 years of use, the steam engine was improved only in efficiency and in detail. Engines were speeded up, and improvements in machining methods made possible more accurate cylinders and bearings. It is worthy of note that the methods of boring cylinders were so crude in the early days of steam engine manufacturing that Watt had to tolerate an error of $\frac{3}{8}$-in. in the bore of a cylinder.

Steam engines did not immediately displace water wheels as a source of power. In the United States the division of power was 64 per cent steam and 36 per cent water in 1880; however, in the next 10 years 90 per cent of the added power was by steam.

The perfection of the water turbine by Fourneyron of Paris in 1832 was regarded as equally important as Watt's steam engine. Prior to this development, water power was limited to relatively crude undershot and overshot water wheels and the amount of power that could be developed by any one unit was limited to around 15 to 20 hp. With the successful construction of the turbine it became possible to increase the power to between 75 and 100 hp. with relative ease and, as we know, the modern turbine has practically unlimited capacity.

It was about this same time, namely 1831, that Faraday discovered the laws of electromagnetism and developed the dynamo, which marked the beginning of the use of electric power. As the dynamo and electric motor gradually improved in efficiency, increased in size, and advanced in operating characteristics, they gradually surpassed all other types of prime movers.

INTRA PLANT POWER TRANSMISSION

The best factual data point to the fact that in the 14th, 15th, and 16th centuries, most power transmission inside of plants was accomplished by open wooden gears such as have been previously illustrated or by means of belt drives using round rope made of rawhide or hemp.

In the 18th century, after the steam engine began to find its place in plants and also as the wooden gears began to be replaced by cast

toothed iron gears, main line shafts carrying considerable horsepower were driven almost entirely by gears, and belts were used only to drive individual machines from the main line shaft. This was largely a result of the inability of the existing belt drives to transmit sufficient power at the low speeds in use at that time. As steam engines became capable of operating at somewhat higher speeds and as the horsepower being transmitted became larger, the crude gears became very objectionable and in about 1850 in Belfast, Ireland, the multiple rope transmission was first tried out and in a very short time became very popular in England and in United States.

It is a curious coincidence that rope drives had almost completely displaced open gear drives in many of the mills in England, particularly for transmission of power from steam engines, just about the time when a more complete understanding of the proper tooth form for gears and also when the technique of cutting gears had become known. However, the trend towards belt drives had accelerated to such an extent that even though it was possible to make gears that were reasonably quiet, the general acceptance of ropes, and the satisfaction with their performance, practically eliminated gears from use. However, there are still some mills in England using open gear drives from the main engine and completely throughout the plant from one lineshaft to another. It is generally felt that the gear drives are more efficient than rope or belt drives. Their disadvantages are that they are noisy, somewhat dirtier and require a fair amount of lubrication. Whereas gears were abandoned in 1850 because of their noise, and inability to handle high speeds, today super precise gears are the only possible means of transmitting power from turbines running as high as 30,000 rpm.

The first flat leather belt for use on a group drive or a drive from a single engine to a multiplicity of line shafts, was made in Lowell, Massachusetts, in 1828. This drive was from a large waterwheel to a series of 14 line shafts. An 18-in. leather belt, over 300 ft. long, was used and the belt inter-twined between the various line shafts so that each line shaft had to carry a total belt pull equivalent to the full horsepower of the water wheel. Since that time it has been learned that it is more economical to employ a series of belts and to drive one line shaft from another, each succeeding belt carrying only the power required by the remaining shafts. To have a separate drive from the engine to each shaft is even better. When electric motors were perfected sufficiently to become the principle type of prime movers, rope drives rapidly gave way to flat belt drives which were better suited to the high speed required by electric motors. During the last fifty years flat belts and group drives have been the principle mode of power transmission, until recently when individual motorization and V-belt drives have been the vogue on most new equipment.

INTER PLANT POWER TRANSMISSION

In the 19th century a number of systems for the distribution of power between plants were tried and discarded. Each served its respective function for a time before electrical power distribution finally was adopted universally and displaced the other systems for inter-plant power distribution.

The five major competing systems other than electricity were: high pressure water, air, cable, steam and gas.

High pressure water as a means of power distribution was started in England by Lord Armstrong about 1840. The pressure most commonly used was 750 lb. per sq. in. The pressure was generated by plunger pumps driven either by water wheels or by steam engines. Plunger type accumulators were used to increase the instantaneous horsepower capacity of the system. The water was used to operate cranes, dock gates, and to drive piston motors. The largest installation was in London, where the number of users was approximately 2000, and the capacity was 2600 hp.

The small amount of power per customer gives a clue to the usage for such a system. Water power was found to be effective and reasonably efficient for intermittent usage, but could not compete with steam engines for continuous usage. Numerous practical difficulties developed, such as: dirty water, and the necessity for filters; corrosion and clogging of the pipes; and freezing of the mains in severe weather. However, the fundamental principle is still in use today on large hydraulic press installations, showing that for particular purposes the system is sound.

Air as a means of power distribution was started in 1845, and the first really significant installation was made for operating the boring machinery for the Mont Cenis Tunnel. Shortly thereafter it was adopted quite generally for use in coal mines, where its natural advantages of safety and ventilation made it the most popular system for many years. In 1885 a large vacuum system was installed in Paris. The motors were worked by atmospheric pressure, and exhausted into pipes in which a vacuum was maintained by air pumps at the central station. The amount of air used was measured by meters. In the 1890's air appeared to be the major contender with electricity as a means of distribution of power throughout a city. It was not seriously considered by factories, since they could afford to develop their own power by steam engines, and distribute mechanically as will be explained later. But, for the small user, air power was superior to water power, largely because air was more readily available, cleaner, and its distribution cheaper. Also, water power had the disadvantage that the amount of water used was the same regardless of load, whereas with air, the amount required would vary with respect to load by altering the cutoff.

One of the most interesting and ingenious uses of air was installed in Paris and Vienna in 1870, for operating over 8000 clocks by means of an air impulse of 20 seconds duration once each minute. The air mains were $\frac{5}{8}$ to $\frac{7}{8}$ in. in diameter and the service pipes were $\frac{1}{4}$- to $\frac{3}{8}$-in. diameter.

The vacuum system sufficed in Paris for domestic use, but as demand for use to operate motors increased a high pressure system of 8000 hp. was installed to operate at about 100-lb. pressure. Numerous other European cities employed air for power distribution, until the economy of electrical distribution became so obvious that the use of air could no longer be justified.

Telodynamic Transmission was the name given to the transmission of power by steel cables passing over large grooved sheaves. This system was first tried out in 1850 by a Mr. M. C. F. Hirn in Colmar, Alsace to supply power to some building approximately 86 yards from the single steam engine available. He first tried using a steel band 2 in. wide and 0.04 in. thick, but the wind set the band into vibrations and the pulley guides tore the belt at the joints. It worked for 18 months delivering 12 hp. He then tried $\frac{1}{4}$-in. wire cable instead of the band using the same pulleys with a $\frac{1}{2}$-in. groove turned in the rim. The drive operated for many years. A second drive to a distance of 256 yards with a $\frac{1}{2}$-in. cable over $9\frac{1}{2}$-ft. pulleys running at 92 rpm. transmitting 50 hp. was erected.

Various installations were made all over Europe and in Russia until electric distribution reached a state of perfection in about 1900. On relatively short drives up to approximately 500 yards; the efficiency might be as high as 93 per cent, but falls to 60 per cent for 5000 yards. The cables and pulleys suffered a good deal from the weather, and the cables had to be replaced almost annually. First cost was high due to the necessity of heavy piers at least every 600 ft. Another disadvantage of the system for distribution of power from a central source to a large number of customers was the impossibility of measuring the amount of power used by each customer.

Steam for heat and power purposes was first distributed on a commercial basis in New York City in 1881 following a study of the problem by Dr. C. E. Emery starting in 1869. The steam pressure was 80 lb. and approximately 20,000 hp. was delivered from two stations. The system was tried out for heating purposes only in 1877 by Birdsill Holly in Lockport, where by 1879, 16,000 ft. of pipe line had been laid. The only advantage of steam distribution was the elimination of boiler investment and maintenance by the consumer, and this of course was only economically sound for small users or intermittent users.

Gas for power purposes as well as for heating and lighting was in use from about 1880 in England and United States. The gas was mostly illuminating gas, and was used in Otto cycle engines for various purposes.

In London, the first current for electric lighting was generated by dynamos, belt driven from two-cylinder Otto cycle gas engines of 60-hp. capacity.

It is significant to note that every one of the systems of power distribution developed in the late 19th century has survived in one form or another. High pressure water with an accumulator is still considered the most efficient means for operating large hydraulic presses. Air, while not used for inter-plant power distribution, is one of the most effective and convenient means of driving hand tools. For operating riveters, hammers and rock drills, nothing has been found to equal it. The convenience of air power on spot welders, jigs, die cushions, paint sprayers, and a host of other uses makes it obvious that it is here to stay. The Telodynamic cable transmission system has survived only in the form of the V-belt which today is the most popular method of power take-off from a motor. However, the system has not survived in any form for power distribution from one building to another or from one plant to another. The cable system for outdoor transmission was followed by rope transmission indoors. Rope transmission was replaced by leather or rubber flat belts, and later and much more gradually plants driven by a single source of power broke up their line-shafts into smaller units individually motor driven, and a great many have eliminated group drives altogether in favor of a separate motor at each machine.

In perspective, it becomes clear that changes in methods of power transmission did not occur because of any whimsical popularity of one form of power transmission over another, or because one manufacturer did a better job of advertising than another, but because the particular system in vogue at any one time represented the best compromise for the industry, the materials, the knowledge, and the economics of the era. It is probable that future history will follow the same course as past history and that we can expect that our methods of power transmission will change as the requirements of industry evolve.

There is a considerable tendency for engineers and manufacturers to be very proud of the advancements of our day and to somewhat assume that our forefathers were pretty backward in almost everything they did. This study has indicated to the writer that our forefathers were extremely brillant mechanics and that considering what they had to work with they were equally as intelligent as our best engineers today.

It was Galileo who said that he was able to do the things he did and make the contributions to science he did, because he was standing on the shoulders of giants. Certainly if Galileo could make that statement, we should humble ourselves to the same extent and recognize that we have achieved our mechanical wonders because we are standing on the developments of our forefathers who gave us a sound platform of achievement on which to build further.

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GRAPHICAL ANALYSIS OF IMPACT OF BARS STRESSED ABOVE THE ELASTIC RANGE *

BY

KALMAN J. DEJUHASZ [1]

ABSTRACT

A number of impact problems of engineering interest are analyzed by means of a graphical method previously developed by the author. Problems considered comprise: impact at constant velocity of finite duration applied to one end of a bar, the length of the bar being infinite; bars of finite length, the other end being free, and the other end being fixed or restrained; impact stresses below yield stress (elastic range), and impact stresses above yield stress (plastic range). The history and gradient of stress and of velocity are represented in three-dimensional diagrams. The energy expended in impact, and its distribution into energy of plastic deformation, and into residual kinetic and elastic stress energy are determined. The total residual permanent strain is determined. The general treatment is supplemented with numerical examples.

INTRODUCTION

Impact is a special case of the broad group of dynamic phenomena in which a force P having a defined history function $P = f(t)$ is exerted on a body of m mass. The present inquiry will be restricted to direct and central impact, that is, the motion is assumed to be perpendicular to the striking surfaces, and the force is assumed to be directed along the line joining the mass centers; furthermore, the impacted body is assumed to be initially at rest. As for any dynamic phenomenon, also for impact, the equality of impulse and momentum holds good and it can be written:

$$P\,dt = d(mv).$$

The distinguishing features of impact, as compared with other dynamic phenomena, are that the force is large, its duration is very short, and it is applied to a point or to an area on the surface of the body.

Impact phenomena formed the subject of theoretical and experimental studies by a long line of investigators, beginning with Galileo and possibly even earlier. The theories have, of necessity, used various degrees of approximation, falling into two groups: considering the bodies (*a*) rigid, and (*b*) yielding.

Impact of Rigid Bodies

In rigid bodies the particles are located at constant and unalterable distances from one another, and under direct and central impact, at

* Presented before the Seventh International Congress of Applied Mechanics, September, 1948, at the Imperial College of Science and Technology, London, England. It is based on work performed for the Department of the Army, Office of the Chief of Ordnance.

[1] Professor of Engineering Research, Engineering Experiment Station, The Pennsylvania State College, State College, Pa.

any given instant of time, all particles will possess the same velocity, that is, any change of velocity applies to the entire body. Therefore the principle of momentum can be written:

$$\int_0^t P\,dt = \int_0^v m dv = m \int_0^v dv.$$

It has been early recognized, however, that the assumption of an ideally "rigid" body introduces conceptual contradictions and inconsistencies. The dimensions of the body are finite (that is, not infinitesimally small) and its particles are located at various distances from the point of application of the force (on the outside surface of the body); therefore an infinite velocity of propagation of the disturbance from particle to particle within the body has to be assumed, and consequently also infinitely large stresses between the particles.

Impact of Yielding Bodies

These contradictions are eliminated by the realistic assumption that the particles of the body are capable of displacement relatively to one

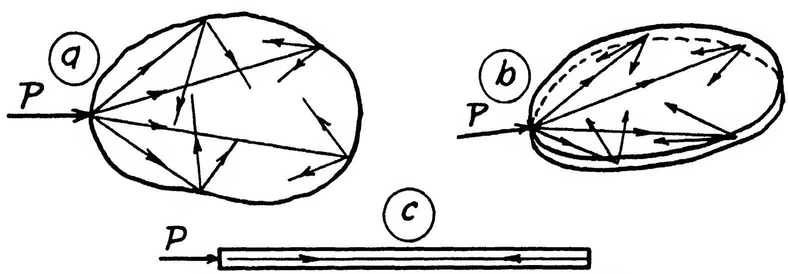

FIG. 1. Reverberations of a disturbance in (a) a three-dimensional body, (b) a two-dimensional disk, and (c) a one-dimensional bar.

another whereby a finite stress is introduced between them. A consequence of these properties is that the velocity of propagation of a disturbance from particle to particle has a finite value. In the case of a yielding body, therefore, the impact can be visualized in a manner that to the particle at the point of application of the force P a finite stress and velocity is imparted, which are transmitted to other adjacent particles with the velocity of propagation, the waves of disturbance radiating in all directions from the point of application of the force as illustrated in Fig. 1(a). As these disturbances arrive at the boundaries of the body the stresses are relieved and the velocities changed in accordance with the boundary conditions of the body, and these new disturbances radiate again in all directions from the points of arrival of the first waves. Thus the waves of disturbance bounce back and forth within the confines of the body. After several reverberations of these waves of stress and velocity, every particle will possess a stress and

velocity which are resultants of the several stress and velocity waves to which the particle has been exposed. Figure 1(*b*) illustrates this phenomenon in a flat, disk-shaped body in which the waves are propagated in two dimensions only, and Fig. 1(*c*) is a bar in which the waves are propagated in one dimension only. Thus, in the case of a yielding body a finite change of velocity is imparted, not to the entire mass, but only to an infinitesimal part of it, and the principle of momentum can be written:

$$\int_0^t P\,dt = v\int_0^m dm.$$

After the cessation of force the body will possess a mean state of stress and velocity, upon which, in general, is superimposed a state of fluctuating stress and velocity between the particles themselves. The energy represented by this superimposed vibration and that due to permanent deformation, is debited against the total energies of the bodies before the impact. For this reason the kinetic energy after the impact will be, in general, less than the energy content of the bodies before impact, even in the absence of damping.

In the dynamics of most bodies of engineering importance (for example, crank and connecting rod mechanisms, and other elements of machinery) the dimensions are small and the duration of the application of force is long relative to the duration of the reverberation of the disturbance within the body. Therefore the stress and velocity condition is rapidly equalized within the body, the phenomena occurring during this equalization are of no interest, the permanent deformation is nil, the superimposed vibrations represent a negligible amount of energy, and no serious error is incurred by assuming the body as rigid.

There are, however, cases in which this so to speak "macroscopic" view of the dynamic phenomenon is inadequate, and it is necessary to study the progress of the phenomena in "microscopic" detail. This is desirable, in particular, whenever the period of reverberations is not negligible in comparison with the duration of force application. This condition prevails in the case of large bodies as the Earth (earthquakes, natural and artificial), and also in engineering devices, such as long trains (starting and stopping), pump rods in oil well machinery (periodic forces and motions), helical springs in valve gears and in guns (surges), and in impact (hammers, projectiles), to mention some specific examples.

For the rational design of such engineering devices the clear understanding of these phenomena is highly desirable. However, experimental investigation is extremely difficult owing to the smallness of motions, and the shortness of time in which they occur. Recourse has to be taken, therefore, mainly to theoretical methods of attack.

In recent years growing recognition has been given to the theory of this group of problems, as evidenced by the literature on impact of

bars, which is probably the simplest species of this group, the conditions of which can be definitely stated, the disturbances in which are essentially one-dimensional and which is also open, to a fair extent, to experimental investigation as well.

Yielding materials are characterized by their stress-strain relationship, part of which, within the elastic range, is linear, beyond which in the plastic range the relationship is curvilinear.

Previous engineering literature on impact problems has treated mainly phenomena below the elastic limit of materials; that is, in the stress range for which the law of proportionality of stress and strain is valid, there is no permanent deformation and the velocity of propagation of a disturbance is constant. In an earlier paper (1942) DeJuhasz[2] has summarized the history of engineering treatments of elastic impact and extended the work of earlier investigators to some novel problems by means of a graphical method of analysis.

Until recently, little attention has been paid to impact phenomena occurring above the elastic limit, that is, in the plastic range, for while the proportionality of stress and strain is no longer valid, permanent deformation occurs, and the velocity of propagation of a disturbance is no longer constant but depends on the value of stress and strain. Probably the earliest work dealing with this field is Donnell's treatise (1930) in which a paragraph is devoted to a brief but basic and lucid exposition of plastic impact phenomena. A later discussion by Angus (1942) points out the need for a treatment of impact in the plastic range and refers to the analogy between this phenomenon and water hammer. Recently, however, an increasing need has been felt for a theory of plastic impact in order to provide a systematic basis for the understanding and interpretation of notched bar impact tests, performance of elements of machinery under dynamic loading, interaction between projectile and armor plate, and phenomena produced by other ordnance devices. Wartime stimulus resulted in a series of papers and official reports on this subject by von Kármán, White, Griffis, Duwez, Clark, Bohnenblust and associates, which contributed to the clearing up of these phenomena by experimental, mathematical, and to some extent by graphical methods. It is felt, however, that these theories and methods need further elucidation, extension and application to complex yet technically important phenomena.

The present paper is an attempt to clarify the theory of plastic impact and to extend its applicability by means of a graphical method of analysis which has been previously developed by the author and used for elastic impact, and which is herewith adapted also for plastic impact.

For the sake of completeness the essential ideas, concepts and procedures of the graphical analysis will be given here.

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[2] References are included in the bibliography appended to Part II of this paper.

GRAPHICAL ANALYSIS

Essential Concepts

It is proposed to investigate the problem of a long bar, having given material properties, to one end of which a velocity in the axial direction according to a given history law $v = f(t)$ is imparted; the mechanical state of the bar is to be determined for every cross section and for every instant of time.

The material properties are defined by the density, ρ, and the stress-strain characteristics, $p = f(\epsilon)$. A cross section is identified by its distance x from one reference cross section, say one end, of the bar. An instant of time is defined by the number of seconds t elapsed after

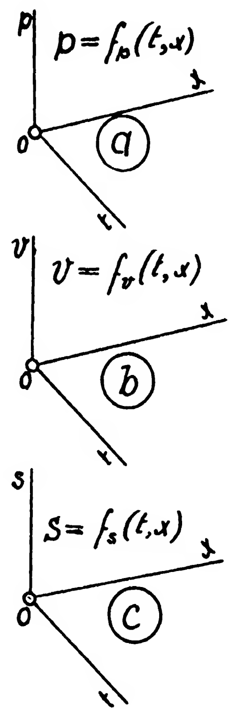

FIG. 2. Coordinate systems for stereograms of (a) stress, (b) velocity, and (c) displacement.

the beginning of time reckoning for which $t = 0$. The "mechanical state" is defined by the stress p, velocity v, strain ϵ, force P, displacement s existing in a cross section. Of these "coordinates of state" main attention will be given to the stress p and to the velocity v, because the other coordinates can be derived from these two as will be shown later. It will be assumed that the mechanical state of all particles in one cross section is the same at one given instant. The change of state of a cross section from an initial state p_1, v_1, to a new p_2, v_2 state will be called a "disturbance".

A dynamic phenomenon in a bar can be completely described by

giving the coordinates of state for every cross section, and for every instant of time. This can be accomplished mathematically by functions of the type:

$$p = f(t, x), \qquad v = f(t, x),$$
$$s = f(t, x), \qquad \epsilon = f(t, x), \qquad P = f(t, x),$$

expressing the coordinates of state as functions of two variables, t and x.

In geometrical terms such functions represent surfaces established within the framework of coordinate systems as shown in Fig. 2 in which the coordinates of state p, v and s are erected, as vertical ordinates, over the t–x plane as base. Such three-dimensional diagrams will be termed "stereograms", and in particular, stereogram of stress, of velocity, of strain, etc.

For technical thinking and imagination such stereograms provide a far clearer visualization of the interrelationships of three variables, than is possible to obtain from a mathematical formula. They even can be actually constructed of plaster or wood to represent a dynamic phenomenon with given data. On these surfaces then lines may be drawn, or the surfaces can be sectioned to define functions of *two* variables which may be of interest. Of these particular interest attaches to the:

(*a*) *history curves* which refer to a given cross section, having only the time, t, as independent variable, e.g.,

$$p = f(t)_x, \qquad v = f(t)_x, \qquad s = f(t)_x,$$

expressing the history of stress, of velocity and of displacement. These are the intersects of the stereograms with planes perpendicular to the x-axis, located at the given x-coordinate.

(*b*) *gradient curves* which refer to a given instant of time, having only the distance, x, as the independent variable, e.g.,

$$p = f(x)_t, \qquad v = f(x)_t, \qquad s = f(x)_t,$$

expressing the gradient of stress, of velocity, and of displacement. These are the intersects of the stereograms with planes perpendicular to the t-axis located at the given t-coordinate.

Construction of Stereograms

The stereograms can be constructed with the aid of two two-dimensional diagrams, namely:

(*A*) The t–x diagram which is termed the "location diagram", in which a point identifies an instant of time t, as reckoned from a reference instant (for example, from the beginning of impact), and a cross section of the bar, located at the distance x, as measured from a reference cross section (for example, from the impact end of the bar).

(*B*) The v–p diagram, termed the "diagram of state" in which a point identifies a "mechanical state", that is, co-existing values of

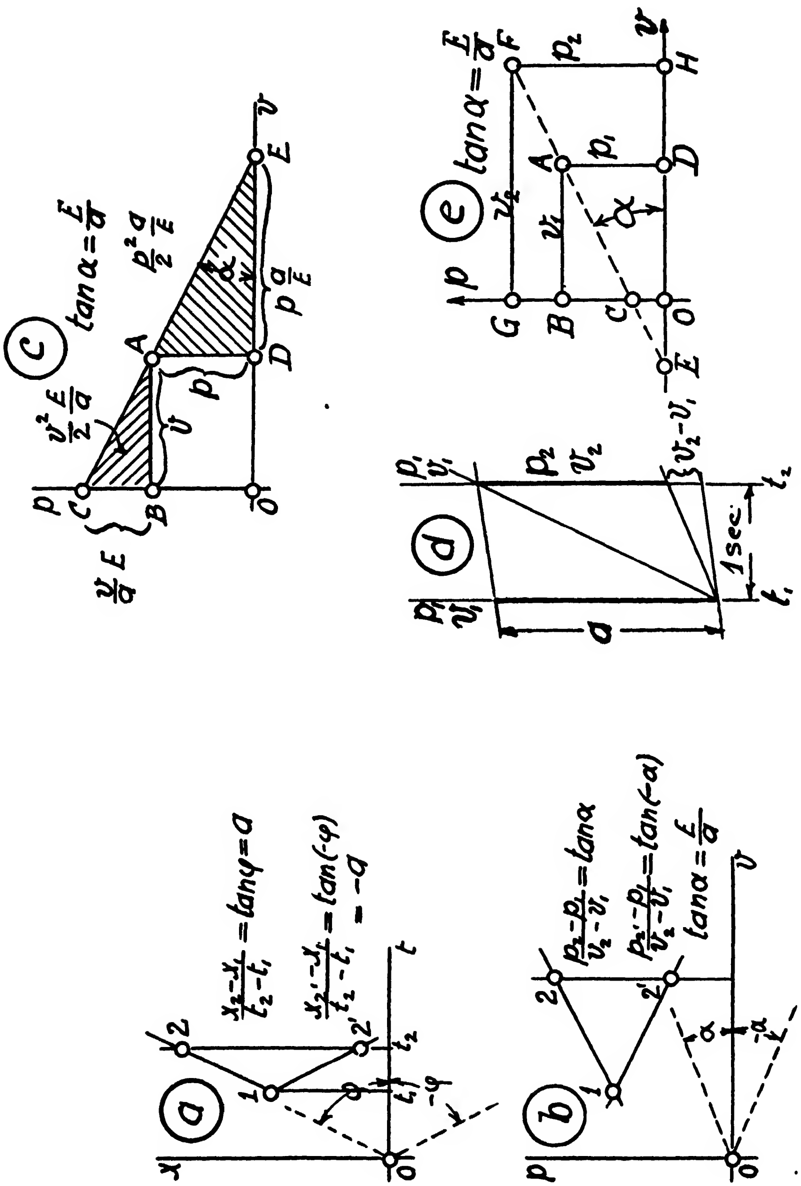

FIG. 3. Representation of an elastic disturbance.

(a) Disturbance lines 1–2 and 1–2′.
(b) Directrices 1–2 and 1–2′.
(c) Energy relations.
(d) Displacement of cross section during change of velocity
(e) Energy change through change of state.

velocity and stress. As stated before, we restrict our inquiry to cases in which the v and p values are the same for all particles in a given cross section, which is tantamount to saying that the cross sections remain plane and parallel to each other at all times.

Thus the construction of the stereograms resolves itself into assigning a point in the v–p diagram to every point in the t–x diagram. It will be seen that under certain conditions the same coordinates of state are valid not only for one *point* in the t–x diagram, but for an area or "domain" of it.

The construction of the t–x and v–p diagrams depends on the course of the phenomenon, which in turn depends on a number of data, namely:

(*a*) The design data of the bar, that is, its cross sectional area F, length L, and its boundary conditions, that is, whether the end is free, fixed, or in some manner restrained.
(*b*) The material characteristics of the bar, that is, its strength and stiffness properties, as defined by its stress-strain curve, and its density.
(*c*) The characteristics of disturbance, in this case that of impact, defined by the force, stress, applied on or by the velocity imparted to the impact end, and its duration.

There are two important relationships on which the graphical analysis is based, one referring to the t–x diagram, and the other to the v–p diagram. These differ slightly for the elastic and the plastic range, owing to the fact that within the elastic range the modulus of elasticity is constant, $E = p/\epsilon$, while for the plastic range it is variable, and is the function of stress, $E = dp/d\epsilon = f(p)$.

For the *elastic range* the first relationship is:

1. In an elastic bar a disturbance (that is, change of a pre-existing v_1, p_1 state to another v_2, p_2 state) is propagated along the bar with a constant velocity, which is the "acoustic velocity", a; written in mathematical form:

$$\frac{\Delta x}{\Delta t} = \text{const} = \pm a = \tan(\pm \varphi). \tag{1}$$

This relationship is represented in the t–x diagram by straight lines, termed "disturbance lines" having a constant slope $\tan(\pm \varphi)$ towards the t-axis, as shown in Fig. 3(*a*). The positive sign is valid for disturbances the location x of which increases with increasing time.

The acoustic velocity is interrelated with the modulus of elasticity E and the density of material ρ by the following equation:

$$a^2 = \frac{E}{\rho}. \tag{2}$$

(For example, for steel the acoustic velocity is about: a = 203,000 in sec.$^{-1}$).

The second relationship is:

2. In an elastic bar a change of velocity is accompanied by a proportional change of stress, and *vice versa;* written in mathematical form:

$$\frac{\Delta p}{\Delta v} = \text{const} = \pm \frac{E}{a} = \tan (\pm \alpha). \tag{3}$$

The value E/a is termed the "surge factor" and it can be expressed by the characteristics of the material:

$$\frac{E}{a} = \sqrt{E\rho} = \rho a. \tag{4}$$

(NOTE: The expression E/a is preferred, because then the numerators,

TABLE I.—*Characteristic Quantities of Materials in the Elastic Range.*

Characteristic \ Material		Steel	Copper Annealed	Aluminum 24 ST	Oak (Red) 80% Moist.
γ		7.8	9.0	2.72	0.57
w	lb. in.$^{-3}$	.282	.325	.098	.0206
	kg. m.$^{-3}$	7800	9000	2720	570
ρ	lb. in.$^{-4}$ sec.^{2}	.000729	.000841	.000254	.0000533
	kg. m.$^{-4}$ sec.^{2}	795	917	278	58.1
V	in.^{4} lb.$^{-1}$ sec.$^{-2}$	1370	1190	3940	18,750
	m.^{4} kg.$^{-1}$ sec.$^{-2}$	.00126	.00109	.00360	.001725
E	lb. in.$^{-2}$	30×10^{6}	14×10^{6}	10×10^{6}	1.3×10^{6}
	kg. m.$^{-2}$	2.11×10^{10}	$.985 \times 10^{10}$	$.704 \times 10^{10}$	$.0915 \times 10^{10}$
a	ft. sec.$^{-1}$	16,900	10,750	16,600	13,000
	in. sec.$^{-1}$	203×10^{3}	129×10^{3}	199×10^{3}	156×10^{3}
	m. sec.$^{-1}$	5150	3270	5050	3960
$\frac{E}{a}$	$\frac{\text{lb. in.}^{-2}}{\text{in. sec.}^{-1}}$	148	109	50.3	83.5
	$\frac{\text{kg. m.}^{-2}}{\text{m. sec.}^{-1}}$	4.10×10^{6}	3.01×10^{6}	1.40×10^{6}	2.31×10^{6}
El. Limit	lb. in.$^{-2}$	30–260,000	~ 3500	$\sim 40,000$	$\geqq 800$
	kg. m.$^{-2}$	$2\text{–}180 \times 10^{6}$	$\sim 2.5 \times 10^{6}$	$\sim 28 \times 10^{6}$	$\geqq .56 \times 10^{6}$

and the denominators, respectively, of both sides of Eq. 3 will have the same dimension.)

This relationship is represented in the v–p diagram by straight lines, termed "directrices" having a constant slope tan ($\pm\ \alpha$) toward the v-axis, Fig. 3(b). The positive sign is valid for disturbances in which the velocity and the stress both increase or both decrease.

(For example, for steel the surge constant is 148 lb. in.$^{-3}$ sec., which means that a velocity change by 1 in. sec.$^{-1}$ imposed on a steel bar is accompanied by 148 lb. in.$^{-2}$ change of the stress.)

The significant strength properties of several materials are given in Table I.

The validity of these relationships (1) and (3) is restricted to such stress values as lie below the elastic limit of the material (for steel about 30,000 lb. in.$^{-2}$, for annealed copper about 3500 lb. in.$^{-2}$) and to such values of velocity which are far below the acoustic velocity, as it will be explained and derived later.

Energy in the Elastic Bar

Areas $v \cdot p$ in the v–p diagram have the dimension of lb. in.$^{-1}$ sec.$^{-1}$, that is, that of work absorbed by or liberated by a bar of unit cross-sectional area during 1 sec. Because during 1 sec. a length a is affected by a disturbance, therefore it is also correct to say that an area represents the work absorbed by or liberated by a bar of unit cross-sectional area having a length a. Triangular areas, defined by a directrix drawn from a v_1, p_1 state (point A, Fig. 3(c)) to its intersection with the v and p-axes represent the potential, and kinetic energy, respectively, contained in a bar of unit cross-sectional area and a length a, in virtue of its stress p and velocity v. Thus, in Fig. 3(c), the potential energy of the bar of unit cross-sectional area and of length a having a stress p (point A) is represented by the hatched triangle $\triangle\ ADE$:

$$H_p = \frac{p^2}{2}\frac{a}{E} = \triangle\ ADE$$

because: $p/E = \epsilon$ = strain = change of unit length; $a\ \epsilon$ = change (displacement) of length a; therefore $H_p = (p/2)\ a\ \epsilon$ = potential energy. Likewise, the kinetic energy is represented by the hatched triangle $\triangle\ ABC$:

$$H_v = \frac{v^2}{2}\frac{E}{a} = \triangle\ ABC,$$

because $E/a = a\rho$ = the mass of a bar of length a of unit cross-section. For a bar of length L, of unit cross section, the corresponding energy values will be L/a-times the hatched areas:

$$U_p = \frac{L}{a}\ \triangle\ ADE, \qquad U_v = \frac{L}{a}\ \triangle\ ABC,$$

L/a being the interval of time necessary for a disturbance to traverse a bar length L, from one end to the other.

These properties of the v–p diagram are useful for determining the energy content of a bar, by multiplying the areas of the respective

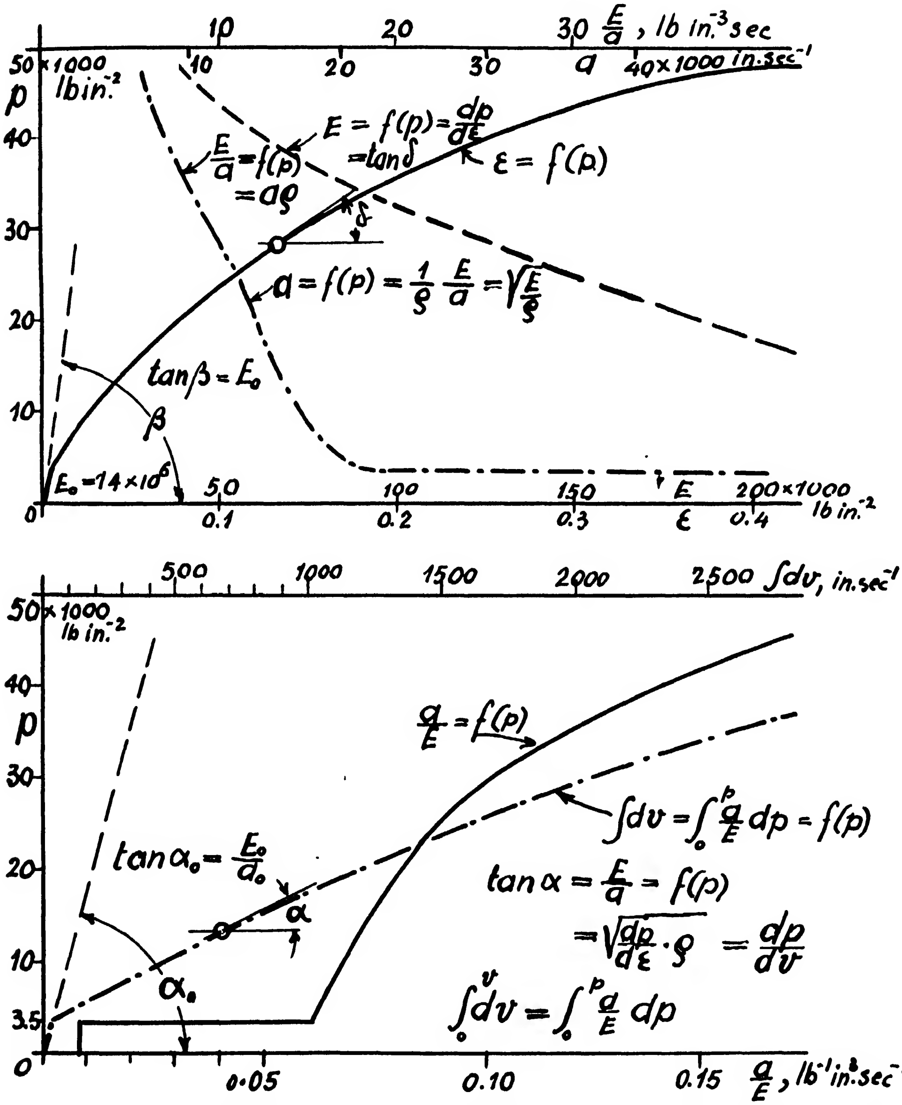

FIG. 4. Derivation of significant elastic and plastic relationships from the stress-strain curve (for annealed copper, according to White and Griffis, ref. 34).

triangles with the cross-sectional areas of the bar, and the length portions characterized by the given v, p coordinates.

For the *plastic range* the Eqs. 1 and 2 have to be written in differential form:

$$\frac{dx}{dt} = \pm a = \tan(\pm \varphi) = f(p); \tag{5}$$

where

$$a^2 = \frac{1}{\rho}\frac{dp}{d\epsilon}, \tag{6}$$

and

$$\frac{dp}{dv} = \pm\frac{E}{a} = \tan(\pm\alpha) = f(p), \tag{7}$$

where

$$\frac{E}{a} = \sqrt{\rho\frac{dp}{d\epsilon}}. \tag{8}$$

In this plastic range neither the velocity of propagation a, nor the surge factor E/a are constants, but both are functions of the stress. The values for any given stress can be determined from the stress-strain curve of the material, either by computation, or by graphical construction, and represented as functions of p. (The interrelationship of these quantities with the stress-strain function is shown in Fig. 4. If the stress-strain function, curve $\epsilon = f(p)$ and the density ρ of material is given, then its slope is the $E = dp/d\epsilon = f(p)$ curve, from which the a, E/a, and a/E functions can be determined. Forming the integral: $\int_a^p (a/E)dp = f(p)$ the $\int dv$ curve can be found, having the property that its slope is E/a for any given value of the variable p. All these operations can be performed either by computation or by known graphical constructions.)

Derivation of the Basic Equations

The derivation of Eqs. 1, 3, 5 and 7 is based on the fundamental principles of conservation of matter, of momentum and energy.

Assume an elastic bar, Fig. 3(d), of modulus of elasticity E, density ρ, of unit cross-sectional area, initially having v_1 velocity and p_1 stress in its every cross section. At the instant t_1, the velocity is changed to v_2 at the point A. Thereby, the stress is changed to p_2 (the value of which is as yet unknown). The new state v_2, p_2 represents a disturbance which is propagated with a velocity a (the value of which is yet unknown). After 1 sec. has elapsed, a length a of the bar will have assumed the new state v_2, p_2. It is our purpose to determine p_2 and a.

After the elapse of 1 sec., the initial length a of the bar is changed to: $a - (v_2 - v_1)$ and it can be written:

$$\frac{a - (v_2 - v_1)}{a} = \frac{E - p_2}{E - p_1}.$$

Assuming an elastic change, in which the values of p_2 and p_1 are very small in comparison with E, it can be written:

$$1 - \frac{v_2 - v_1}{a} = 1 - \frac{p_2 - p_1}{E};$$

denoting $p_2 - p_1 = \Delta p$ and $v_2 - v_1 = \Delta v$ it can be written:

$$\frac{\Delta p}{\Delta v} = \frac{E}{a},$$

which is identical with Eq. 3. From the postulatum of conservation of momentum, it can be written:

$$(p_2 - p_1)\Delta t = m(v_2 - v_1).$$

Substituting:

$$\Delta t = 1 \text{ sec.}; \qquad m = a\rho; \qquad p_2 - p_1 = \Delta p; \qquad v_2 - v_1 = \Delta v,$$

we obtain:

$$\frac{\Delta p}{\Delta v} = a\rho,$$

which combined with Eq. 3 yields:

$$a^2 = \frac{E}{\rho},$$

giving the velocity of propagation in terms of the properties of the material.

It is to be shown that these relationships satisfy the postulatum of conservation of energy.

For a length of bar a having coordinates of state v, p the energies can be expressed:

$$\text{Kinetic energy} \quad H_v = \frac{mv^2}{2} = a\rho\frac{v^2}{2} = \frac{E}{a}\frac{v^2}{2},$$

$$\text{Potential energy} \quad H_p = \frac{ap}{E}\frac{p}{2} = \frac{a}{E}\frac{p^2}{2},$$

that is, expansion of bar from p stress to zero stress, by a travel: $a(p/E)$. If the initial state is $p = 0$, $v = 0$, then: $p = (E/a)v$ and the kinetic energy is equal to the potential energy.

When the state of the bar is changed from state v_1, p_1, to state v_2, p_2 the work done per second (on length a because during 1 sec. a length a of the bar is affected) is p_2v_2. Part of this work is expended in moving the bar against p_1 stress with v_1 velocity, that is, p_1v_1. The balance appears as the change of Kinetic plus potential energies in the bar. It can be written:

$$p_2v_2 - p_1v_1 = \frac{1}{2}\frac{E}{a}(v_2^2 - v_1^2) + \frac{1}{2}\frac{a}{E}(p_2^2 - p_1^2)$$

$$= \frac{1}{2}\frac{E}{a}(v_2 - v_1)(v_2 + v_1) + \frac{1}{2}\frac{a}{E}(p_2 - p_1)(p_2 + p_1).$$

Substituting:

$$p_2 - p_1 = \frac{E}{a}(v_2 - v_1),$$

we obtain:

$$p_2v_2 - p_1v_1 = \tfrac{1}{2}(p_2 - p_1)(v_2 + v_1) + \tfrac{1}{2}(v_2 - v_1)(p_2 + p_1),$$

which, expanded, yields to an identity, q.e.d.

In the v–p diagram for a length of bar a, the kinetic energy is represented, for v_1p_1 state (point A, Fig. 3(e)) by the triangle $\triangle ABC$; and the potential energy by the triangle $\triangle ADE$. Similarly, for the state, point F, the respective triangles are $\triangle FGC$ and $\triangle FHE$. It is readily seen that the difference $p_2v_2 - p_1v_1$ is $FGOH - ABOD = ABGFHD$.

The above derived basic relationships and formulas have been developed by several authors by means of calculus. But the elementary derivation used here is just as rigorous, and it is believed to be easier to follow and visualize.

The previously used mathematical methods are based on the same principles and formulas and therefore the results of mathematical analysis are bound to be the same as those of the graphical analysis, within the limits of computational and constructional errors. In contrast with mathematical procedures the utilization of these formulas for graphical construction of the t–x and v–p diagrams endows the graphical method with a greater clarity of visualization and with wider applicability to complicated cases, as will be illustrated by the examples to follow.

EXAMPLES

General Remarks

Based on the previously given derivations the graphical method will be applied to numerous impact problems: first, involving stresses in the elastic range; second, involving stresses in the elastic and plastic range. Summarizing the previously obtained results the procedure will be:

1. To set up two coordinate systems, the t–x diagram and the v–p diagram.
2. To determine, from the given data of the material, the velocity of propagation $a = \tan\varphi$ and the wave factor $E/a = \tan\alpha$, and to establish in the t–x diagram the direction of disturbance lines, $\tan(\pm\varphi)$, and in the v–p diagram the directrices $\tan(\pm\alpha)$.
3. From the design data of the problem, that is, the dimensions of the bar, the applied impact stress or velocity, to follow through the phenomena from instant to instant and from point to point in the bar, with the aid of the disturbance lines and directrices, and to determine thereby the points or domains in the t–x diagram, for which a given state in the v–p diagram holds good.

4. To erect the v and p coordinates, as ordinates, over the plane as base, and thereby to obtain the stereograms of velocity and stress; to obtain from the stereogram of velocity, by integrating according to time, the stereogram of displacement.

5. To determine the impact energy, and the energy content of the bar for various portions of the bar and from instant to instant of time.

The examples will be divided into two groups, those below and those above the elastic limit.

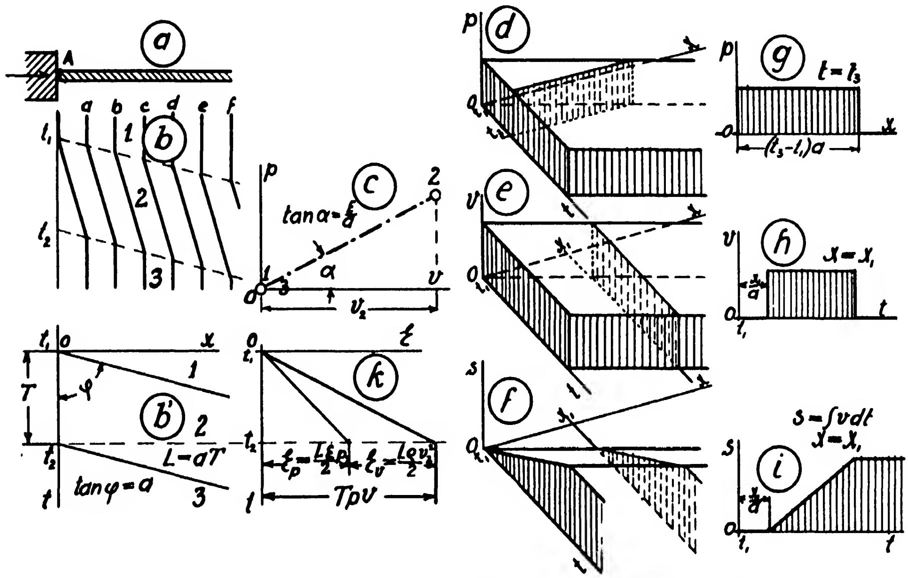

FIG. 5. Elastic impact in compression, bar of infinite length.

(a) Schematic arrangement.
(b) Displacement of cross sections.
(b') Location diagram (t-x diagram).
(c) Diagram of state (v-p diagram).
(d) Stereogram of stress.
(e) Stereogram of velocity.
(f) Stereogram of displacement.
(g) Gradient diagram of stress, taken at instant t_3.
(h) History diagram of velocity, taken at cross section x_1
(i) History diagram of displacement of cross section x_1.
(k) History diagram of energy relations.

I. IMPACT IN THE STRESS RANGE BELOW THE ELASTIC LIMIT

In this range the E, a, and E/a quantities are constant, and therefore the tan φ disturbance lines and the tan α directrices are represented by straight lines of constant slope. The procedures of graphical analysis are, therefore, simple as will be shown in the examples that follow.

Problem 1. Elastic Impact in Compression, Bar of Infinite Length (Fig. 5)

Referring to Fig. 5(a) we shall consider a bar of unit cross-sectional area one end of which is subject to impact, and which extends in the

other direction to infinite length, of a material of elastic modulus E, acoustic velocity a, and surge factor E/a. The bar is supported without friction in such a manner that it can move only longitudinally in the direction of its axis. Initially it has zero velocity and zero stress, that is, its coordinates of state are: $p_1 = 0$; $v_1 = 0$. (Positive sign is assigned to velocity towards the right, and to compressional stress.) At $t = 0$ instant a comparatively large mass, having v_2 velocity, comes into contact with the end of the bar to which it imparts the same v_2 velocity, for a time duration T. This disturbance is characterized by the velocity of the impact end of the bar changing from $v_1 = 0$ to $v = v_2$; this change is accompanied by a proportional change of stress from $p_1 = 0$ to $p = p_2$. It is assumed that the v_2 is small enough for p_2 not to exceed the elastic limit; therefore it can be written (from Eq. 1):

$$\frac{p_2 - p_1}{v_2 - v_1} = \frac{p_2}{v_2} = \frac{\Delta p}{\Delta v} = \frac{E}{a} = \tan\alpha.$$

This disturbance is represented in the v–p diagram, Fig. 5(c) by a tan α directrix drawn from point 1 defining the initial state p_1, v_1, to point 2 at the intersection of the directrix tan α with the v_2 = const time, defining the state of the bar affected by the impact. This disturbance travels away from the point of impact with the acoustic velocity a for which it can be written (from Eq. 3):

$$\frac{x}{t} = \frac{\Delta x}{\Delta t} = a = \tan\varphi.$$

This is represented in the t–x diagram, Fig. 5(b'), by a tan φ slope. At the cessation of the impact at $t = T$ instant (according to our initial assumptions), the velocity of the impact end of the bar is reduced to zero, $v_3 = 0$, whereby also the stress is reduced to zero:

$$v_3 = v_1 = 0, \qquad p_3 = p_1 = 0.$$

Figure 5(b) shows the initial cross sections $a, b, c, d \cdots$ as displaced by the impact, the particle velocity of the cross section (v_2 = slope of the heavily drawn lines) and the velocity of propagation (dash-lines). The heavily drawn lines are closer together (measured in the x-direction) in the portion of the bar affected by the impact, indicating the presence of compressive strain, hence compressive stress. Figure 5(b) shows impressively the nature and progress of the impact phenomenon, but it is exaggerated and is not in true scale inasmuch as actually the particle velocity is far smaller than the acoustic velocity. (In the case of steel the acoustic velocity is a = 203,000 in. sec.$^{-1}$, while the particle velocity corresponding to the elastic limit of 30,000 lb. in.$^{-2}$ is only about 200 in. sec.$^{-1}$, that is, 1/1000th part of the acoustic velocity.) Therefore the location diagram Fig. 5(b') is closer to the truth, giving

the boundaries of the domains in which state 1, state 2, and state 3 (as defined by the v- and p-coordinates of points 1, 2, and 3 of the diagram of state, Fig. 5(c)) exist.

The stereogram of stress, Fig. 5(d), and of velocity, Fig. 5(e), are constructed by erecting the v- and p-ordinates of points 1, 2, and 3 taken from the v–p diagram, Fig. 5(c), over the respective domains 1, 2, and 3 of the location diagram, Fig. 5(b'). The intersect of the stress stereogram, Fig. 5(d), with a plane perpendicular to the t-axis at t_3 instant is the gradient of stress, Fig. 5(g), giving the stress of the entire bar at the t_3 instant. The intersect of the velocity stereogram, Fig. 5(e), with a plane perpendicular to the x-axis at the x_1 cross section is the velocity history, Fig. 5(h), giving the velocity for the cross section for all instants of time. Forming the integral of the history curve: $s = \int_{x_1/a}^{t} v\,dt$ gives the history of displacement, Fig. 5(i), for the x_1 cross section; carrying out this integration for all x-values yields the stereogram of displacement, Fig. 5(f). This is obtainable also by integrating the strain gradient curves according to distance: $s = \int (P/E)dx$ for all t-values.

The stereograms of stress, velocity and displacement give full information on the state of the bar for all cross sections and for all instants of time. A stereogram of force F could be constructed, by multiplying the p-values by the cross-sectional area of the bar; but this would be identical with the stress stereogram in another ordinate scale. Likewise, a stereogram of strain would be identical with the stress stereogram, at an $1/E$-times the ordinate scale.

It can be visualized from these stereograms, that to length $L = Ta$ of the bar velocity v_2 and stress p_2 is imparted by the impact; this wave of length L is propagated along the bar with the acoustic velocity a; when the front of the wave passes over a cross section it changes the state of the cross section from v_1, p_1 to v_2, p_2; after the rear of the wave passes over a cross section, it changes the state of the cross section from v_2, p_2 to v_3, p_3 leaving it again at rest, and free of stress. In the process the cross section had advanced by the displacement $s_2 = Tv_2$.

The history of energy relations is represented in Fig. 5(k); energy is being fed into the bar at a rate of p_2v_2, until the termination of the impact period T; thus the total energy input is: $\mathcal{E}_T = Tp_2v_2$. One half of the energy appears as compression energy: $\mathcal{E}_p = (L\epsilon_2 p_2/2)$, where $\epsilon_2 = p_2/E$; therefore:

$$\mathcal{E}_p = L\frac{p_2}{2E} = \frac{Tap_2^2}{2E} = \frac{1}{2}Tp_2v_2.$$

The other half appears as kinetic energy:

$$\mathcal{E}_v = \frac{L\rho v_2^2}{2}, \qquad \text{where} \qquad \rho = \frac{E}{a^2}; \qquad \frac{L}{a} = T; \qquad \text{and} \qquad \frac{E}{a}v_2 = p_2;$$

hence

$$\mathcal{E}_v = \tfrac{1}{2}Tp_2v_2.$$

Problem 2. Elastic Impact in Tension, Bar of Infinite Length (Fig. 6)

The assumptions are similar to those in the previous example, Fig. 5, with the exception that the stress is tensional, that is, negative, and the impact velocity is to the left, that is, also negative. Therefore the

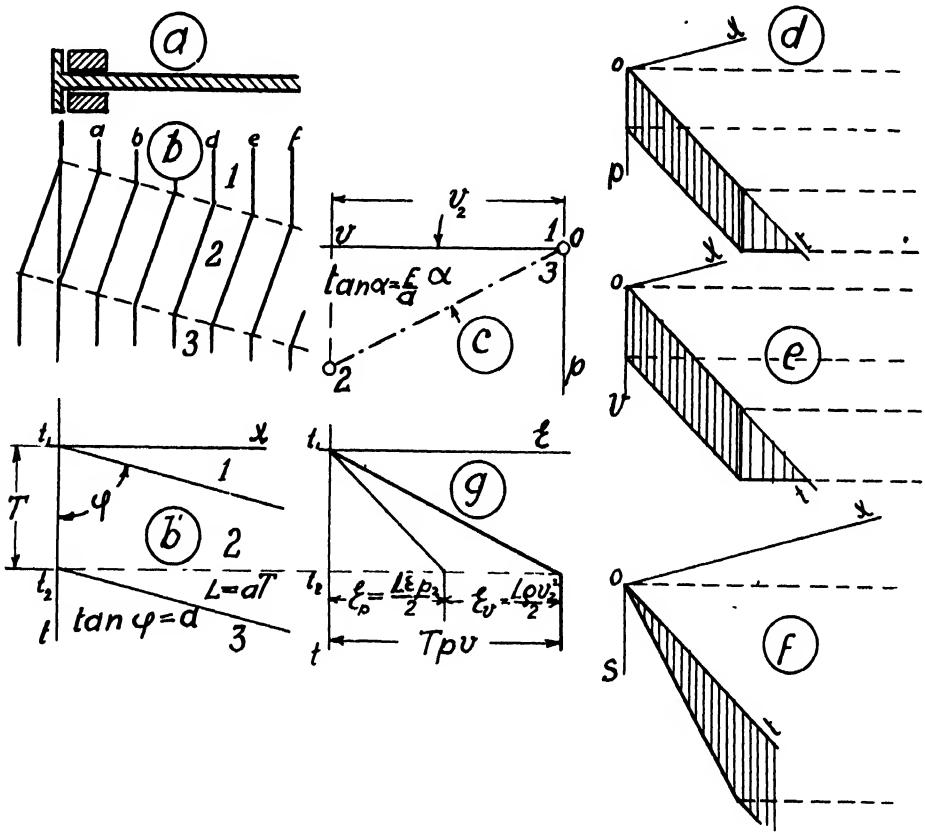

FIG. 6. Elastic impact in tension, bar of infinite length.

(*a*) Schematic arrangement.
(*b*) Displacement of cross sections.
(*b'*) Location diagram.
(*c*) Diagram of state.
(*d*) Stereogram of stress.
(*e*) Stereogram of velocity.
(*f*) Stereogram of displacement.
(*g*) History diagram of energy relations.

point 2 in the diagram of state, Fig. 6(*c*) will be situated in the lower left quadrant, instead of the upper right quadrant of the diagram. In the exaggerated displacement diagram, Fig. 6(*b*), it is visible that the distance between the neighboring cross sections *a*, *b*, *c*, *d* · · · is increased (measured in the *x* direction) in the domain 2 affected by the impact, indicating the presence of tensional strain, hence tensional stress. The stereograms of stress, velocity and displacement, Fig. 6(*d*), (*e*) and (*f*)

are also similar to the corresponding ones in the previous example, with the difference that the ordinates are directed downwards, in accordance with the negative sign of the stress, velocity and displacement. The energy diagram, Fig. 6(*g*), is of identical form as for compressive impact, because both p and v being negative their product is positive.

Problem 3. Elastic Impact in Compression at Variable Velocity, Bar of Infinite Length (Fig. 7)

A compressive impact is assumed, at variable velocity, the history of impact velocity being of triangular shape as shown in Fig. 7(*b'*);

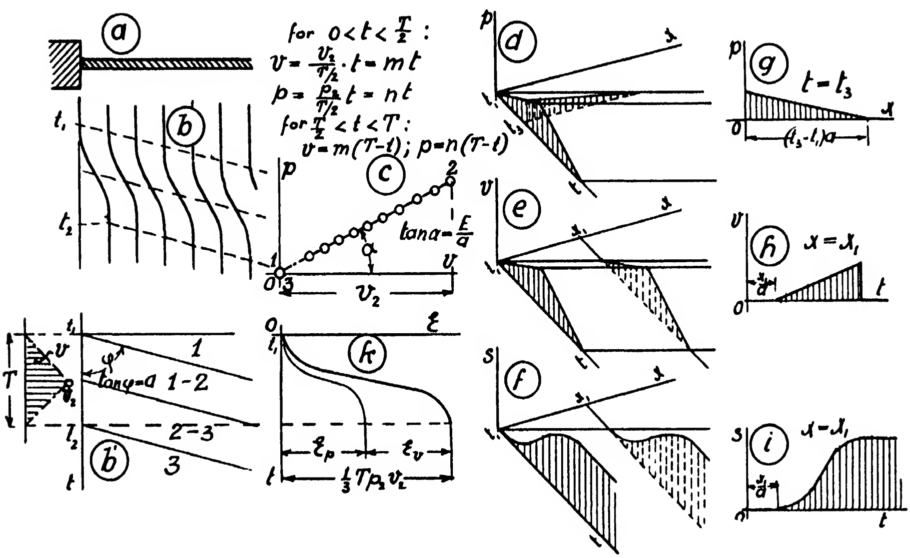

FIG. 7. Elastic impact in compression at variable velocity, bar of infinite length.

(*a*) Schematic arrangement.
(*b*) Displacement of cross sections
(*b'*) Location diagram.
(*c*) Diagram of state.
(*d*) Stereogram of stress.
(*e*) Stereogram of velocity.
(*f*) Stereogram of displacement.
(*g*) Gradient diagram of stress, taken at instant t_3.
(*h*) History diagram of velocity, taken at cross sections x_1.
(*i*) History diagram of displacement of cross section x_1.
(*k*) History diagram of energy relations.

according to the v–p diagram Fig. 7(*c*), also the impact stress has a triangular history diagram; mathematically stated:

for $0 < t < (T/2)$:

$$v = \frac{v_2}{T/2}t = mt; \qquad p = \frac{p_2}{T/2}t = \frac{E}{a}mt = nt$$

and for $T/2 < t < T$:

$$v = m(T - t); \qquad p = n(T - t).$$

The stereograms of stress and of velocity, Figs. 7(*d*) and (*e*), as well as the gradient of stress, Fig. 7(*g*), and the history of velocity, Fig. 7(*h*), are constructed in the manner previously explained and do not need further elucidation. The displacement history diagram, Fig. 7(*i*), is composed of two parabolas with common tangent:
for $0 < t < (T/2)$:

$$s_0 = \int_0^t vdt = \frac{2v_2}{T}\int tdt = \frac{2v_2t^2}{T2} = \frac{v_2}{T}t^2,$$

for $t = (T/2)$:

$$s_{T/2} = \frac{v_2T}{4},$$

and for $t = T$:

$$s_T = \frac{v_2T}{2}.$$

From these displacement curves the stereogram of displacement, Fig. 7(*f*), can be constructed.

The energy relationships are given in Fig. 7(*k*). The history curves of energy input, $\int_0^t pvdt$, as well as its distribution between potential energy: $\int_0^x (p_2/2E)dx$, and kinetic energy: $\int_0^x (\rho v^2/2)dx$ are represented by cubic parabolas. For $0 < t < (T/2)$ the energy input:

$$\int_0^t pvdt = \frac{2p_2}{T}\frac{2v_2}{T}\int_0^t t^2dt = \frac{4p_2v_2}{T^2}\frac{t^3}{3}$$

the value of which at the $T/2$ instant $= \frac{1}{6}p_2v_2T$.

For the total impact period, T, the energy input is $\frac{1}{3}p_2v_2T$. The energy distribution between potential and kinetic can be calculated in a similar manner.

Problem 4. Elastic Impact in Compression, Bar of Finite Length, with Free End, Duration of Impact Equal 2L/a (Fig. 8)

It is assumed that the bar length is finite, L, and that the duration of impact is $T = 2L/a$. It is seen in Fig. 8(*b*) and (*c*), that at the middle of the impact period, that is, at the $t_1 + L/a$ instant the entire length L of the bar is in a state 2, that is, has p_2 stress, and v_2 velocity. The far end of the bar, being free and unrestrained, cannot remain under stress; therefore the stress is reduced to zero (negative directrix drawn from point 2 to point 3, the intersection with the $p = 0$ line, that is, the abscissa axis) whereby the velocity is doubled: $v_3 = 2v_2$. This disturbance wave (change of state 2 to state 3) is reflected towards the impact end of the bar, where it arrives at the instant at which according to our assumption the impact ends. Thus, at the $t_1 + T$ instant the final state of the entire bar will be state 3, that is, $v_3 = 2v_2$; $p_3 = 0$.

The stereograms of stress, of velocity and of displacement are given in Fig. 8(*d*), (*e*), and (*f*).

The history of energy relations is given in Fig. 8(*g*). It is seen that until the middle of the impact period, $t_1 + L/a$, one half of the

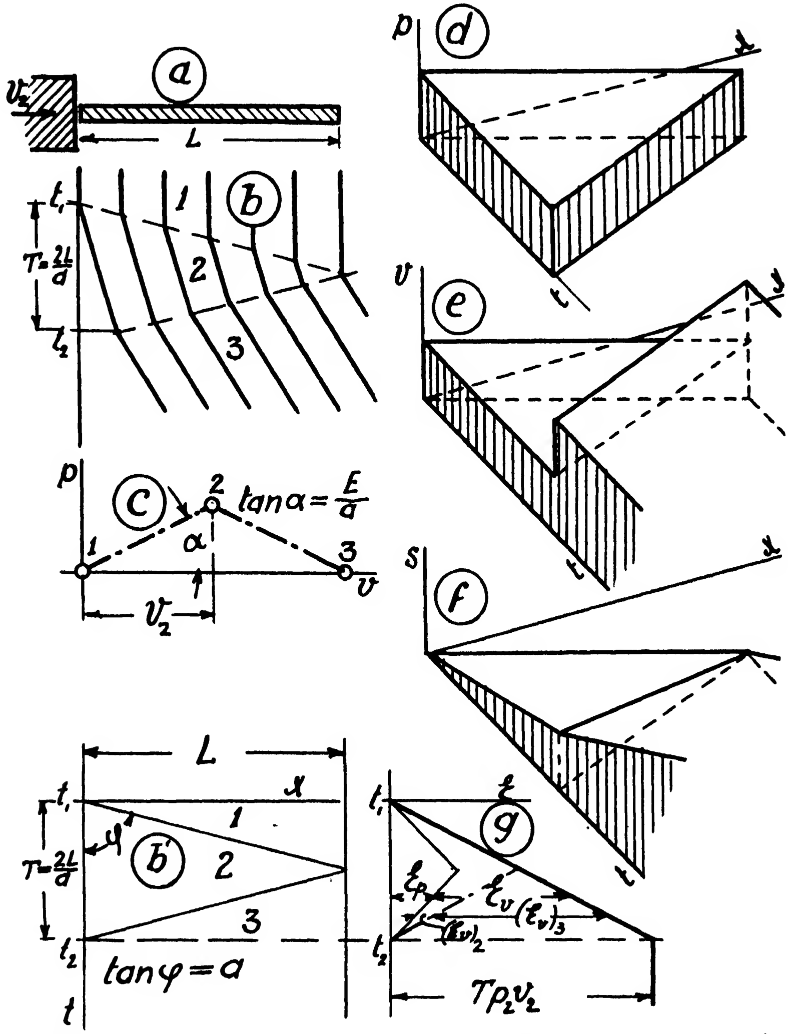

FIG. 8. Elastic impact in compression, bar of finite length, with free end; duration of impact $2L/a$.

(*a*) Schematic arrangement.
(*b*) Displacement of cross sections.
(*b'*) Location diagram.
(*c*) Diagram of state.
(*d*) Stereogram of stress.
(*e*) Stereogram of velocity.
(*f*) Stereogram of displacement
(*g*) History of energy relations.

input energy appears as potential energy (stress energy), the other one half of the input energy appears as kinetic energy (velocity energy). Subsequently the stress energy diminishes, and entirely disappears at

the end of the impact period, $t_1 + T$, when the entire input energy appears as velocity energy of the bar. Input energy: $\mathcal{E} = Tp_2v_2$.

Final kinetic energy

$$\mathcal{E}_{v,f} = \frac{L\rho v_3^2}{2},$$

with

$$\frac{L}{a} = \frac{T}{2}; \qquad \rho a = \frac{E}{a}; \qquad v_3 = 2v_2; \qquad v_2 \frac{E}{a} = p_2;$$

it can be written:

$$\mathcal{E}_{v,f} = \frac{L}{a}\frac{a\rho}{2} 4v_2^2 = \frac{T}{2}\frac{4}{2} p_2v_2$$
$$= Tp_2v_2.$$

Problem 5. Elastic Impact in Compression, Bar of Finite Length, with Free End, Duration of Impact is Not Equal $2L/a$ (Fig. 9)

Figure 8 depicted a special condition, in so far as the duration of input was closely related to the length of the bar: $T = 2L/a$. In Fig. 9 the more general case will be treated, in which this equality does not hold, but $(L/a) < T < (2L/a)$. The phenomenon is identical with that of the previous example, until the end of the impact period T, at which instant the impact ceases and the end of the bar comes to rest (that is, state 2 changes to 3). At the far end of the bar, which is free and unrestrained, the stress 2 is released and the bar assumes the state 3′. The two disturbance waves 3 and 3′ travel towards each other and meet at a point in the bar and at an instant of time which is determinable from the location diagram, Fig. 9(b'). The resulting new state will be defined by point 4 in the v–p diagram, Fig. 9(c), this point being the intersection of the positive directrix, tan $(+\alpha)$, drawn from point 3′ and the negative directrix, tan $(-\alpha)$, drawn from point 3. When the disturbance wave 4 arrives at the two ends of the bar the state 4 is changed into state 5 and 5′, respectively, both being located at the $p = 0$ line, that is, the abscissa axis (because at a free end the stress is zero). Thus the states change, each state defined by a number in the v–p diagram being valid for a domain of the t–x diagram, which is marked by the same number. The resulting stereograms are given in Fig. 9(d), (e), (f). It is seen that the bar does not attain one state over its total length, but, superimposed on its velocity imparted by the impact, it undergoes also variations of velocity and stress. The interchange of energies, kinetic and potential, is even more clearly shown in the energy diagram, Fig. 9(g). Until the termination of impact, that is, the end of the T period, the energy history is the same as in the previous example. From there on, there is a continuous interchange

between the energy of the middle portion of the bar (valid for the domains 4, 6, etc., of the t–x diagram) which is divided equally between

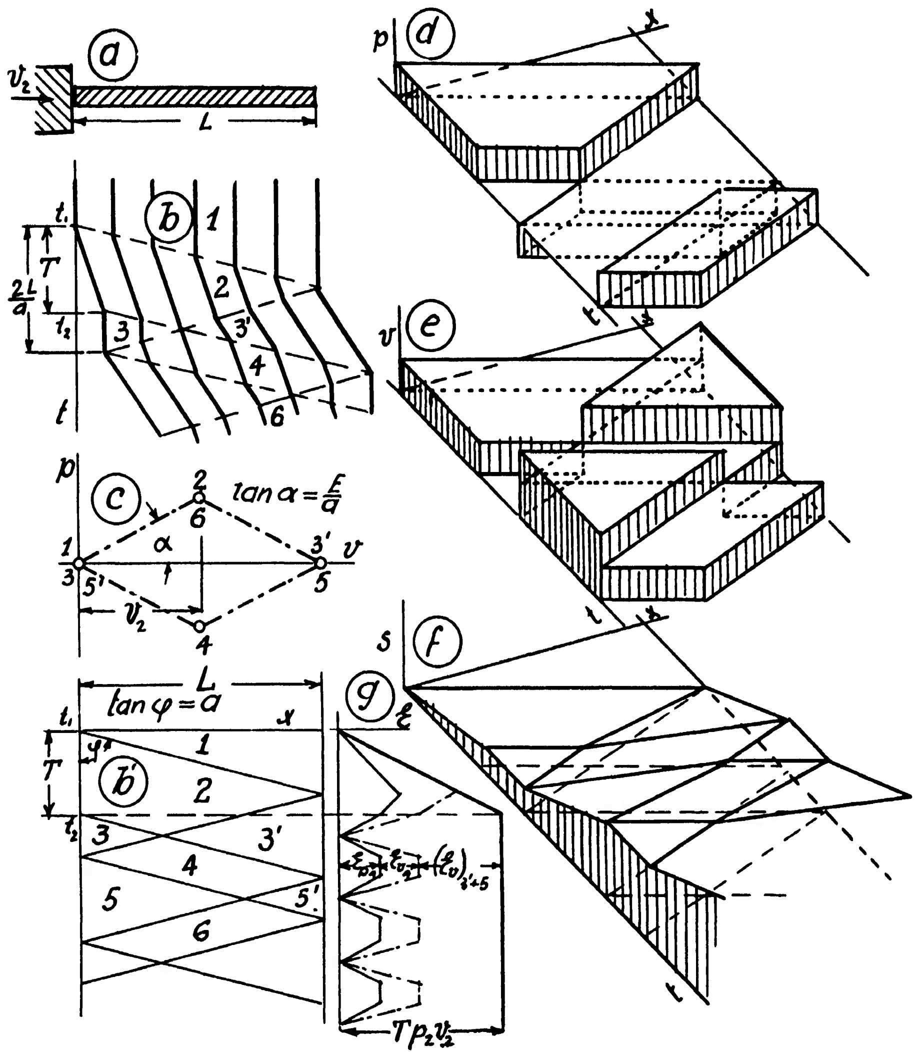

FIG. 9. Elastic impact in compression, bar of finite length, with free end; duration of impact less than $2L/a$.

(*a*) Schematic diagram.
(*b*) Displacement of cross section.
(*b'*) Location diagram.
(*c*) Diagram of state.
(*d*) Stereogram of stress.
(*e*) Stereogram of velocity.
(*f*) Stereogram of displacement.
(*g*) History of energy relations.

kinetic and potential energy, and the energy of the end portions of the bar (domains 3, 3', 5, 5', etc.) which is of purely kinetic form (the stress being zero).

Problem 6. Elastic Impact in Compression, Bar of Finite Length, with Fixed End, Duration of Impact Equal 2L/a (Fig. 10)

It is assumed that the motion of the far end of the bar is prevented of motion by means of a stationary infinite mass; furthermore, it is

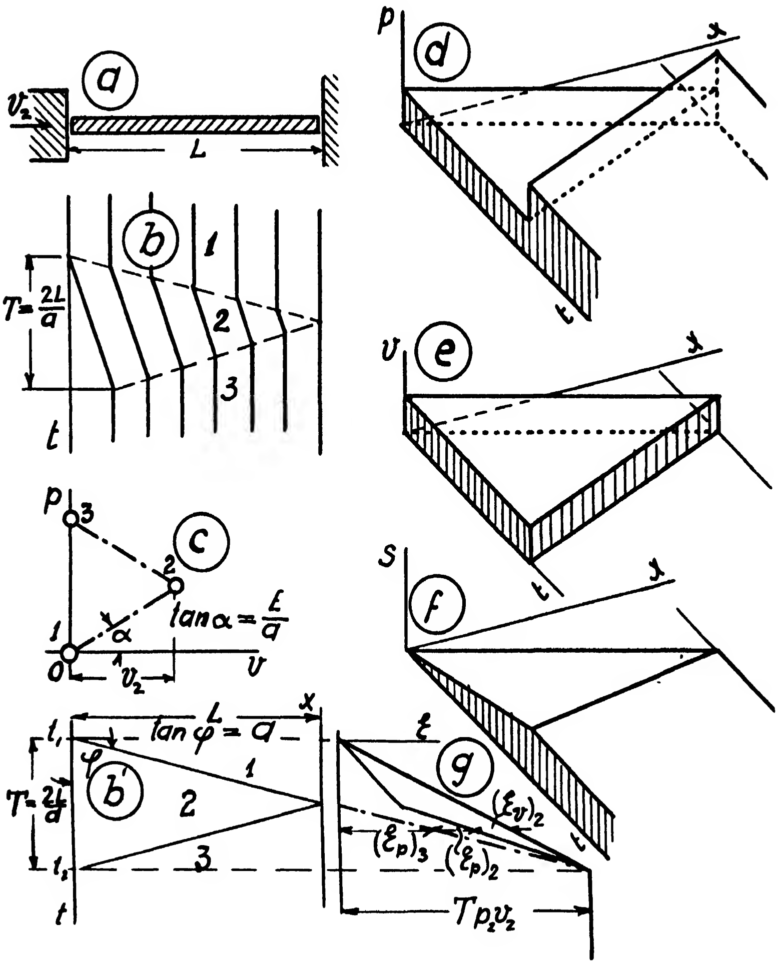

FIG. 10. Elastic impact in compression, bar of finite length, with fixed end; duration of impact $2L/a$.

(a) Schematic arrangement.
(b) Displacement of cross sections.
(b') Location diagram.
(c) Diagram of state.
(d) Stereogram of stress.
(e) Stereogram of velocity.
(f) Stereogram of displacement.
(g) History of energy relations.

assumed that the duration of impact is $T = 2L/a$. It is seen in Fig. 10(*b*) and (*c*) that at the middle of the impact period, that is, at the $t_1 + L/a$ instant the entire length of the bar, L, is in the state 2, that is, it is under stress p_2, and is moving to the right with the velocity v_2.

Owing to the stoppage by the stationary infinite mass the state of this far end changes from 2 to 3, the latter being located at the intersection of the negative directrix tan $(-\alpha)$ (because the wave is moving *toward* the impact end) with the $v = 0$ line (that is, with the ordinate axis). On its arrival at the impact end the wave 3 ($p = p_3; v = v_3 = 0$) is not disturbed, that is, it is not changed into a new state, because also at this end the bar's motion is stopped by the impacting mass (which is also at rest). Thus, the entire length L of the bar will remain at rest, ($v = v_3 = 0$), at the compressed state $p = p_3$.

The stereograms of stress, velocity and displacement are given by Fig. 10(d), (e), and (f).

The history of energy relations is given in Fig. 10(g). It is seen that until the middle of the impact period, that is, until the instant $t_1 + L/a$ one half of the input energy appears in kinetic, the other half in potential form, just as in a bar with free end, Fig. 8(g). From this instant onwards the kinetic portion of the energy will diminish, and the potential part increase, until, at the termination of impact, that is, at the instant $t_1 + 2L/a$, the entire energy will be in potential form (that is, $v_3 = 0$, kinetic energy is zero).

Problem 7. Elastic Impact in Compression, Bar of Finite Length, with Fixed End, Duration of Impact is Not Equal 2L/a (Fig. 11)

Similarly to the previous cases, with free end, Figs. 8 and 9, also for the fixed end condition the case will be treated in which the duration of impact is less than $2L/a$. This case, Fig. 11, is the same as the previous one, until the termination of impact period, that is, until the instant $t_1 + T$, at which the impact end of the bar comes to rest (state 2 changes to state 3, in the diagram of state, Fig. 11(c)). Meanwhile, the wave 2 has arrived at the far end of the bar and was stopped there by the obstruction of the stationary infinite mass. Thereby the state 2 has been changed to state 3′, in the diagram of state, Fig. 11(c). The two disturbance waves 3 and 3′ travel toward each other (see Fig. 11(b')) and meet at a certain cross section of the bar at a certain instant. The resulting new phase will be defined by point 4 in the v–p diagram Fig. 11(c), located at the intersection of the positive directrix tan $(+\alpha)$ drawn from the point 3′, and of the negative directrix tan $(-\alpha)$ drawn from the point 3. At its arrival at the two ends the wave 4 changes into states 5 and 5′ respectively, both being located on the $v = 0$ line, that is, on the p-axis. Thus the states change from domain to domain, as it can be easily followed in the location diagram Fig. 11(b'), and in the diagram of state, Fig. 11(c).

The stereograms of stress, of velocity, and of displacement are given in Fig. 11(d), (e), and (f), which can be easily constructed from Figs. 11(b) and (c).

The history of energy relations is given in Fig. 11(g). There is a similar analogy between Fig. 11(g) and 9(g), as was pointed out between Figs. 10(g) and 8(g).

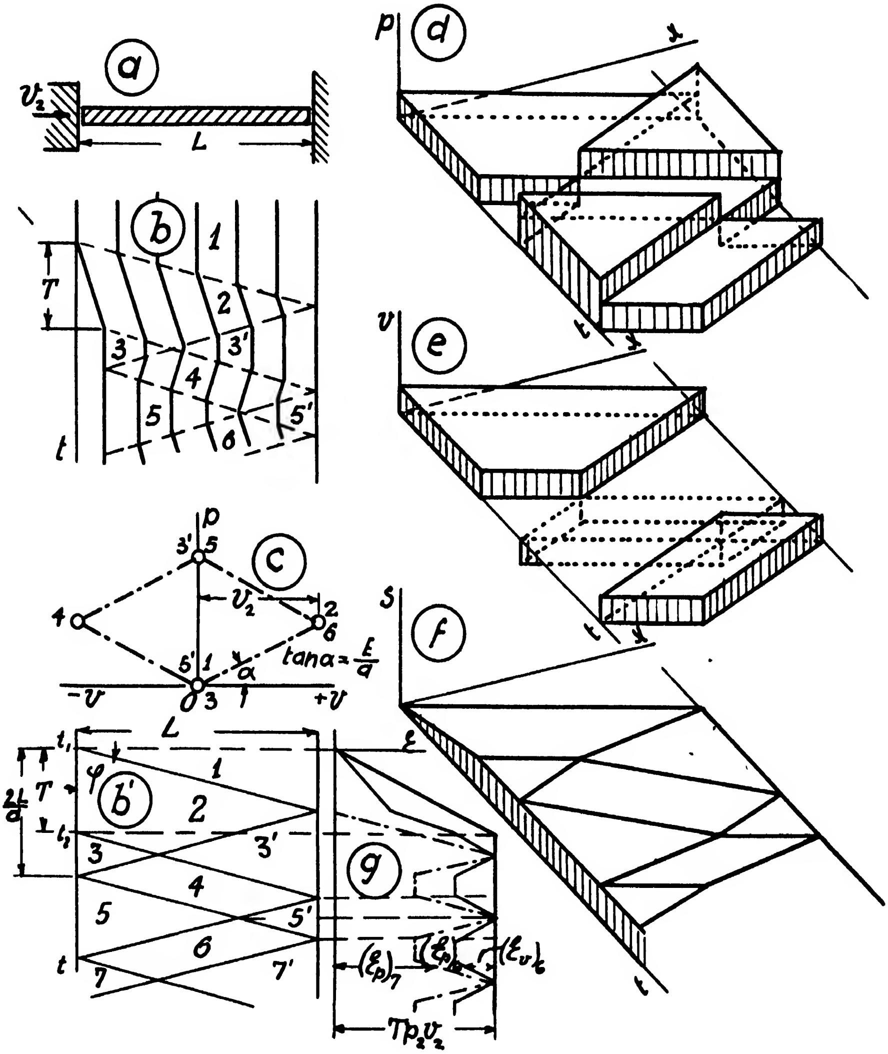

FIG. 11. Elastic impact in compression, bar of finite length, with fixed end; duration of impact less than $2L/a$.

(a) Schematic arrangement.
(b) Displacement of cross section.
(b') Location diagram.
(c) Diagram of state.
(d) Stereogram of stress.
(e) Stereogram of velocity.
(f) Stereogram of displacement.
(g) History of energy relations.

Elastic Impact in Compression, Bar of Finite Length, with Restrained End

In the previous examples, Figs. 8, 9, 10 and 11, the far end of the bar was considered either as entirely free, or completely fixed. Inter-

mediate conditions can be considered in which the motion of the bar end is restrained. The nature of the restraint may be friction, in which the resisting force is constant, and damping, in which the resisting force is a function of velocity. For both types of restraints various degrees can be considered, varying from entire free motion to completely fixed condition. Examples will be given for both types of restraint, and it will be assumed that the duration of impact is $2L/a$.

Problem 8. Elastic Impact in Compression, Bar of Finite Length, End Restrained by Friction (Fig. 12)

The condition of friction is analyzed in Fig. 12. The friction is considered to be a constant resisting force, corresponding to a unit stress r, which the stress existing at the friction-end of the bar must surpass before motion can result. Therefore, at the arrival of the disturbance wave 2 at the friction end, the state 2 will be changed to state 3 (in the diagram of state, Fig. 12(c)) situated at the intersection of the directrix tan $(-\alpha)$ drawn from point 2 with the r = const line. At the arrival of disturbance wave 3 at the impact end, the impact has *just* terminated, $p_4 = 0$, and the state of the bar changes to point 4 (because at the end of the period T the impact has terminated). On the arrival of the disturbance wave 4 at the friction end the state changes to 5; on the arrival of this wave at the impact end (which is now a free end, $p = 0$) the velocity will assume a negative value, point 6, which produces a negative stress, tension, point 7. But this negative stress is smaller than the stress r corresponding to the friction, hence no further motion at the friction end will take place, but henceforth residual energy will produce an interchange of fluctuating velocities and stresses, corresponding to points 6–7–8–9–etc., in the v–p diagram, Fig. 12(c). The stereograms of stress and of velocity are given in Fig. 12(d) and (e), which are self-explanatory.

Interest attaches to the total penetration of the friction end produced by the impact, that is, to the displacement of the friction end. This is determined by integrating the $v = f(t)$ line for the friction end: $s = \int v dt$, as it is shown in Fig. 12(f). It can be found also mathematically:

$$s = \frac{2L}{a}(v_3 + v_5).$$

The successive velocity values at the friction end form a decreasing arithmetic series, having a decrement of $2r(a/E)$, that is:

$$v_3 = 2v_2 - r\frac{a}{E}; \quad v_5 = 2v_2 = 3r\frac{a}{E}; \quad \cdots \quad v_{2n+1} = 2v_2 - (2n-1)r\frac{a}{E},$$

ending with the last member of positive sign.

The total penetration is:

$$s = \frac{2L}{a}(v_3 + v_5 + \cdots v_{2n+1}) = \frac{2L}{a}\left(2v_2 - nr\frac{a}{E}\right)n.$$

The history of energy relations is given in Fig. 12(g), showing that after the termination of impact there remains a residual energy in the

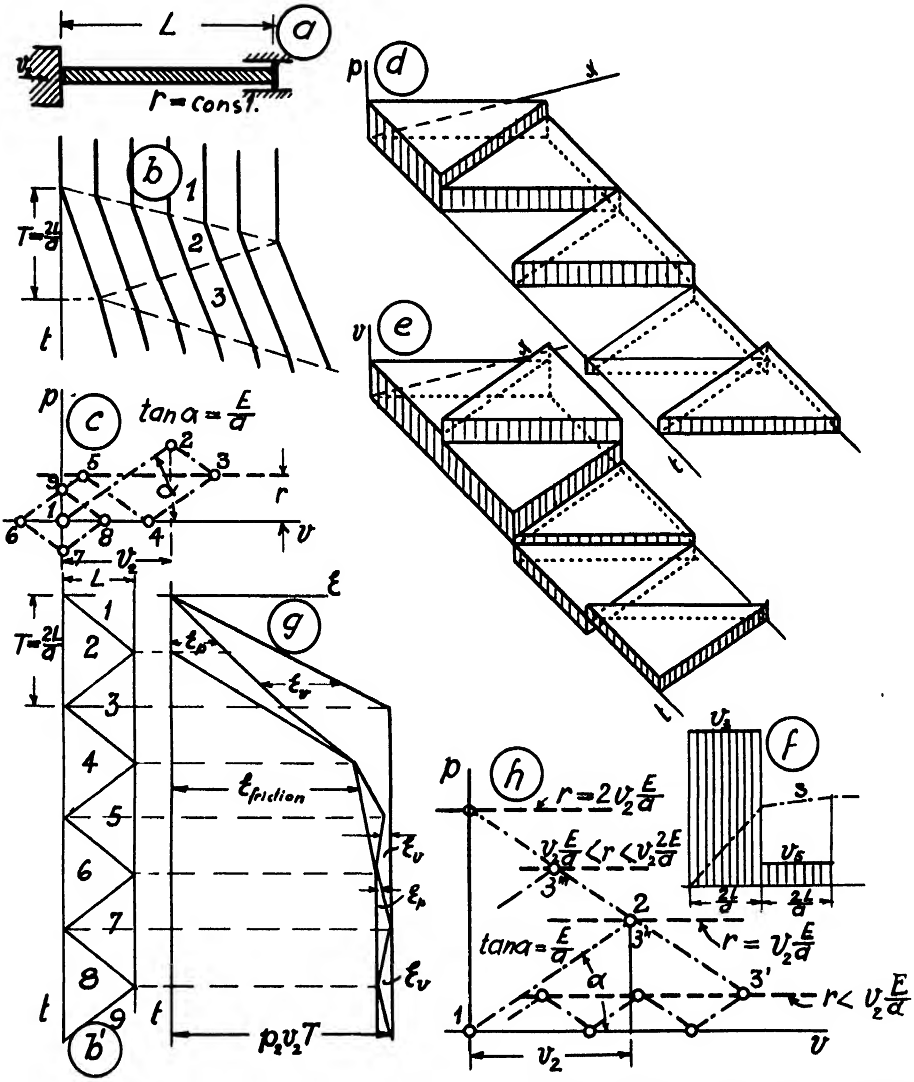

FIG. 12. Elastic impact in compression, bar of finite length, with end restrained by friction; duration of impact $2L/a$.

(a) Schematic arrangement.
(b) Displacement of cross sections.
(b') Location diagram.
(c) Diagram of state.
(d) Stereogram of stress.
(e) Stereogram of velocity.
(f) Velocity and displacement of bar end.
(g) History of energy relations.
(h) Effect of varying friction on the p–v relations.

bar which constantly undergoes a transformation from kinetic to potential and *vice versa*. In particular cases, however, for which $(E/a)(v_2/r) = n$ is an integer number, there will remain no residual energy in the bar, but the total energy of impact will be consumed by the friction. Figure 12(*h*) shows the diagram of state for various relative magnitudes of the frictional stress r. Denoting $(E/a)(v_2/r) = n$ several cases can be distinguished:

If $n > 1$ there will be one or more than one penetration periods $2L/a$, the velocity of penetration decreasing in arithmetical progression;

if $n = 1$ there will be only one penetration period, and there will be no residual energy left in the bar after the termination of impact;

if $1 > n > \frac{1}{2}$ there will be one penetration period, and there will remain residual energy in the bar;

if $\frac{1}{2} \geqq n$ there will be no motion at the friction end, and the condition is equivalent to a fixed end.

Problem 9. Elastic Impact in Compression, Bar of Finite Length, End Restrained by Damping (Fig. 13)

The condition of damping is analyzed in Fig. 13. The damping force (or its value for unit area: the damping stress) is assumed to be proportional to the velocity. Also in this case, it is assumed that the duration of impact is $2L/a$.

Figure 13(b') represents the location diagram, and Fig. 13(c), the diagram of state. The damping stress is represented by a straight line OA having with the v-axis, a $\tan \rho$ slope for which: the damping stress $r = v \tan \rho$. The impact wave 2 arriving at the damped end is changed to state 3, being located at the intersection of the negative directrix drawn from point 2 with the damping line $\tan \rho$. The successive states 4, 5, 6, 7 ... can be readily followed in the diagram of state. It will be realized that the $\tan \rho$ line takes the role of the r-line in the case of friction. The stereograms of stress and velocity are omitted in this case, though they could be easily constructed with the aid of the location diagram, Fig. 13(b') and the diagram of state Fig. 13(c').

Interest attaches to the penetration of the damped end. The penetration is the integral of the velocity values of the damped end:

$$s = \frac{2L}{a}(v_3 + v_5 + v_7 + \cdots),$$

as shown in Fig. 13(f). The velocities represent a decreasing infinite geometrical series, for which:

$$v_5 = v_3 \frac{1 - A}{1 + A}; \quad v_7 = v_3 \left(\frac{1 - A}{1 + A}\right)^2; \quad \cdots \quad v_{2n+1} = v_3 \left(\frac{1 - A}{1 + A}\right)^{n-1},$$

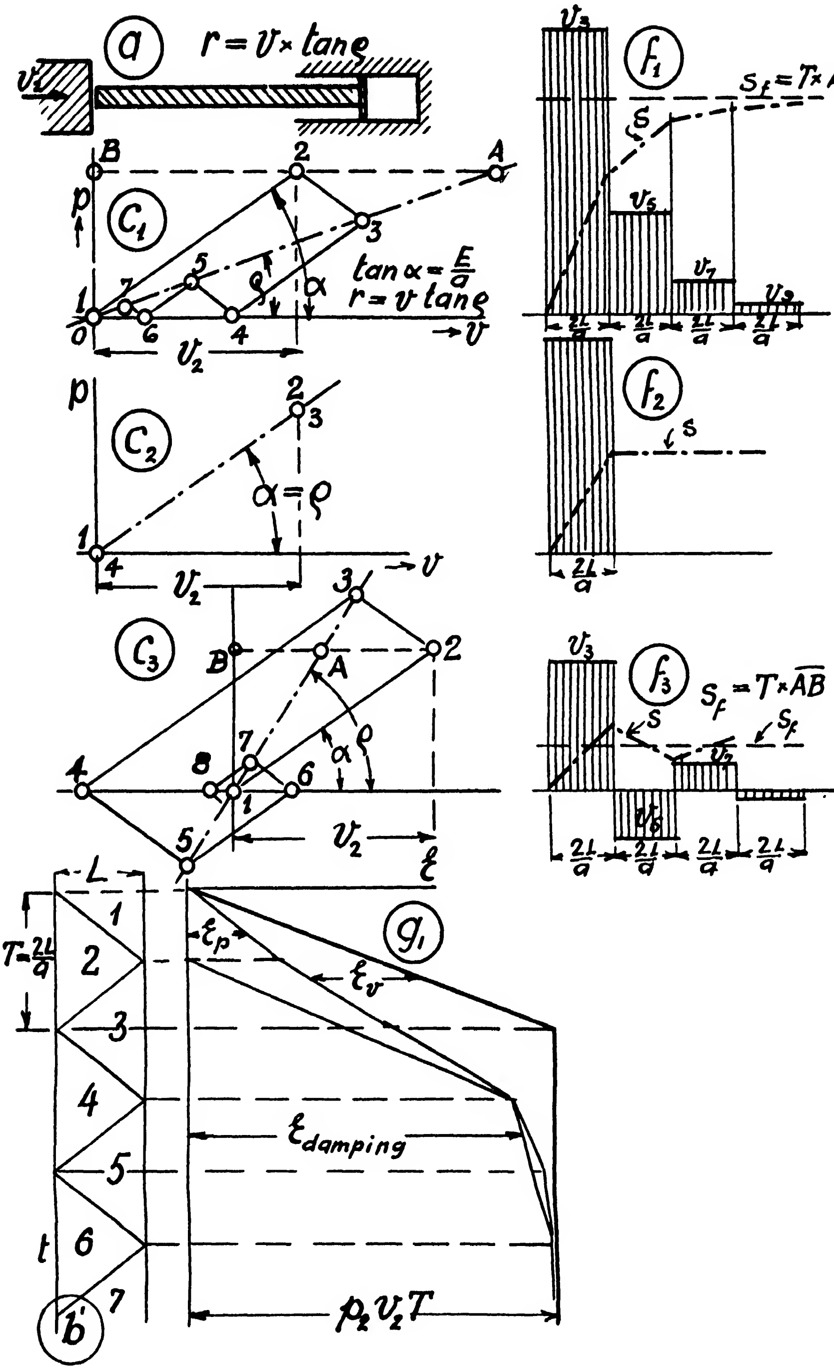

FIG. 13. Elastic impact in compression, bar of finite length, with end restrained by damping; duration of impact $2L/a$.

(a) Schematic arrangement.
(b') Location diagram.
(c_1) (c_2) (c_3) Diagrams of state for various degrees of damping.
(f_1) (f_2) (f_3) Velocity and displacement of bar end for various degrees of damping.
(g) History of energy relations.

in which

$$A = \frac{\tan \rho}{\tan \alpha},$$

and

$$v_3 = \frac{2v_2}{1 + A}$$

The sum of the above infinite series is:

$$v_3 + v_5 + v_7 + \cdots = \frac{1}{1 - \dfrac{1 - A}{1 + A}} \frac{2}{1 + A} v_2 = \frac{v_2}{A},$$

and therefore:

$$s = \frac{2L}{a} \frac{v_2}{A}.$$

The value

$$\frac{v_2}{A} = \frac{v_2 \tan \alpha}{\tan \rho} = v_A$$

is represented in the v–p diagram by the distance AB read in the v-scale as it can be readily seen by inspection. Point A is the intersection of the horizontal line drawn through point 2 with the tan ρ line.

The history diagram of velocity and of displacement for the damped end is shown in Fig. 13(f), and the history of energy input and distribution is given by Fig. 13(g).

According to the relative magnitude of the damping tan ρ and the wave factor E/a the progress of the phenomenon can be of different types:

(1) If $\dfrac{\tan \rho}{\tan \alpha} = \dfrac{r/v}{E/a} = A < 1$, that is, $\rho < \alpha$, then the velocity for the successive $2L/a$ periods will decrease in geometrical progression. This case was treated above, in connection with Fig. 13(c_1) and (f_1). This case is analogous to a periodic damping.

(2) If $A = 1$, that is, $\rho = \alpha$, then the bar will penetrate into the damping medium with the velocity v_2 (= impact velocity) during one $2L/a$ interval, after which the bar will come to rest in its total length (state 3) as shown in Fig. 13(c_2) and (f_2). This case is analogous to critical damping.

(3) If $A > 1$, that is, $\rho > \alpha$, then the motion of the bar will be in alternating directions with ever decreasing amplitude, as it is shown in Fig. 13(c_3) and (f_3). The total penetration of the bar into the damping medium will be the sum of these positive and negative velocities:

$$s = \frac{2L}{a}(v_3 + v_5 + v_7 + \cdots) = \frac{2L}{a}\overline{BA},$$

where it is to be noted that $v_3, v_7, \ldots$ are in the positive direction, while $v_5, v_9, \ldots$ are in the negative direction. The equivalent mean velocity is represented by $v_a = \overline{AB}$ read in the v-scale, where point A is found by a similar construction as explained above for Fig. 13(c), and with which:

$$s = \frac{2L}{a} v_a.$$

(Derivation is as follows:

$$v_3 = \frac{2v_2}{1+A}; \qquad v_5 = \frac{1-A}{1+A} v_3; \qquad v_7 = \left(\frac{1-A}{1+A}\right)^2 v_3;$$

$$v_9 = \left(\frac{1-A}{1+A}\right)^3 v_3; \qquad \ldots \qquad v_{2n+1} = \left(\frac{1-A}{1+A}\right)^{n-1} v_3.$$

It is to be noted that, $(1 - A)$ being negative, the fraction $\left(\frac{1-A}{1+A}\right)^i$ will be negative for odd values of i and positive for even values of i.

The sum is:

$$s = \frac{2L}{a} v_3 \frac{1}{1 - \dfrac{1-A}{1+A}} = \frac{2L}{a} v_3 \frac{1+A}{2A} = \frac{2L}{a} \frac{v_2}{A},$$

as in the previously treated case.)

Problm 10. Elastic Impact in Compression, Bar of Finite Length Having Discontinuities, End of Bar Free (Fig. 14)

If the bar is not of uniform cross section and material but it is composed of portions having different cross sections, or of materials of different elastic properties then at the cross sections at which the change is located the disturbance wave will also change from the previous state to a new state which change will be reflected in both directions from the cross section of discontinuity. Thus, a cross section of discontinuity arises, if on its two sides the wave factor FE/a is not equal. In this sense a free end is a cross section of discontinuity (because the cross-sectional area of the bar changes to zero, $FE/a = 0$) and a fixed end is a discontinuity (because the cross-sectional area of the bar changes to infinite, $FE/a = \infty$). In the following a treatment will be given to impact of bars having discontinuities between these limiting cases, that is, for which $0 < FE/a < \infty$.

As an illustration, a simple case will be analyzed, shown in Fig. 14(a), in which the bar is composed of three portions, AB, BC, and CD of different cross sections F_1, F_2 and F_3, and/or of different material prop-

erties E_1, E_2 and E_3 and a_1, a_2 and a_3, such that:

$$\left(\frac{FE}{a}\right)_{AB} : \left(\frac{FE}{a}\right)_{BC} : \left(\frac{FE}{a}\right)_{CD} = 1:0.75:0.50,$$

and

$$\frac{L_1}{a_1} = \frac{L_2}{a_2} = \frac{L_3}{a_3}.$$

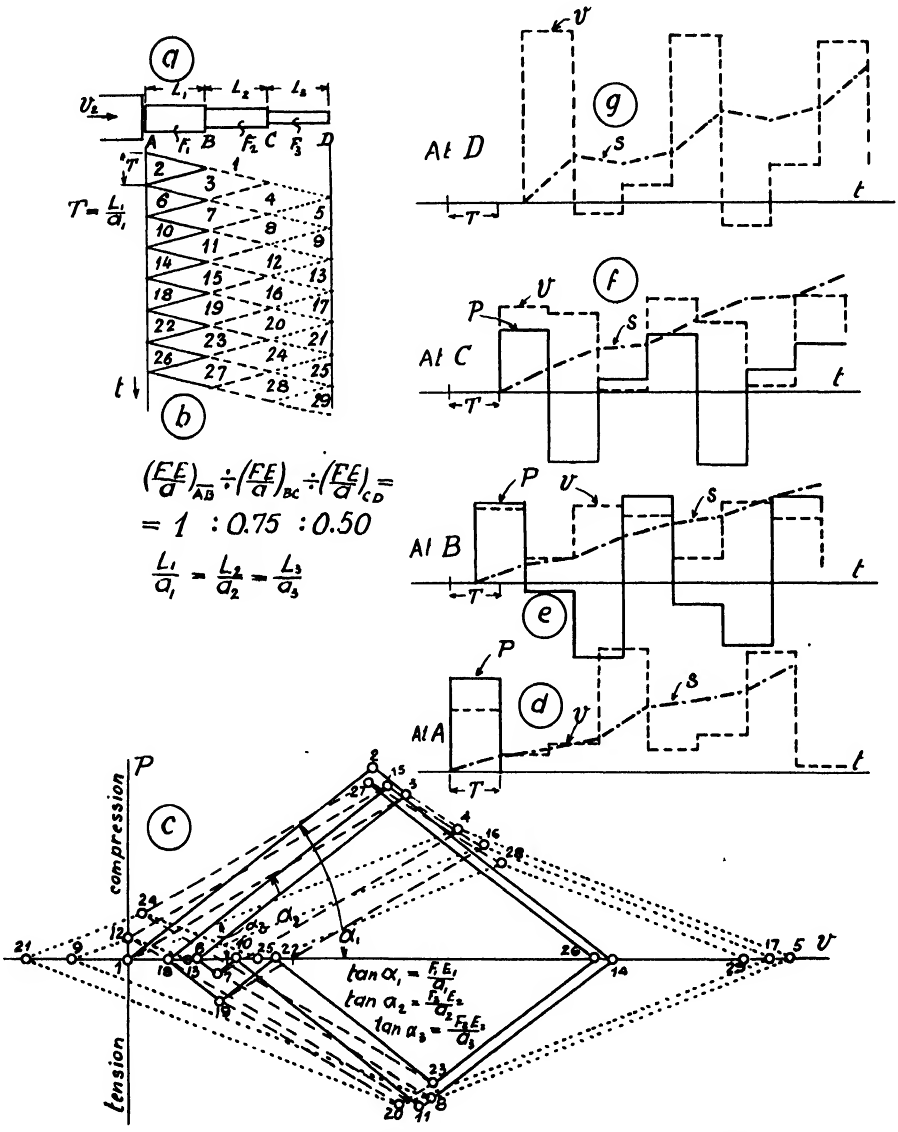

FIG. 14. Elastic impact in compression, bar of finite length having discontinuities, with free end; duration of impact L_1/a_1.

(a) Schematic arrangement.
(b) Location diagram.
(c) Diagram of state.
(d) (e) (f) (g) History diagrams of force, velocity, and displacement at the discontinuities.

It is assumed that the end A of this bar is subjected to an impact velocity v_2 lasting $T = L_1/a_1$ time. At the boundary B this state 2 is changed to state 3 (Fig. 14(c)) which is at the intersection of the negative directrix tan $(-\alpha_1)$ drawn from point 2 with the positive directrix tan $(+\alpha_2)$ drawn from point 1. When this wave 3 arrives at the boundary C the state changes to 4, being at the intersection of the negative directrix tan $(-\alpha_2)$ drawn from point 3 with the positive directrix tan $(+\alpha_3)$ drawn from point 1. At the free end, cross section D, the wave changes from 4 to 5, the latter being located at the intersection of the tan $(-\alpha_3)$ directrix drawn from point 4 with the abscissa axis $(P = 0)$.

It will be noted that in the diagram of state, Fig. 14(c), the ordinate axis represents forces P, instead of stresses p. This is done because the *forces* balance each other at the cross sections of discontinuity, and not the *stresses*, owing to the difference in the cross sections.

The successive progress of the phenomena can be readily followed on the v–p diagram, Fig. 14(c). It will be noted that points 3, 7, 11, 15, 19, etc., defining the change of state at the boundary B are located at the intersections of directrices tan α_1 (characterizing the portion AB) and directrices tan α_2 (characterizing the portion BC). Similarly, points 4, 8, 12, 16, 20, etc., defining the change of state at the boundary C, are located at the intersections of directrices tan α_2 (characterizing the portion BC) and directrices tan α_3 (characterizing the portion CD). Finally, the points 5, 9, 13, 17, etc., defining the change of state at the free end D, are located at the intersections of the directrices tan α_3 (characterizing the portion CD) with the abscissa axis (for which $P = 0$).

The stereograms could be readily constructed, but are omitted in this case. Instead, the history diagrams of force P, velocity v and displacement s are given for the two end-cross sections A and D, and the two intermediate boundaries B and C, Fig. 14(d), (e), (f), and (g).

The initial assumption $L_1/a_1 = L_2/a_2 = L_3/a_3$ was made solely for the sake of simplicity, but it does not represent a limitation of the method. Any other relations between the lengths can be treated without difficulty, at the cost of more labor.

(NOTE: A somewhat similar problem has been treated in a paper by Langer and Lamberger by calculation and in a discussion to this paper by DeJuhasz and Yorgiadis by graphical analysis. This refers to a long vertical rod, of length portions having different cross sections, which is initially loaded by a weight suspended at its end and is in a state of static stress; the load is suddenly released. The ensuing phenomenon is analyzed by the graphical method.)

The previous paper of the author treats elastic impacts of bars against other bars of various lengths, cross sections and restraint conditions. Therefore it is believed that the foregoing examples cover sufficiently the analysis of problems of elastic impact.

(*To be continued in the August issue.*)

VAPORIZATION COOLING OF LARGE ELECTRICAL MACHINES

BY

TH. DE KONING[1]

SYNOPSIS

This article describes a method of cooling as revolutionary as hydrogen cooling was in the twenties. A much wider applicability, considerable savings and other advantages are expected of it. The principles of vaporization cooling are so different from those customary for other media (air, hydrogen and fluid) that for a proper evaluation one must make oneself free from any preconceptions. The fundamentals, the far-reaching influence upon the design of machines and machine-sets, -stators and -rotors; the methods of supply of the cooling means, the conditioning of the machines and finally the influence of the cooling-means upon the insulation and other machine-parts (corrosion) will be discussed in this order in a condensed form.

1. FUNDAMENTALS OF VAPORIZATION COOLING

Introduction

For the "cooling" of large machines of all types—the word "ventilation" leads to undesirable associations—a much more universal solution than the present is possible. The cooling as the decisive factor must be considered first of all. The method of cooling to be discussed is simpler than hydrogen cooling and allows a natural integration with the machines coupled (for example, turbines and exciters). The result is a saving in housing, end shields, seals, bearings, couplings, etc. The auxiliary equipment for the cooling, which represents a much larger percentage of the cost of small than of large machines, has been in wide use on identical apparatus for a long time. The cost of this standard equipment is small and it can be used, moreover, in part, for all the machines of the set.

Over the present machines with the best utilization of the materials of construction one can expect: a saving of 5–15 per cent in copper- and punching-volume without increasing the current- or flux-densities; a somewhat better efficiency; improved simplicity and reliability; and a saving of 25–40 per cent in weight and 15 per cent in cost of large machines. Just as hydrogen cooling cuts the space needed for air-cooled units in half, so will vaporization cooling cut the space needed for hydrogen-cooled units in half. The overload capacity is as good as that of a transformer.

The better heat resistance of the iron laminations rather than of the windings is taken advantage of. No air vents, usually 4–12 per cent of the core length, are needed. Such a solid stator core is explosion- and corrosion-proof. The cooling has the advantages of hydrogen

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cooling (small windage losses and increased insulation life) but not the disadvantages (a large heavy shell, hydrogen purity and shaft-sealing equipment). Filling or exhausting the machines becomes a matter of minutes instead of hours. Careful supervision becomes unnecessary and the yearly outlay for lost hydrogen and scavenger gas can be saved. The machines are suitable for any size and environment and for any constant or variable speed.

Basic Principles

The essential parts of an electrical machine are the magnetic and winding materials. These are arranged in five concentric rings of iron, iron and copper, air, iron and copper, iron. Those adjoining the airgap generate about four fifths of the full load heat losses. The heat losses throughout the core length are the same, so for cores without air vents we have a two-dimensional heat field. The ring design is valuable for axial assembly and disassembly. All other parts but those mentioned are of secondary importance. Whether they are really needed or not has to be carefully looked into.

The air gap is the decisive place for the cooling and also for the supply of the cooling means. The relatively unprotected winding ends use up what remains of its cooling power. The heat resistance between the conductors and the cooling medium must be reduced to the utmost. The basic formula for the heat transmission is $1/U = \Sigma b/K + \Sigma 1/h$ with U = overall heat transmission coefficient, Σ = sum of the individual values, b = wall thickness in feet, K = heat conductivity coefficient, and h = heat transfer coefficient. K in Btu/sq. ft/hr/° F. is, for copper, 224; aluminum, 118; iron, 35; iron laminations (air gap) 25–15; laminations (air vents) 1–0.4; insulating materials 0.3–0.15; and trapped air in slots, 0.025. For the most favorable cooling speeds h in Btu/sq.ft/hr/°F. is, for air, 25–15; hydrogen, 40–25 for atmospheric pressures. The increase of the tooth temperature away from the air gap keeps the core at elevated temperature and makes the cooling of the stator-core outer circumference more effective. For thick cores and/or great winding heights, the air-gap cooling can be supplemented by means to be discussed later in detail.

The heat absorbed by air or hydrogen depends upon the temperature difference. About 20–30° C. (36–54° F.) maximum rise in gas temperature is permissible. Fans and circuit resistance can reduce this available temperature span up to 8° C. As the maximum temperature rise for the windings is 55° C. the temperature rise of the air will soon make it lose much of its effectivness. Moreover, an even air circulation cannot be guaranteed. The mean over-temperature is therefore taken at 0.7–0.8 instead of 0.5 of the maximum temperature rise. All this makes the temperature anywhere in the machine guesswork. Clear-cut conditions are essential and these can only be obtained with a cooling

medium of a temperature which remains constant during its heat absorption, of outstanding heat capacity, excellent heat-transfer coefficient, etc.

Constant Cooling-Means Temperature

The heat losses per unit of core length are the same, therefore the insulation stresses will be, too, with constant cooling-medium temperature. A sizeable decrease in cooling surface needed or in insulation strain is the result. In an evaporating medium the temperature remains constant until it is fully vaporized. As this only holds true at constant pressure we have a means of regulating the cooling temperature.

As the cooling is to be limited to the air gap (no dead corners, plugged passages, air vents) the cooling fluid is equally and fully utilized. With the high rotor surface speed, vapor and fluid form a finely divided mixture. As a housing is superfluous and the core explosion-proof, the end shields can be small, sturdy and light. Vacuum pressures are desirable because of the surface-friction. The vaporization temperature of the most effective fluids is around 70–100° C. For water at 0.8 psia. and a vaporization temperature of 34.2° C., 1 lb. has a volume of 418 cu. ft. and absorbs 962.5 Btu of heat. At 1.6 psia., and a temperature of 47.3° C., 1 lb. has a volume of 214 cu. ft. and absorbs 962.5 Btu.

Now 1 lb. of air, about 13.6 cu.ft., absorbs about 12.9 Btu with a temperature increase of 30° C. The heat-transfer coefficient of evaporating liquids is about 50 times that of gases. Even with a cooling-surface reduction the whole temperature difference, instead of only part, can be used for cooling the windings. As will be proven later in this paper in detail, water and steam, when properly applied are at least as good an insulating means as air or hydrogen. In a closed cycle-cooled machine the fundamental requirements can be easily fulfilled.

Closed Cycle Cooling

Closed cycle cooling is desirable with fluids to be kept at high purity. As only small quantities are required, small diameter piping for the fluid and a highly utilized vapor space of ring volume are sufficient. Most of the cooling is done in the air gap. The end rings and end shields fit closely over the winding ends. Even over-dimensioned condenser(s) will be much smaller than the cooler for hydrogen-cooled machines. The heat-transfer coefficient of cooling water is about 400–1200; that of vaporizing water or condensing steam, 2000 and more; air, 15–25; hydrogen, 25–50.

During condensation, air and all other undesirables are being immediately removed (compare hydrogen cooling); moreover, a fluid can be much better cleaned than a gas. Dust from any source (atmosphere, brushes, collectors, commutators) is removed as sludge by any fluid

filter. With a well-conducted vapor stream, the interior of the machine becomes progressively cleaner. No scavengers are needed, neither long drying-out periods or large air filters. Fans are not needed. Any reason for not having the exciters in the main housing disappears and one gets rid of one shaft seal.

The winding ends can be cooled by the vapor-mist mixture, leaving the air gap at high speed and small water content. The steam will become slightly superheated which keeps the winding ends dry, end shields and end rings above air gap- and certainly above condenser-temperature. Stator core and bearings are naturally above vaporization temperature. When out of service the outer air can be let in through a filter. This prevents water absorption of the windings even then.

A Chemically Inert Cooling Medium

The explosion danger of hydrogen requires a complicated auxiliary equipment and highly skilled attendants, while with an evaporating fluid the standard equipment is easily maintained. The vacuum pump prevents gas accumulations. Generator fires are unlikely with hydrogen, impossible with water-vapor cooling. Neither ozone nor nitrous oxides, both dangerous to the insulation, can be found in hydrogen or vaporization-cooled machines. Corona does not cause damage. As 1 lb. of water requires 5720 Btu, its dissociation in hydrogen and oxygen at low pressures and temperatures is unlikely. It is to be expected that the insulation will behave at least as well in vacuum steam as in hydrogen. This will be the case for short circuits, too.

Principles of Cooling-Means Supply

Every surface unit must get exactly the quantity required for its needs. The fluid stream must therefore be subdivided and broken up by rapid motion in the air gap, the stream hitting a hard surface, atomizing nozzles, over-temperature and -pressure or other means.

The smaller the fluid drops the larger their surface and the easier it is for the vapor to free itself. Absence of air and clean surfaces permit heat-transfer coefficients of 2000 and more. Even an over supply of fluid is no objection in a well-guided vapor stream flowing towards the condenser. There are no vents in which it can accumulate. A collection of fluid in the rotor is impossible (centrifugal force), and is practically impossible in the stator core. The teeth tips are coolest, the teeth roots hottest. Any fluid penetrating between teeth and coils cannot get any further because of the vapor it creates and is quickly dissolved. The same holds true for fluid trying to penetrate the insulation. The lowest temperature in the machine (except for the condenser) is the air gap. Superheating the steam before it is condensed is an

insurance against too much fluid being supplied. The heat absorption of vapor is about twice that of air; also, with too little water the temperature rises rapidly.

Principles of Cooling-Means Control

The cooling of vaporization-cooled machines is independent of speed (speed range), outside temperature and atmospheric pressure. Only those fluid quantities have to be supplied which are needed to remove the heat created by the losses. Superfluous and undesirable cooling at no load or small loads is done away with. The heat movements can be much better controlled and are therefore smaller and less abrupt. With a two-dimensional heat field exact calculations are possible, the basic condition for a useful temperature control. With all losses to be removed by vaporization cooling a 50,000 kva machine with 2 per cent losses needs at full-load, 400 gal. of water, instead of 40 million cu. ft. of air every hour. With 7 ft. per sec. water speed, the interior diameter of the main supply tube is only $\frac{5}{8}$ in.

A small regulating valve or pump is sufficient, therefore, to adjust the supply to the momentary demand. A thermostat in the vapor flow just before it reaches the condenser can take care of the regulation. Heavy overloads are taken care of by increasing the fluid supply (practically useless with air), if so desired aided by a better vacuum and increased condenser-water supply. Rapid load changes can produce heat strains and relative movements, for example, of the insulation in the slots. With slow movements the weakest material has time to adjust itself, tears and other surface damages can be prevented. Insulation breakdowns are often due or aggravated by such an initiation. With a sensitive and accurate temperature control these strains can be dampened and decreased in magnitude.

2. ELECTRICAL MACHINES

In brief, the derived advantages are: the heat losses are removed as evenly as generated without an increase in cooling-means temperature; the vaporization-cooled surfaces can be limited to the air gap; the cores are without air vents and the stator-core circumference as the outside of the machine is naturally cooled; the cooling fluid is only needed in small quantities; it has when evaporating an excellent heat-absorption and heat-transfer coefficient, a small surface friction, is explosion proof, non-inflammable, does not penetrate the winding surfaces and removes any kind of dust and undesirable substances fully; the cooling fluid is supplied according to the momentary need; fans and gas filters become superfluous; the cooler becomes considerably smaller; auxiliary apparatus is hardly needed and is only of the simplest and fully developed kind. All this indicates that a far-reaching simplification in machine construction must be possible.

Fluid Supply and Vapor Flow

The cooling fluid can be supplied through small radial channels, in the stator core, from outer circumference to air gap. The smooth air-gap surfaces (unbroken through air vents) prevent water accumulations and make the nearly saturated vapor flow with increasing speed towards the winding ends. This reversal of the usual direction of flow and the increasing vapor speed are of the greatest importance as will be shown later. For the 50,000 Kva. machine with 2 per cent losses and 25 points of supply, 7 ft./sec. water speed in the connecting tubes and 20 ft./sec. in the hair tubes in the core, interior diameters of $\frac{1}{8}$ and 5/64 in. are needed. The hair tubes equalize the water supply independent of supply tube length and are so small that they can easily be led through the teeth to the air gap. Any number of supply points can be chosen. One is free in the spacing. The openings themselves are so small that they will hardly be visible. The number of supply points used at any time can be made dependent upon the quantity of fluid needed.

An equalized distribution of the cooling medium over the air-gap surfaces independent of the quantity of heat to be removed is desirable. The volume of the vaporizing fluid to be supplied to the air gap will vary widely dependent upon the load and other factors. Now the shape, wetted surface and droplet size of the fluid coming out of an opening are influenced more or less by the fluid speed. What is needed is a method of supply similar to that used for the better fuel-injection systems. It injects carefully measured oil quantities in accordance with the load at accurately predetermined time intervals independent of the motor speed to the individual cylinders. A water-injection system built to less exacting specifications can be used. One is much freer in the injection duration and instead of one hair tube a whole row can be supplied. Moreover, the timing between the rows does not have to be so accurate as between cylinders. As a result the number of supply points can be taken at will.

The vapor speed midway between the exhaust rings (usually the core sides) will be zero. The flow resistance is therefore one third of the one of a constant vapor flow of exhaust ring velocity. Because of the rotor-surface speed of 300 mph. and more, an axial component of the vapor speed of considerable magnitude hardly increases the surface friction at the rotor side. Also, fairly high exit velocities can be used. From an aerodynamic point of view vaporization cooling, compared with air- or hydrogen-cooling, is outstanding.

Figure 1 is a sketch for a vaporization-cooled machine. The closed housing contains from right to left the fluid supply tubes, stator and rotor, the thermostatic regulator and the condenser cooling coil. The condensed vapor collects in the lower half of the condenser and passes through a cleaning filter in the refillable water container at atmospheric pressure. In service the vacuum in the machine sucks the water by

way of the thermostatic regulator and the supply tubes into the machine in the quantity needed for the cooling. The regulator is operated by the temperature of the vapor flowing in the condenser. The air collects in the upper part of the condenser and is continuously removed.

To put the machine in service, the hand control closes the air-inlet valve and the electric circuit. This circuit starts the machine and the pumping set automatically. As the vacuum space is small the pump will create the necessary vacuum from atmospheric pressure in no time. This is in marked contrast to preparing a machine for hydrogen cooling. When shutting down the machine the hand control opens the electric circuits and the valve which lets dry air in the machine. It is all very easy for the attendant.

The apparatus for the condensation and fluid supply consists also of pumps, squirrel-cage motor, contactor, fluid valve and fluid filter, all mass-production articles. As only small sizes are needed, the equipment can be build in or upon the machine.

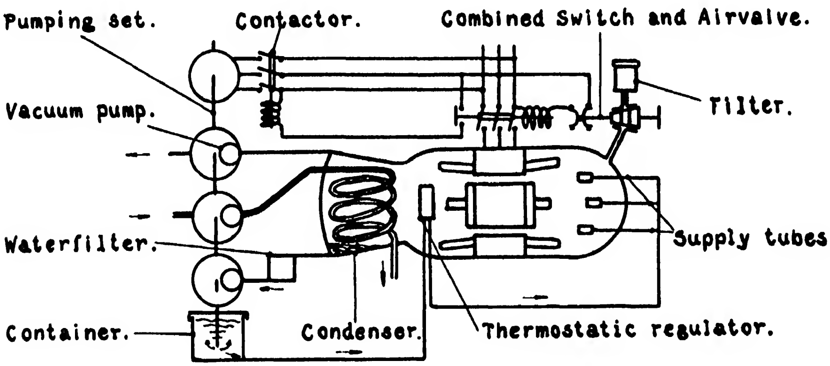

FIG. 1. Sketch of the cooling machine in principle.

Electrical Machine and Auxiliary Equipment in One Unit

The auxiliary apparatus needed is the same as for the simplest surface condensers. For a 200-Kva motor with 92 per cent efficiency, four fifths of the losses to be removed by vaporization, one fifth by natural cooling, only $\frac{1}{2}$ gal. of water is needed at full load every hour. As the condenser can be placed between stator end windings and outgoing shaft the machine dimensions need not be increased even when the condenser is designed for large overload capacity, shaft sealing and bearing cooling. Figure 2 shows such a vaporization-cooled machine in side view, front view and longitudinal section.

The vapor blast increases in sealing ability with the load. The condensate passes an exchangeable filter which removes carbon dust, dust, oil, etc. and flows through the pump in the supply reservoir. At atmospheric pressure this reservoir can be refilled at any time. If cast in the lower end shield the water in the supply tubing will automatically

flow back when the air valve is opened. Excepting the thermostat placed between rotor and condenser, all apparatus can be mounted on the outside of the machine. For small machines a combined air- and condensate-pump and a valve for the cooling water may be sufficient. If the natural pressure difference (fluid supply dependent upon vacuum pressure) must be increased, a small pressure pump can be useful. At the place the machine is going to be used only electrical and water connections have to be made.

Electrical Machine with Air-Cooled Condenser

Where water is objected to for the condenser cooling or the needed quantities of cooling water are not available, air cooling can be used.

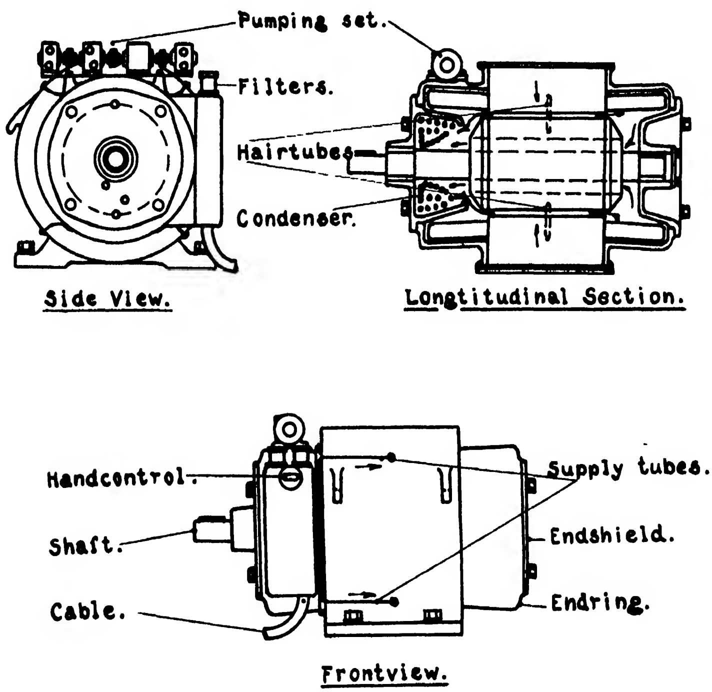

FIG. 2. Self-contained water-cooled machine.

As large vapor velocities are permissible, the vacuum-steam pipe can be small. The condenser can be placed in the most favorable position, especially since with modern welding airtightness can be assured. A ventilator, if so desired, in an aerodynamically designed air duct that turns in wind direction can be used and finned tubing (see Fig. 3). The tubing will be under external pressure (no inter-crystalline corrosion.) Even for the largest machines little water is needed for the vaporization cycle. With water cooling this quantity is maintained or even increased by means of entrained water and because of the fact that the water temperature is below the air temperature. This is not

the case with air-cooled machines, so the water level has to be checked every three months or so (vapor in the air removed by the vacuum pump).

Because of the thickness of the stator core the sun can burn upon vaporization-cooled machines without increasing the burden upon the cooling. This is not so with air- or hydrogen-cooled machines. It is an additional safeguard in preventing condensation in the end rings or end shields. The air condenser being small in size, its outer surface cannot absorb much heat either.

When using air cooling the water cooling of the stator core (to be discussed later) becomes a closed circuit independent of or interconnected with the vaporization cooling. An interconnection has the advantage of decreasing the water reserve needed and also of allowing

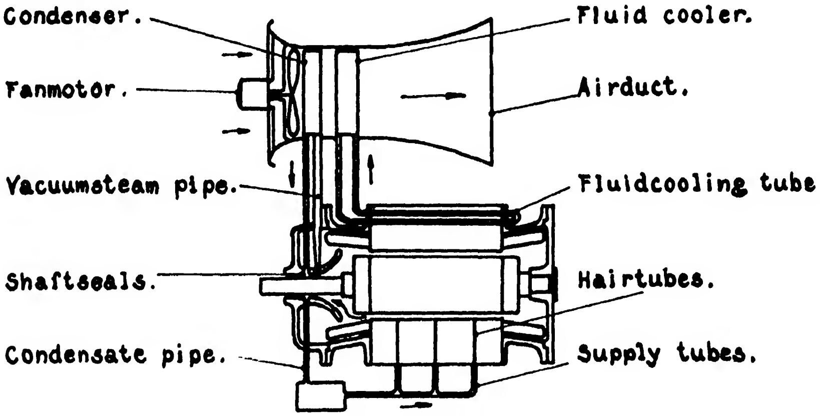

FIG. 3. Air-cooled vaporization machine.

the water for the vaporization cooling to be tapped at the highest temperature. Furthermore the condenser and the cooler can be put behind each other in the air-cooling circuit. Only one ventilation fan and air duct are needed in that case. The total air resistance will not be much increased.

The vacuum seal for the outgoing shaft of air-cooled machines can be like the one for machines with a water-cooled condenser around the shaft. The vapor stream will be directed towards the bearing and the vapor intakes arranged in a circle before it. The condensed vapor out of the condenser forms a ring seal between atmosphere and machine vacuum. Afterwards it is pumped in the water container from where it flows regulated by the thermostat to the tubes opening in the machine-airgap.

3. ELECTRICAL MACHINES AND MACHINE-SETS

The larger sizes of electrical machines usually form a unit with driving or driven machines, for example, in speaking of frequency con-

verters or turbo-generators, the complete equipment including exciters is meant. With such expensive sets it is desirable to know whether any single part, as end shields, bearings, couplings, etc., is really needed.

A Vapor Space in Common to Two or More Machines

As vaporization cooling and a satisfactory vapor guidance remove carbon dust fully, all electrical machines including exciters can have

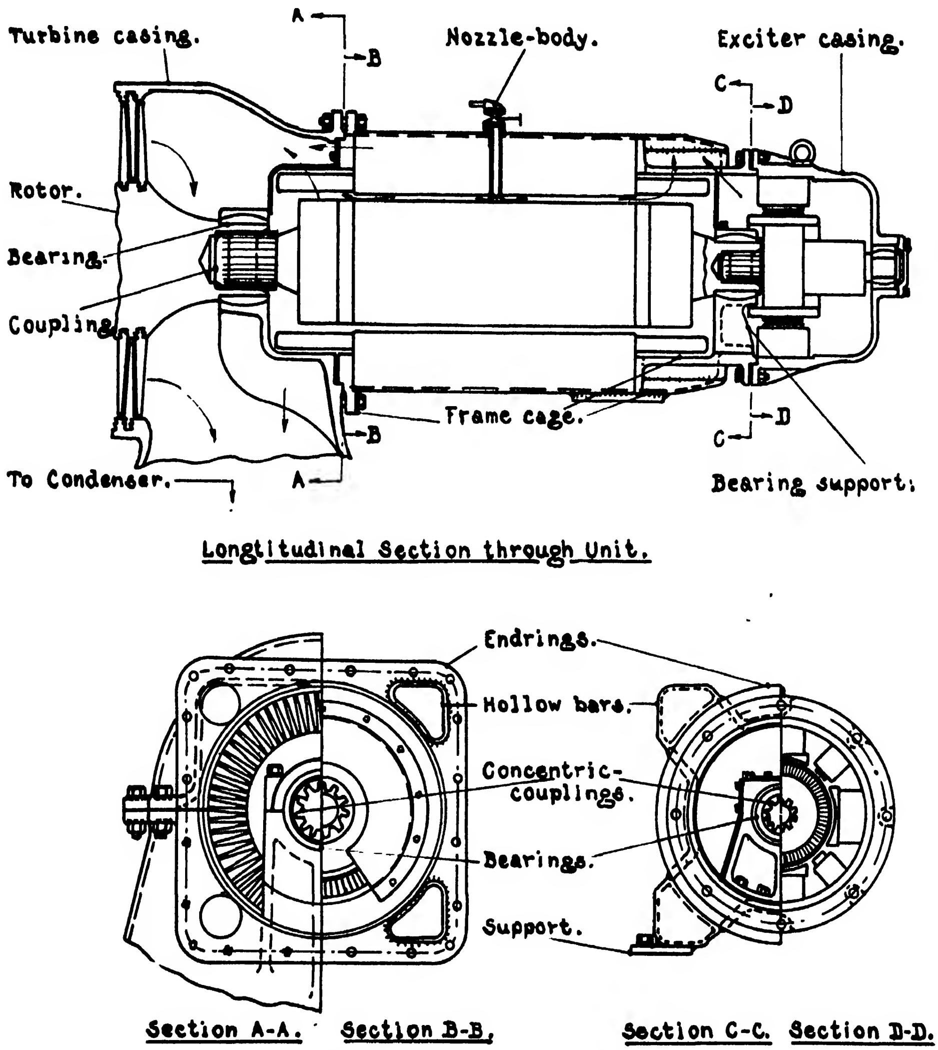

FIG. 4. Turbo-generator unit.

a vapor space in common. Contrary to hydrogen cooling, for example, opening a machine for a brush renewal means not any or only a short intermission, as the air pump which can be assisted by an ejector has the vacuum back to normal in no time. The necessary humidity for the brushes is always maintained. Water vapor is an excellent insulator.

The advantages of a vapor space in common are: no shaft seals, one closed housing suffices, only one condenser and its set of pumps are needed for the whole housing, the over-all length decreases. As exciter losses are at the most a couple of per cent of the main machine losses an enlargement of the condenser or pump-sizes is superfluous. The one part needed for each machine in the housing is a thermostatically controlled regulating valve, all parts for the condensation are in common (see Fig. 4).

The vacuum pressure of condensing steam turbines is suitable for electrical machines, also a vapor space in common can be used. The two remaining shaft seals disappear and the ideal condition is attained. Neither the turbine condenser nor its auxiliary equipment will notice the 1 per cent load increase. The time out is not influenced as one machine out of commission means that the whole set is.

Contrary to air- or hydrogen-cooled machines there is no objection to having the bearings in the common housing as the vapor flow out of the air gap and proper design will prevent any water in the bearing oil or oil vapor from getting at the windings. They will be forced directly into the condenser. As the oil enters the bearings with about 15–30 lb. over pressure and only a 30° C. rise in temperature is permissible, foaming of the oil and the creation of oil vapor are unlikely. A bearing functions as well in vacuum as under high pressure.

Combined Intermediate Bearings and Couplings

Any bearing is a potential source of trouble, requires excellent supervision, a continuous stream of oil of the right temperature and quality and mostly a cooling system. The bearings must be very carefully adjusted and every time the machine is taken apart the positive alignment and functioning must be accurately checked.

The fewer the bearings the better. The bearing size does not matter as long as the dimensions are in accordance with the load. Now the shaft in the bearings is bigger than in the shaft ends. Furthermore, the rotor of an electrical machine can be removed in axial direction. This is not possible with turbine rotors of normal construction.

The shaft in the bearing can be bored out quite a bit without losing its strength, a cubic function of the diameter. With teeth on the inside of the boring, the shaft to be coupled with teeth on the outside, both fitted together with the fine tolerances of high-grade gearing, only one bearing is needed and no additional coupling. With a sliding- or push-fit the necessary clearance is small compared to a running fit (bearing clearance); moreover, the clearances are dependent upon the diameter. Even if they act as solid couplings at full speed this will not interfere with the assembly, disassembly and warming up. Only for those, axial motion is desired. The turbine shaft is short and better suited for direct support. The long rotor shaft can be removed axially. The

teeth tips at the ends can be shaped for easy assembly and disassembly (see Fig. 4).

A check-up for proper alignment is easy. Removal and even exchange of the rotor of the electrical machine can cause no damage to the sensitive bearing surfaces. No bearing-adjustment or even a check-up is needed. There is no wear on the coupling-teeth. One bearing fixes a rotor axially. Overall length and engine room space for the removal of the main electrical rotor are saved.

Spigot Type Exciter with Plug Contacts

The larger synchronous machines have their own exciter mounted on the shaft between one of the main-bearings and a third bearing with its stator on its own pedestal. With overhanging exciter rotors the commutation is easily disturbed. With the bearing and coupling construction mentioned the exciter can be fastened, instead of an end shield, spigot-fashion on the stator end ring (see Fig. 4).

The flange insures excellent centering and support. With the small contact pitch circle even a slight angle between the shafts is permissible. The commutator being near its own bearing is unlikely to be affected. An emergency shaft seal and connecting valves can be used to facilitate maintaining the vacuum when an exciter must be opened for a brush inspection or renewal. For the current leads, disconnecting switch contacts or plug contacts can be used. The replacement of such an exciter only takes the machine out of service for a few minutes. Pilot exciters can use the same construction. The short main rotor length and the self contained units of construction are further advantages.

4. STATOR AND ROTOR

Machines for vaporization cooling are fully self contained, do not require openings in the foundation and are, because of the vacuum, as noiseless as hydrogen-cooled machines. The end shields fit closely over the winding ends; vapor space and machine dimensions are small. The ventless stator core becomes the outside of the machine.

The Nozzle Bodies in the Stator Core

The air gap is especially suitable for applying the cooling fluid, as the windings are well protected in the cores on all sides. The thin supply tubes contain but little fluid. As fluids are incompressible there is no time lag between control valve and supply openings.

The nozzle bodies can be led through and sealed in radial channels in selected groups of laminations. For inspection, a tube can be sealed in the core with the nozzle body fitting closely in it (see Fig. 4). With a copper connecting tube of sufficient length it can be taken out at any time, inspected, adjusted, cleaned or even exchanged and put back again. Nothing has to be disconnected. The removal of 1 body out of 25, for

instance, hardly influences the cooling at all. The vacuum will not change much even if the opening is not plugged. A cleaning of air-cooled machines takes the machine out of service for a long period of time and still the proper distribution may not be attained or a constriction gotten at.

The direction and shape of the water jets can be according to requirements. The water droplets will be thrown against the rotor and move over it cooling the surface at the same time until they have attained about rotor speed. What remains of the water droplets is thrown against the stator laminations and cools these and the windings.

The Cooling of the Stator Core

With thick cores and high silicon laminations and where a predetermined maximum outer temperature is desired or a controlled temperature at the bottom of the winding slots, fluid-cooled holes in the length of the stator offer a very acceptable solution. High core temperature and low winding temperatures are basically impossible with air- or hydrogen-cooled machines. Insulating the hole walls prevents leakage or rusting. Non-conductive plastic tubing (eddy currents) can be bent or welded and made to form a good contact with the punched holes.

The excellent heat absorption and heat transfer of liquids makes only small diameter holes necessary. Furthermore, a much higher surface temperature can be allowed as for the windings. High initial fluid temperatures can be used. Used bearing oil or used cooling water from the condenser can be taken. As about a third of the losses can be removed this way the condenser size and the quantity of cooling and vaporization water can be reduced by the same amount. This is not possible with air- or hydrogen-cooled machines. The latent heat of the stator is to be kept high at all times so the fluid content of the holes must be small. Return bends are useful because of the slight conductivity of the water and to keep the heat field as two-dimensional as possible (see Fig. 3).

The solid laminated stator core can be enclosed in a frame cage (like for a squirrel-cage rotor) consisting of end rings and longitudinal bars (hollow for strength and vapor guidance). End rings and end shields can be externally or internally insulated, if so desired, if condensation is feared in them. As the laminated core is somewhat elastic in axial direction the balancing of expansion coefficients and temperatures of conductors and bars will prevent insulation damages (see Fig. 4).

The Cooling of the Rotor- (and Stator-) Winding Ends

The stator-core end windings are cooled by the vapor coming out of the air gap passing centrifugally outward through them on its way to the condenser. To facilitate this flow and to prevent the short sides

of the rectangular cross-section of the winding ends from getting more then their share of the wet vapor, the winding ends can be given a drop-shape or hooded.

For the rotor coils at each rotor-end the end windings can be cooled by the fluid spray out of tubes with their openings between rotor shaft and end windings. The naked innermost turns are cooled at their flat side by the spray; the sides of the staples of turns are cooled with decreasing intensity towards the rotor end caps. Thus the cooling effect gradually decreases toward the rotor circumference. This progression is contrary to the one of the embedded length of the coils. The rotor-surface cooling can never be so effective because of the slot insulation and the keys. Furthermore, the vapor flowing out of the air-gap ends will move radially outward as soon as possible so the rotor end caps will be poorly cooled. All these factors together make the mean-temperature gradient in radial direction in the coils quite small in comparison to those in air- or hydrogen-cooled machines.

5. THE VAPORIZATION FLUID AND THE WINDINGS

Introduction

The rules for applying vaporization cooling are different from those applying to air cooling and hydrogen cooling. The quantity needed only depends upon the heat losses to be removed and is independent of temperatures and temperature differential of cooling substance and cooling surfaces. With the latent heat of evaporation the total heat absorption between two practical temperatures is the same. With constant fluid regulator setting the pressure difference between supply and vacuum determines the flow. An instantaneous fluid reduction as well as a surface-temperature rise is the result of a vacuum breakdown.

If the vaporization temperature (vacuum pressure) increases, the temperature difference for the removal of the heat out of the windings decreases. Temperature gradients become less and maximum temperatures increase. This makes it even more difficult for the fluid to penetrate and damage the insulation. The exposed winding ends are cooled by nearly dry clean vapor instead of the wettest and dirtiest gases as with air cooling. No surface damage can be caused by creepage. The disruptive strength of water is 56.5 kV. compared to 25 kV. for atmospheric air. The resistance of water is about 4×10^{-8} ohms per cu. mm., the one of petroleum 3×10^{-13}. However in the mist stage this is of little importance and, in the superheated vapor-stage, of no importance at all. After dry air has been valved in a totally enclosed machine no water can condense or leak on the windings. The filter will keep the breathing air dry, too. Furthermore, there is a lot of latent heat in cores and windings which can be used for insulation drying and the vaporization fluid supply can be cut off when desired.

In air- or hydrogen-cooled machines water on the windings can

cause damage because of a through-wetting. A superficial judgment will attribute the same to vaporization-cooled machines. However, what are the facts? Take the insulation resistance. It is used as a criterion to judge whether the insulation is in satisfactory condition. Also a resistance-temperature curve taken of hygroscopic paper fiber will be instructive (see Fig. 5). It shows a falling-off with increasing temperature to a dangerous low at about 85° C. The maximum remains about constant at 115–130° C. The explanation is that the increased vapor tension frees absorbed water and forms conducting paths. Only around the vaporization temperature of 100° C. can this moisture be expelled. In vacuum machines the vaporization temperature (at 1 lb. 100° F.) is near the lowest temperature the machine will assume in or out of service. This makes water absorption next to impossible. The insulation remains in top condition over the entire temperature range. This fully justifies a smaller insulation thickness. The electrical insulation value rapidly decreases with an increase in

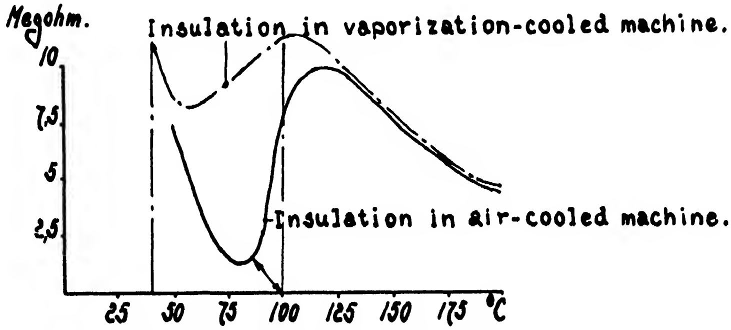

FIG. 5. Resistance-temperature curves for hygroscopic insulation.

thickness (mica: 0.1 mm., 150 kV/mm.; lmm., 50 kV/mm.). This proves the value of a thin insulation for high-tension windings for a high power rating and long life of the machine, as the heat resistance is proportional to the insulation thickness.

Diffusion of Water and Vapor in the Insulation

The regulation of the vaporization fluid supply according to need keeps the insulation surface temperature practically constant and at least at vaporization-pressure temperature. The temperature increases gradually towards the conductor so diffusion of water in the insulation is impossible and furthermore, steam is even a better insulator than air. The electrical machine out of service is filled with dry air or inert gas of atmospheric pressure.

The pores (holes) in the insulation can be divided into those connecting to the outside and fully closed pores. The last need not be considered here especially as the percentage of the insulation volume taken up by non-solids is negligible. All pores have in common that they are of

microscopic size in undamaged insulation. The smaller their size the less damage equalizing vapor- or gas-currents, etc., can cause. The connecting pores will be filled with air, and there is no reason to assume that this air will be more than saturated with steam according to temperature and pressure. When letting dry air in the machine when it is taken out of service, part of the pores and connecting channels will be closed by the pressure. The other ones will fill themselves with dry hot air, thus precluding an increase in moisture content.

In air-cooled machines (daily temperature cycles, small load, moisture laden air) the pores of the insulation can gradually suck themselves full of water by condensation at the inside and so on. To dry the insulation (temperature about 80–100° C.) a volume of vapor thousands of times that of the water is created. The vapor flow or vapor pressure (clogging) can therefore seriously weaken the insulation and initiate insulation breakdowns. Dirt getting in the insulation can do much damage, too. The absolutely clean controlled vacuum atmosphere in vaporization-cooled machines prevents all this.

Excessive temperatures created during overloads or shorts initiate most insulation breakdowns which can happen a long time afterwards. With other advantages, too, a special winding protection can be used. Its discussion can be made the subject of another paper as its application is not limited to vaporization cooling.

The Machine Out of Service

The auxiliary machines can be shut down when no heat losses are produced that must be removed or intercepted. To prevent water- or moisture-laden air from creeping through pumps, seals, etc., an easier way is to be provided to attain atmospheric pressure (see Fig. 1). The dry air admitted through a valve and a filter absorbs all the vapor and any water particles left, while assuming the mean machine temperature.

As the condenser is still for some time the coldest part of the machine, the air will give off most of its moisture there. Under exacting circumstances the auxiliary machines can be kept running for a while (time element). The winding surfaces will assume the mean temperature and become hotter than in service. This assures a thorough drying out, as no vaporization fluid is supplied any more. By the rate of temperature increase of the vapor flowing in the condenser an experienced attendant can judge closely how dry the machine is. By cutting of the vaporization water supply for the time needed for the experiment this is possible whenever desired with the machine in service. A connection to the atmosphere through a filter keeps a closed cycle cooled machine dry independent of time and atmospheric changes when out of service.

Under unusual circumstances a trickle of water in the condenser coil can fix a coldspot. The air volume is only a fraction of that in

gas-cooled machines relative to the exposed winding surfaces. In the latter, 65 per cent saturation must be counted upon with higher temperatures.

It is further to be noted that under normal operating conditions the fluid level of the atmospheric water container remains constant whatever the load. The fluid contents of the closed cooling system can be small so the slightest leak outward in the atmosphere or a fluid accumulation in the machine will soon change the short-time position of the float in the water container. This float can actuate an alarm or take all measures desired. Vaporization cooling makes for simplicity of the means employed.

The Hair Tubes In or Out of Service

If any break in the supply tubing occurs it must be outside the core. With hair tubes even thin walls have great strength. An automatic

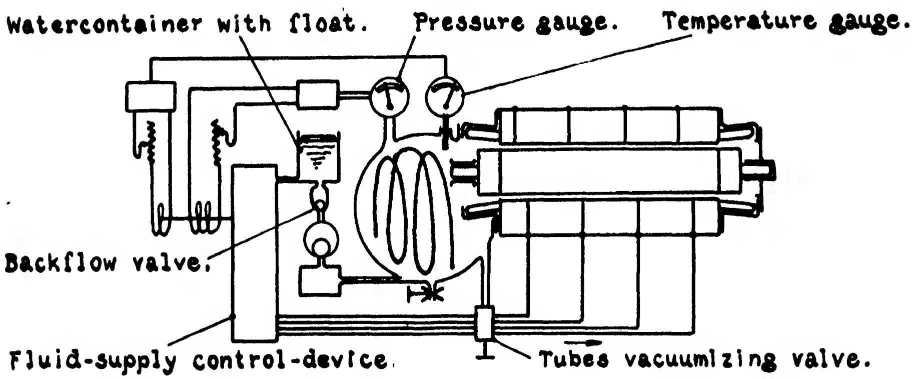

FIG. 6. Control equipment for machine.

equalizing of the fluid supplied to each opening is desirable especially with connecting tubes of different length. A slight difference in connecting- and hair tube diameters makes individual regulating valves unnecessary and the nozzle bodies simple.

High fluid pressures (small tube volume) will make for high exit velocities and a fine spray on the rotor surface. Moreover, the large tube surface in relation to the volume in the core, kept at elevated temperature, will superheat it relatively to the vacuum. This causes a finely divided mist to be spread evenly over the air-gap surfaces.

All water will be rapidly sucked back out of the tubing when connected to vacuum at the supply end. Vapor or air will fill the tubes in the core, and so leakage of the tubing not in use becomes impossible. The water flows in the condenser and is pumped through a filter in the container. A backflow valve can be used. In or out of service there cannot be any leakage in the machine (see Fig. 6).

A Regulating Device Controlled by the Temperature

With superheated vapor flowing in the condenser one can be fairly sure that all the fluid supplied to the air gap is used up. As the specific heat is small the heat quantity used is negligible compared to the heat of vaporization. Also the vapor temperature allows an accurate fluid regulation. The difference between air-gap temperature, surface temperature, and vaporizing temperature is only of academic interest.

A fluid is heavier than its vapor. With poor distribution the air gap at the bottom will be richer than at the top. This affects the stator cooling only. Instead of using one control thermometer, two or more at one or both air-gaps sides can be used. The regulating valve can be actuated by them in any combination. Special protection actuated by temperature differences and so on can be used.

Control by Temperature and Vacuum-Pressure in Combination

The vacuum pressure determines the temperature of vaporization. In emergencies so much air can get in the machine that the vacuum pump cannot handle it. The vaporizing temperature can become higher than the temperature for which the fluid regulator has been set. The temperature of the conductor and all other temperatures increase (also water cannot get into the insulation) until the air-gap surface temperature allows the vaporization to be resumed. Still an instantaneous correction for the new vacuum is of value. The regulating valve can be actuated by a temperature- and a vacuum pressure-element in combination (see Fig. 6). No more water is then supplied at any time than needed. With an increase in vacuum pressure the fluid supply is not only reduced but cut off. It is not restored before the vapor flowing in the condenser at the now desired temperature assure one of the proper air-gap surface temperatures.

With condensing turbo-generators the maximum load decreases. Thus the maximum temperature will hardly change. Obviously the higher vacuum pressure slows down the vapor flow for an instant so turbo-steam cannot get into the electrical machines. The guide rings for the wet steam supplemented by a fine gauze partition stop even the finest water droplets and allow a rapid pressure interchange.

6. CORROSION IN VAPORIZATION-COOLED MACHINES

Introduction

The oxygen, dissolved in the water, which comes in contact with metal surfaces can cause undesirable corrosion and under unfavorable circumstances erosion. The pertaining chemical formulas are Metal + HOH = MOH + H and 2H + O = H_2O. The water molecules in a fluid or surface below condensation temperature are much closer together then in a vapor, especially in a vacuum. The formation

of metal-hydroxide and hydrogen will be reduced in the last and the basic protective hydrogen layer on the metal surfaces better kept. Corrosion products and atmospheric impurities can absorb moisture at humidities down to 65 per cent and form an invisible water layer. They must be prevented from forming.

In vaporization-cooled machines the temperature can be kept at any level desired independent of the load. The vaporization temperature is between 20–40° C. instead of 100° C.; moreover, the machine is fully independent of varying and often unfavorable atmospheric conditions. Vaporization cooling and the admittance of dry air in the machine when not in service do not only keep the insulation in top condition, but the metal interior corrosion-free.

The formation of a thin protective coating on the rotor can be tolerated. The individual droplets of the water sprays will hit different surface spots all the time. Oxidation, if it occurs, will be in an even layer. The oxides will be dry most of the time. We have also the same favorable conditions as in dry hot air. Furthermore, when superheated water comes under vacuum pressure it explodes and liberates about 95 per cent of the air it contains. The oxygen will hardly come in contact with the rotor surface, therefore, and need not come in contact with the stator surface as the vaporizing water will act as a shield. The small quantity of cooling fluid needed to remove the heat losses, the high maximum closed cycle speed possible, the continuous removal of air by the air pump and impurities by the filter, they all all assist in keeping the water at a high purity and the desired slight alkalinity to inhibit corrosion. The treated boiler feedwater used for high-pressure steam installations can be used so no additional equipment is needed.

Basic Data

What are the water and air quantities involved for a 50,000 Kva machine with 2 per cent losses, to be removed by vaporization cooling only? Assume a rotor diameter of $2\frac{1}{2}$ ft. and a rotor length of 10 ft. This gives a surface of 70 sq. ft. Take the average heat losses reduced to full-load losses equal to one third of the number of hours in a year and the rotor losses to one half of that. Then the rotor surface has to evaporate 20,000 gallons per sq. ft. a year. With 1 cu. cm. of air with 35 per cent O_2 per liter, 0.0245 lb. of O_2 are liberated each year per sq. ft.

For an evaluation, compare these figures with those pertaining to boilers. With a full-load cooling-cycle speed of 6 min., only 40 gal. of water are needed for the machine. These values (the vaporization cooling will not have to remove the bearing- and stator-core losses and the oxygen content will usually be smaller in practice) show better than words how little corrosion and erosion have to be feared. Machines

with a shaft seal are usually smaller in size and the cooling surfaces larger in proportion to the fluid need.

The rapid change from vapor pressure at the pertaining water temperature (vacuum) to a high pressure (water column) causes the destructiveness of cavitation. There is also no danger in vaporization-cooled machines; moreover, the water is supplied in microscopic droplets. The surrounding rotor material will, therefore, help in making any blows harmless. This is not possible with solid water jets as in turbines. As to erosion, the rotor-surface speed is about half the turbine-blade speed. The water-droplet size is controlled (not in steam turbines where sudden water cataracts are known). The metal surfaces are always at higher temperature than the fluid or vapor.

Since the induced rotor and stator currents vary continuously in intensity and direction, electrolytic corrosion is unlikely.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS*

THE FALLACY OF THE BEST TWO OUT OF THREE

In scientific and engineering investigations a single measurement of an unknown quantity is seldom considered sufficient. Two or more measurements are usually made in order to establish a check on instrumental errors, operator's errors in making readings, and the reliability of the sample. These multiple measurements have two principal advantages: They reveal by their concordance the precision of the measuring process, and they make possible the use of an average of several measurements which will, in general, have a higher precision than one measurement alone. If three measurements are made, it is fairly common practice for students to take the "best two out of three" —averaging the two values closest together and discarding the other. Recently, however, Dr. W. J. Youden of the Statistical Engineering Laboratory at the National Bureau of Standards has shown that this procedure very often leads to less precise results than the averaging of all three measurements together.

Experimental work frequently creates new situations in which the precision of the observations is not known in advance and must be determined from the same data that establish the estimated or average value assigned to the quantity being measured. While a single measurement cannot yield any estimate of the reproducibility of the value, two measurements do give a primitive indication of their precision. But in an entirely new experimental situation, two measurements may not give a reliable estimate of the precision, since any marked disagreement between the two readings may be due either to the inherent crudeness and inaccuracy of the measurement process or to some accident, such as the gross misreading of an instrument scale, which makes at least one of the measurements greatly in error. With two discordant observations and no other information, it is impossible to decide between these alternative interpretations.

Three measurements is the minimum number that can conceivably reveal one of the measurements to be unreliable in a new experimental situation. Intuition suggests that if two of the three measurements are in close agreement while the third stands apart considerably removed from either of the others, then there may be grounds for suspecting and perhaps rejecting the third value. In terms of the difference between the two in good agreement, how different may the third measurement be before it should be suspected? Since this problem is important to all

* Communicated by the Director.

who make and interpret measurements, it is a little surprising that an answer has only recently been found.

An approximate solution has been obtained in the Bureau's statistical Engineering Laboratory through an empirical study of triads drawn at random from a large group of measurements constructed to conform to the characteristics of a normally distributed set entirely free from any gross errors. In this way, it is possible to examine a great many sets of three measurements and to determine, for example, how often the two differences between adjacent values in a set of three measurements will bear a ratio of 5 to 1, 10 to 1, 20 to 1, or any other ratio that might be considered unlikely in the normal course of events. If only 1 out of 100 sets of three measurements contained a measurement differing from the others by as much as five times the difference between the two closest, then such an observation might reasonably be discarded in actual experimental data. But the empirical study actually showed that a rather unbalanced spacing between three measurements occurs quite frequently. In fact, the ratio of the two differences was as much as 16 to 1 in 10 per cent of the sets of three measurements. In this connection, it is important to note that high ratios can result when two of the readings are very close or coincident while the third is not far removed.

The mathematical solution of this problem has been obtained and the frequency of occurrence for various ratios of the two differences has been calculated. The following table shows for certain ratios the results of an empirical sampling study of 400 sets of three measuresments compared with those predicted by the exact mathematical solution of the problem.

Ratio of Large to Small Difference	Frequency in 400 Sets	Theoretical Frequency
4.0 or more	149	145.2
9.0 or more	76	69.4
19.0 or more	38	33.9

These results reveal that in an average of one out of every twelve sets, one of the measurements will be at least 19 times farther away from its neighbor than the difference separating the two closest. Since in every 12 sets 1 shows such a spacing for measurements with no gross observational errors, it appears that measurements which should be retained are often dropped from the record. The problem of deciding on standards for the rejection of observations is one of long standing. Statisticians are again attacking this problem and, in the light of recent advances in the theory of small samples, considerable progress can be expected.

THE FRANKLIN INSTITUTE

ANNUAL REPORT OF THE BOARD OF MANAGERS FOR THE YEAR 1948

Again this year it was found impracticable to ready the Annual Report of the Board of Managers for presentation at the Annual Meeting in January. That report is now presented by the President in behalf of that Board.

Membership

The total membership at the end of 1948 was 5448. During the year 763 persons were elected to membership, for a net gain of 23 in total membership during 1948 as compared to a net loss of 137 in 1947.

Our members have cooperated with the Membership Department's efforts to counterbalance the effect of the many abandonments of membership by proposing a substantial number of their friends for membership. The popular talks, under the heading of "Science is Fun" have continued to attract more and more of our members and their friends to these informal gatherings.

During the year of celebration of our 125th Anniversary continued effort will be made to increase the interest and participation of our members in the many Institute activities and to increase the membership of the Institute.

Friends of Franklin

The group of 212 individuals and companies comprising the Friends of Franklin has continued its generous support. The amount contributed by them in 1948 was $22,150.00.

Meetings

During the year seventeen stated and joint meetings with technical societies were held at which the average attendance was 232. The dinners preceding these meetings were well attended and were enjoyed by our members and their guests. Dr. Hubert N. Alyea of Princeton University gave a most interesting and popular ralk on the Chemistry of the Atom Bomb; and the film entitled "Atomic Physics" was shown here (for the first time in the Philadelphia area) and drew two large audiences.

The eleventh series of piano lecture-recitals by Mr. Guy Marriner, Director of Music, continued into the spring of 1948, with an extra recital in February, in addition to the usual request recital given in April. The twelfth series opened in October 1948, and an extra recital was given in that month. There was an increase in attendance at these lecture-recitals during 1948 over 1947, indicating continued appreciation by the public.

Scientific organizations and technical societies continued to make use of our lecture hall and other meeting rooms for their regular and special meetings. It is our policy to encourage organizations whose aims and purposes parallel our own to use our facilities for such meetings, for which only a nominal charge is made. During 1948 thirty-six such organizations used our meeting rooms for a total of 157 meetings, which, in addition to the 101 meetings held with relation to Institute activities, shows a total of 258 meetings for the year 1948.

Committee on Science and the Arts

The entire Science and Arts Committee met nine times, while thirty-six Sub-Committee meetings were held. Fourteen Medals were awarded at the annual Medal Day Excercises, two of which were Franklin Medals, one being awarded to Dr. Wendell Meredith Stanley, and one to Dr. Theodore von Karman. Four George A. Hoadley Certificates were presented to

Committee members during the year as follows:

Dr. James Barnes
Mr. Charles E. Bonine } For thirty-five years of service.
Mr. Benjamin Franklin
Dr. Frederic Palmer –For twenty-five years of service.

Journal of the Franklin Institute

Our Journal, one of the oldest continuously published technical journals in the country, maintained its high standards of technical excellence in the 41 papers which appeared in the twelve issues for 1948. During the year, 62 manuscripts were submitted for consideration, of which 10 were published and 20 were added to the lengthy list awaiting publication. Each month approximately 4658 copies were distributed—3177 to members, 617 to domestic subscribers, 380 to foreign subscribers, 112 to "Friends of Franklin," and 372 in exchange for other publications.

During 1948 the following changes in Journal policy were effected: (1) the incorporation into each issue of one "general interest" paper, intelligible to the lay reader and yet of interest to technical men and scientists; (2) the expansion of the Book Reviews section, with all reviews by technically qualified contributors; and (3) the addition of a section of "Museum Notes" designed to place on permanent record the outstanding activities of that department of the Institute.

As evidence of the esteem in which our Journal is held, several hundred dollars worth of back files have been sold to libraries, industrial plants and government agencies during the past year.

Library

The additions to the Library during 1948 were as follows:

	Bound Volumes	*Unbound Volumes*	*Pamphlets*	*Maps*
By gift	328	324	876	224
Binding	816			
Purchase	626	46		
	1770	370	876	224

Total additions of all kinds: 3240.

Gifts were received from members and friends of the Institute, and through exchange from many scientific societies and institutions.

Complete specifications of patents from United States, Great Britain and Switzerland were received throughout the year. 800 periodical publications were received regularly through exchange and subscriptions, and 672 books were purchased. 13,184 photostat prints were furnished from material on file in the Library. 11,278 persons consulted the Library collection. Of this number 8033 were members and 1580 were members of the staff of The Institute Laboratories. There were 5458 telephone calls for technical information.

The total contents of the Library as of January 1, 1949 were:

131,101 Volumes
47,593 Pamphlets
4,667 Maps

Museum

Attendance:

The following tabulation shows the attendance at the Museum and Planetarium:

	Paid	*Free*	*Total*
1947	178,724	72,247	250,971
1948	160,428	71,559	231,987

As in prior years all public and parochial school children from Philadelphia continued to be admitted to the Museum and Planetarium without charge when attending in groups accom-

panied by a teacher. When attending individually, the children paid only the Federal admission tax.

Exhibits:

This year has been marked by unusual activity in major changes of exhibition space. Owing to changes in the location of some research laboratories, an area in the Electrical Communications Section was restored for exhibit purposes and a portion of the Physics Section had to be sacrificed. Advantage was taken of the former change to clear the entire room in order to provide space for the Atomic Energy Exhibition furnished by the Brookhaven National Laboratory. The original exhibits displayed in the room were distributed throughout other sections of the Museum.

As was anticipated, the Atomic Energy Exhibition proved to be very popular. As indicative of the appeal which the subject makes to young people, nearly 400 high school students accepted the invitation to attend a lecture in which an appeal was made for volunteers to serve as demonstrators of the individual exhibits. Some 175 volunteered, and from these a selection was made. The enthusiasm of these young people is deserving of the highest praise.

The Physics Section has been re-organized to accomodate its exhibits within a smaller space.

Toward the end of the year a very attractive exhibit room provided by the Lee Rubber and Tire Corporation was opened to the public, in which the story of rubber is told. The sources, distribution, and application of natural and synthetic rubber are graphically shown, while well designed exhibits of the push-button type demonstrate the properties of rubber, and explain its use in the manufacture of an automobile tire.

The issue of the Franklin half-dollar was marked by the arrangement of an exhibit illustrating how coins are made from the original designs. In the preparation of this exhibit we had the cooperation of the U. S. Mint.

Additions:

The policy of carefully screening gifts and loans offered to the Museum, enforced upon us through restrictions upon the exhibit space, has been rigidly followed, but 57 new exhibits were accepted. Among the gifts were: a model of a television studio from the Philco Corporation; an Adams reflex camera from Mr. M. G. Curtis, Bala; a five-stringed violincello from Mrs. Karapetoff, New York; original wind tunnel devices bequeathed by Orville Wright; and a fluorescent lighting exhibit from the General Electric Company. Noteworthy among the loans were: a display of Fiberglas from the Owens-Corning Fiberglas Corporation; a squeeze press from the Tabor Manufacturing Company for the Steel Castings exhibit; an operating television set from the Radio Corporation of America; a Ramage Press from the John Wanamaker Company; an automatic film projector, a primary flight trainer, and other material from the United States Navy, Special Devices Division; a dump truck model from Mr. W. L. Aiken, Ardmore; an iron chest with Franklin associations from Mr. R. Riling; a recognition-type electronic reading machine for the use of the blind from the Veterans Administration; and a signed plaster cast of the Houdon bust of Benjamin Franklin from Mr. A. L. Wolfe, New York.

Special Activities:

The monthly exhibitions of industrial photographs in the Photographic Salon and the showing of industrial films in the Little Theatre was continued with satisfactory results.

A monthly television program over station WPTZ has been conducted by Mr. A. D. Hollingsworth in which museum exhibits have been freely introduced. The fifth series of radio programs prepared with the cooperation of station WFIL and the public, parochial, and private schools of the area opened in October. Mr. Armand Spitz again planned the programs and supervised the script writing. This series of broadcasts, called "Science is Fun," has been selected by the Education Section of the Supreme Headquarters Staff for the Allied Powers for translation into Japanese and broadcast in Japan.

The figures covering the Museum attendance of groups and individual students (included in the totals given above) are as follows:

	1944	1945	1946	1947	1948
Paid	27,225	38,368	64,396	65,527	73,500
Free	33,633	42,675	52,140	57,207	48,816
Total	60,858	81,043	116,536	122,734	122,316

Work has continued with the Schools of Education of the Universtiy of Pennsylvania and of Temple University and with the State Teachers' Colleges. This cooperation has taken the form of lectures and conferences in the colleges and also of informal vistis to the Museum by groups of student teachers. In-service classes, affiliated with the Museum's activities, have been held and the Department of Museum Education has been consulted in connection with curriculum planning.

There has also been continued cooperation with the Philadelphia Board of Education through its Curriculum Office, Visual Aids Department, Adult Education, and other functions. The well established association with the parochial and private schools in the area, as well as with the Boy Scouts, Girl Scouts, and similar organizations has been maintained.

The number of special planetarium demonstrations designed for school pupils has greatly increased. In connection with the work of the Science Council, tests and interviews to recognize young people of outstanding scientific ability have been held, with a view to assisting them in securing employment, admission to college, or attempts to obtain scholarship aids. Preliminary work of the Science Fair has been undertaken and co-sponsors found in the proprietors of the Philadelphia Inquirer.

The normal program of the Fels Planetarium had to be curtailed because of a major repair job undertaken on October 26th to replace the main bearing of the instrument. This repair, together with the re-painting of the dome, and the refinishing of the floor was completed on November 25th. This is the first time to our knowledge that any major repair work to an instrument has been undertaken outside the Zeiss plant. It may be noted that on completion of the work the operation of the instrument was noticeably quieter and more satisfactory. We were fortunate in having the cooperation of SKF's expert engineers in replacing the bearing and in having it presented to us as a gift by the SKF Industries.

In spite of this handicap, 901 planetarium demonstrations were given during the course of the year, attended by 159,401 visitors. As in the past the planetarium demonstrations were of two categories—the conventional demonstration in which fundamental astronomical information is conveyed in non-technical language, and the more spectacular type of demonstration which is intended to arouse unusual attendance and promote interest through the use of special effects and devices.

As in former years extensive extra-mural lecture engagements were filled by members of the Planetarium staff.

Staff:

At the end of the year Dr. Roy K. Marshall resigned from the staff in order to accept a post with the University of North Carolina. Dr. I. M. Levitt, who has been associated with the Fels Planetarium since its inception, was appointed to succeed him as Director of the Fels Planetarium and Associate Director of Astronomy, Seismology, and Electrical Communications in the Museum.

Personnel

On January 1, 1948 there were 469 on our payroll. During the year 168 new employees were taken on our staff and 164 were separated, leaving 473 as of January 1, 1949. There were five deaths in the course of the year: James Brett and Clarence Taylor (both of whom joined the Institute in 1933 before the opening of the Museum), William Brauner (who was killed in an airplane accident while engaged in research work for the U. S. Air Forces), Joseph Berry and Joseph Sepsey.

During the year a study was made of ways and means of giving greater economic security to our employees, particularly as a substitute for our not being subject to the Federal Social Security Act. In line with this, a contributory group insurance plan was authorized by the

Board of Managers and became effective January 1st (on January 31st, at the end of the eligibility period, 88 per cent of those eligible had joined). This insurance plan provided $1000 life insurance for each employee and sickness and accident benefits depending upon the salary classification of the employee.

Hostess Committee

The bi-monthly luncheon meetings of the Hostess Committee have featured this season visits to various sections of the Museum and Laboratories.

In December when Beatrice Fox Griffith's new book *Historic Stars and Comets* was released a special luncheon was tendered Mrs. Griffith. Mrs. Griffith's interest in the Institute was indicated by her desire that all profits from sales of her book at the Institute be turned over to the Hostess Committee for use in connection with Institute activities.

The committee also continued its work on projects begun in the preceding season. In December the newly decorated Girls Recreation Room was made available to the women employees of the Institute. The expressions of appreciation received from the girls and their daily use of this very attractive room indicate how worthwhile this project was. The results of the efforts of the Garden Committee are beginning to show and by summer it is hoped that the appearance of the lot beside the Laboratory Building will be greatly improved.

Research Laboratories

Franklin Institute Laboratories for Research and Development:

Perhaps the most interesting feature of the steady progress made by these Laboratories during 1948 was the increase in industrial work, while the Government work continued at a level only slightly higher than in 1947. The staff, exclusive of shop personnel, part-time employees, and consultants, was composed of 246 members as of December 31, 1948 compared with 260 on December 31, 1947. The three operating Divisions continued to function in their respective specialized fields.

The entire Laboratory Annex, formerly known as the Court Building, was occupied in March, 1948. The additional space thus acquired, together with the installation of a well equipped Electronics Laboratory on the second floor of the Museum Building, brought the total space occupied by the Laboratories to 55,994 sq. ft. The former Electronics Laboratory on the second mezzanine was retained as a stock room and engineers' laboratory, while the space on the third floor previously used for research work was released for a Girls' Recreation Room, referred to more fully in the report of the Hostess Committee. Since the total space occupied as of January 1, 1948 was 49,107 sq. ft., there was an expansion of 6,887 sq. ft. during the year.

New Equipment.—The principal items of new equipment acquired during the year include altitude and cold test chambers; testing ovens with very accurate control; the addition to the former photoelastic equipment in the Halstead Laboratory of a strain indicator and strain recorder for use with electric strain gages and a set of equipment for determining stresses by using brittle lacquers; a complete unit for preparing high polymers, especially GR-S synthetic rubber; a spectrograph of high light sensitivity, using the Image Orthicon to obtain rapid time-resolution in combustion studies; an efficient rubber mill; and a ten-channel infrared spectrograph designed for extremely rapid coverage of the infrared region. The last two items were designed and developed in the course of project work. In addition to these major items, the supply of equipment and test facilities has been enlarged so that a variety of problems in the various fields can be attacked successfully.

Work Performed in 1948.—There has been a gradual inclination toward specialization in certain categories, rather than a tendency to attempt each kind of scientific problem that may be offered for consideration. Some of the specific kinds of work performed during 1948, which are particularly adaptable to meeting the needs of industry, included:

a. Physics of the Solid State: Research on thermocouples, carbon film resistors, photoconductivity, friction and wear, and stress corrosion and fatigue. New techniques with the electron microscope were applied to metallographic and other problems.

b. High Polymer Physics: Properties and preparation of synthetic rubber of improved quality; development of rubbers for low temperature applications; application of ultrasonics to such problems as polishing castings; and research on asphalts, sealing compounds, and abrasives.
c. Radiation and Thermodynamics: Research on gaseous ignition and combustion; thermal properties of gases; and development of sensitive and stable infrared detectors and the application of infrared techniques.
d. Electrical Instrumentation: Basic studies on magnetic recording, techniques, and equipment; the development of an automatic recorder of voltage and current levels for use in electrolytic research; and the construction of a machine for determining the magnetic characteristics of samples.
e. Instrumentation other than Electrical: Development of a resonance method for testing the quality of abrasive wheels; and the study of the physical action of nebulizers used in treating respiratory conditions.
f. Aeronautics: The design of an interlocked air traffic control system.
g. Stress Analysis: By use of electrical strain gages and brittle lacquer, determinations were made of stress in plates of spotwelded joints.
h. Photoelasticity: Studies of stress trajectories and isoclinics in models of spot-welded joints; applications to gear teeth and other mechanical items; and determinations in double glazing units.
i. Sensory Devices: Continued development of sensory devices for the physically handicapped, including a projection magnifier for the semi-blind. A major requirement of this program is for a design which can be produced at a price within the reach of the majority of those afflicted.

It is difficult to convey even the semblance of a picture of the work performed for the Government during 1948 because of its confidential classification. However, some of the accomplishments were:

a. Aeronautics: The development of pressure and speed recorders, special guidance and control systems, and airborne fire control equipment; research in instrumentation for supersonic aircraft; and the establishment of design criteria for aeronautical use. Improved recording accelerometers also were developed. Moreover, the Air Technical Index was established in the large file of aeronautical publications.
b. Electrical Instrumentation: The analysis of control stability requirements, and the design of a controller for a marine turbine installation.
c. Mechanical Designing: Mechanical designers and draftsmen were engaged in designing and developing intricate ordnance mechanisms, and in assisting in the upper air research program.

New Features.—There has been a noted increase in the number of technical papers prepared by members of the staff and published in scientific magazines, many of which were presented in person before regional or national meetings of such organizations as ASME, AIEE, Textile Research Institute, and others. This increase of technical publications is due in part to the new policy of The Franklin Institute to encourage their preparation and to procure reprints of the papers published for distribution to a selected group of approximately five hundred commercial firms, research laboratories, and scientific individuals, crediting the author, the medium of publication, and the Institute. Plans were made, for 1949, to have abstracts of the papers published as a regular monthly feature in the JOURNAL OF THE FRANKLIN INSTITUTE.

Other new features introduced during the year which contribute toward better public relations and/or employee relations include:

a. Members of the technical staff continued to take active part in various technical and scientific organizations by serving as committee members or officers.
b. The Franklin Institute, during the past eight months, has been the headquarters for the Aviation Writers Association.
c. Invitations have been extended to the Army, Navy and Air Forces for the establishment

of research and development groups of reserve officers under the auspices of The Franklin Institute.

d. Establishment of conducted tours of special groups through the Laboratories, including scientific men and women who may be attending special seminars or meetings at The Franklin Institute, such as were arranged for the members of the Economic Mobilization Course in November 1948. Enlargement of this policy was adopted to include the staff and members of special committees of the Institute; a selected group of heads of industrial organizations by invitation; and an "Open House" to be held in the Spring, which may be attended by anyone interested and which should be a means of acquainting the people of Philadelphia with the types of service available.
e. Sponsorship of "The Filter." This paper, published semi-monthly, has proven an effective agent for disseminating information of interest to all Institute personnel.
f. Considerable work was devoted to publishing a new brochure which was ready for distribution in the spring of 1949 and which is expected to serve the purpose of familiarizing industry with the facilities of the Laboratories.

From the above composite report of the Laboratories for the year 1948, it is evident emphasis has been placed on basic research for industrial organizations. In some cases, the Laboratories are actually operating as the research laboratories for small concerns, in addition to performing specialized services for large companies who maintain their own laboratories. Continued opportunity to serve industry at a constantly increasing rate seems assured.

The Bartol Research Foundation:

The Bartol Foundation is engaged in work under the following heads:

1. Researches in nuclear physics and cosmic rays.
2. Researches in matters pertaining to thermionic work.
3. Work having to do with the development and construction of a new Van de Graaff generator for positive ions and for a voltage range between 5 and 10 million volts.
4. Miscellaneous researches sponsored by the Bartol Foundation itself, including the development of a small linear accelerator.

Cosmic ray work has involved researches using a B-29 bomber; researches up to very high altitudes employing radiosonde balloons; and investigations up to 15,000 ft. on Mount Evans.

The present Van de Graaff generator (a two million volt machine) has been functioning admirably during the year and has been used in many researches.

Work on the new Van de Graaff generator has progressed very satisfactorily. A great many of the parts have been completed and are awaiting installation. Also much research concerned with the development of the machine has been carried out. Preliminary plans for the building have been drawn and we are awaiting final decision as to the site. When this is made, plans for mounting the large tank, which is already completed and tested, will immediately be put in operation.

The Biochemical Research Foundation:

The year 1948 was a good year for The Biochemical Research Foundation, one marked by the finding of some new and very interesting leads in the study of cancer and by a closer application of chemical knowledge to the biological problems in hand, with considerable success.

Great value was derived from the closeness of association of the Foundation with the University of Delaware, seven Ph.D. students of which have been working on projects pertinent to the Foundation's research under the direction of Professor W. A. Mosher of the University, the chemical consultant of the Foundation. New assignments are expected to replace the students who graduate, with continuance of the benefit to the Foundation.

The Foundation has found its personnel to be in great demand by other institutions, and has a substantial replacement problem to solve in consequence. Satisfactory replacements have been found, however, and absorbed into an integrated and correlated force now numbering forty-seven persons.

The only major replenishment of equipment has been the replacement of a 19-year old X-ray apparatus, now becoming obsolete, by newly-purchased equipment.

A number of the Foundation's publications have attracted considerable attention, notable among them being the book "Neutron Effects on Animals" which gives promise of selling out its first edition this year. The requests for reprints of some items have been so numerous as to constitute a minor embarrassment from the financial standpoint, and a substantial stock of those most in demand has been assembled to meet this situation.

Public Relations

During 1948 there continued, with some intensification, sustained effort towards promotion of knowledge of the Institute's activities and general functioning among the several publics to whom its varied potentialities might appeal. This was done through the usual publicity channels under the auspices of the Public Relations consultants who have been retained to guide the Institute's program in this field since 1943, with two members of the Institute staff assigned to assist them.

Public Relations counsel was readily available to, and was increasingly sought and used by, the Institute's other agencies in basic planning, in promotion, and in special problems of all kinds.

In addition to the normal meetings and the usual annual events, special promotions of one kind or another were launched as occasion offered, among them being the ceremonies held at the Institute, with the Director of the Mints as guest, to launch the new Benjamin Franklin half-dollar.

Finances

The financial position of The Franklin Institute at December 31, 1948, as compared with December 31, 1947, is indicated in the following comparative summarized statements of assets and liabilities:

Assets	*December 31, 1948*	*December 31, 1947*
Current assets	$ 692,054	$ 662,753
Fixed assets	4,466,598	4,366,250
Less: Reserve for depreciation	(313,943)	(139,137)
Endowment fund investments (including Notes Payable contra)	1,711,285	1,681,215
	$6,555,994	$6,571,081
Liabilities		
Notes payable (included in Endowment funds contra)	$ 470,000	$ 319,000
Liabilities, deferred income, and reserves	219,785	249,195
Endowment funds	1,711,285	1,681,215
Capital stock and contributed funds	3,962,514	3,965,494
General appropriation fund (surplus)	192,410	356,177
	$6,555,994	$6,571,081

The results of operations for the year 1948 as compared with 1947 are shown in the following comparative statements of income and expenses:

	1948	*1947*
Operating Revenue:		
Members dues	$ 66,035	$ 59,092
Museum admissions and other income	93,590	101,410
Other Institute activities	34,616	30,527
Laboratory research projects	1,747,302	1,436,903
Total operating revenue	1,941,543	1,627,932

	1948	1947
Operating Costs and Expenses:		
Museum	$277,815	$279,302
Library, less amounts transferred	51,247	46,749
Other Institute activities	66,992	46,998
Laboratory research projects	1,726,694	1,426,864
Administrative and general expense, less amounts transferred	105,067	115,586
Total operating costs and expenses	2,227,815	1,915,499
Operating Loss	286,272	287,567
Other Income:		
Income from endownment funds	70,314	68,956
Donations from Friends of Franklin	22,150	24,071
Appropriation, Commonwealth of Pennsylvania	34,500	31,583
Service fees from Bartol and Biochemical Foundations	13,453	14,869
Miscellaneous	1,310	1,354
	141,727	140,833
Deficit before other expenses	$ 144,545	$ 146,734
Other Expenses:		
Interest (included in endowment income above)	12,665	4,132
Miscellaneous	8,577	11,386
	21,242	15,518
Net deficit for the period	$ 165,787	$ 162,252

As will be seen from the foregoing statement, the operations for the year 1948, after including in costs a charge for depreciation of buildings and equipment amounting to $174,843, resulted in a deficit of $165,787. The deficit for the preceding year of 1947 was $162,252 after a charge for depreciation of $139,137. The present budget for 1949 indicates a deficit of $80,000 after depreciation of $174,000.

In our report for last year the need of the Institute for unrestricted endowment and gifts was pointed out. It is with gratitude that we acknowledge the gift of $2,000 from Dr. George W. Outerbridge in memory of his father, the late Alexander E. Outerbridge, and the gift from Mrs. Edward G. Budd and Mr. Edward G. Budd, Jr., for the establishment of the annual Edward G. Budd Lecture.

Our financial needs continue. It is hoped that there are those who have faith in the educational and scientific work in which we are engaged who will help us to meet those needs more adequately.

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in the Journal within the next few months:

MINORSKY, N.: Energy Fluctuations in a van der Pol Oscillator.

MARIN, JOSEPH: Stress-Strain Relations in the Plastic Range for Biaxial Stresses.

DEVLIN, JAMES A., WALLACE M. MCNABB AND FRED HAZEL: Preparation of Vanadium Pentoxide Sols by Ion Exchange.

COULSON, THOMAS: The Story of Aids to Navigation.

DUBILIER, WILLIAM: Development, Design and Construction of Electrical Condensers.

FANO, ROBERT M.: Theoretical Limitations on the Broadband Matching of Arbitrary Impedances.

FANO, ROBERT M.: A Note on the Solution of Certain Approximation Problems in Network Synthesis.

JACOBSEN, LYDIK, R. L. EVALDSON AND R. S. AYRE: Response of an Elastically Non-Linear System to Transient Disturbances.

LIBRARY

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. The average cost for a print 9 × 14 inches is thirty-five cents.

The Library and reading room are open on Mondays, Tuesdays, Wednesdays and Fridays from 9 A.M. until 5 P.M.; Thursdays from 2 P.M. until 10 P.M.; and Saturdays from 9 A.M until noon.

RECENT ADDITIONS

ARCHITECTURE AND BUILDING

MAGNEL, GUSTAVE. Prestressed Concrete. 1948.

ASTRONOMY

ROSSELAND, SVEIN. The Pulsation Theory of Variable Stars. 1949.

CHEMISTRY AND CHEMICAL TECHNOLOGY

Advances in Catalysis and Related Subjects. Volume 1. 1948.

BURK, ROBERT EMMETT AND GRUMMITT, OLIVER. High Molecular Weight Organic Compounds. 1949.

BURK, ROBERT EMMETT AND GRUMMITT, OLIVER. Recent Advances in Analytical Chemistry. 1949.

THE CHEMICAL CORPS ASSOCIATION. The Chemical Warfare Service in World War II. 1948.

DEWAR, M. J. S. The Electronic Theory of Organic Chemistry. 1949.

GROSS, MARTIN AND GREENBERG, LEON A. The Salicylates; a Critical Bibliographic Review. 1948.

PANNELL, ERNST V. Magnesium. 1948.

SMITH, HENRY MONMOUTH. Torchbearers of Chemistry. 1949.

WEISER, HARRY BOYER. A Textbook of Colloid Chemistry. Ed. 2. 1949.

CIVIL ENGINEERING

KNIGHT, BERNARD H. Soil Mechanics for Civil Engineers. 1948.

ELECTRICITY AND ELECTRICAL ENGINEERING

BRIGHT, ARTHUR A. The Electric Lamp Industry. 1949.

HARNWELL, GAYLORD P. Principles of Electricity and Electromagnetism. Ed. 2. 1949.

MORROW, L. W. W. Electric Power Stations. 1927.

ELECTRONICS

ZWORYKIN, VLADIMIR KOSMA AND RAMBERG, EDWARD G. Photoelectricity and its Applications. 1949.

ENGINEERING

ALDEN, JOHN L. Design of Industrial Exhaust Systems. 1948.

DIETZ, ALBERT GEORGE HENRY. Engineering Laminates. 1949.

GODDARD, ROBERT H. Rocket Development; Liquid Fuel Rocket Research. 1948.

MORLEY, ARTHUR. Theory of Structures. Ed. 5. 1948.

O'NEILL, JOHN JOSEPH. Engineering the New Age. 1949.

FOOD

SHERMAN, HENRY CLAPP. Food Products. Ed. 4. 1948.

MANUFACTURE

GUYÉ, RENE P. AND BOSSART, MAX. Horologerie Électrique. 1948.
International Industry Yearbook. 1948.

MATHEMATICS

COLLATZ, LOTHAR. Eigenwertprobleme und ihre Numerische Behandlung. 1948.
FISHER, RONALD AYLMER. The Design of Experiments. 1949.
TARSKI, ALFRED. Cardinal Algebras. 1949.

MECHANICAL ENGINEERING

ANDERSON, J. W. Diesel Engines. Ed. 2. 1949.
HØEGH, CARL. The Cylinder Wear in Diesel Engines. 1949.

METALLURGY

BURKE, J. E. ET AL. Grain Control in Industrial Metallurgy. 1949.
STANLEY, JAMES K. Metallurgy and Magnetism. 1949.

PHARMACY AND PUBLIC HEALTH

Modern Drug Encyclopedia and Therapeutic Index. Ed. 4. 1949.

PHYSICS

CARLIN, BENSON. Ultrasonics. 1949.
EYRING, CARL FERDINAND. Essentials of Physics. 1948.
FERRI, ANTONIO. Elements of Aerodynamics of Supersonic Flows. 1949.
MILNE, EDWARD ARTHUR. Kinematic Relativity. 1948.
YAGODA, HERMAN. Radioactive Measurements with Nuclear Emulsions. 1949.

SANITARY ENGINEERING

BABBITT, HAROLD EATON AND DOLAND, JAMES J. Water Supply Engineering. Ed. 4. 1949.
SOUTHGATE, B. A. Treatment and Disposal of Industrial Waste Waters. 1948.

SCIENCE

STIMSON, DOROTHY. Scientists and Amateurs, a History of the Royal Society. 1948.
SULLIVAN, J. W. N. The Limitations of Science. 1949.

MUSEUM

A MASTERPIECE RESTORED

When The Franklin Institute moved into the present building generous donors offered exhibits to furnish it. Among the gifts was a blackened mass of cams, gears, and levers which had obviously passed through a destructive fire. The donor could give little information concerning the fragments other than that they had formed an automaton capable of writing and drawing. Only a few incomplete pictures had survived the fire but it was clear from these that the automaton or android was of an unusual nature. One of our mechanics, the late Charles Roberts, undertook to rebuild the mechanism. After several months of patient labor he succeeded in reconstructing the mechanism, and the staff of the Museum were aware that they possessed a masterpiece of the mechanic's art. It was attributed by its donor to Robert Mältzel, inventor of the metronome.

Then one day a reader in the Library chanced upon an account in the *Edinburgh Encyclopaedia*, published in 1812, of an android made in London by Henri Maillardet, manager of the famous Jaquet-Droz firm of watchmakers. The only difference was that the early description was of a kneeling boy writing, whereas The Franklin Institute figure represented a

seated lady. However, a member of the staff submitted the writings and drawings, seven in number, to a very careful scrutiny, with the result that what had formerly been regarded as decorative detail on one drawing proved to be the statement, written in French, that the drawing was "done by Maillardet's automaton."

It was known that Maillardet's writer and draughtsman had passed through the hands of several well known showmen in the nineteenth century. It had last been reported as being in the Peiping Museum. However, further research here and in Switzerland has served to prove that The Franklin Institute automaton is Maillardet's masterpiece.

The principal part of any writing or drawing mechanism is that which initiates and controls the movements of the hand. The other bodily movements that contribute to realism can be simulated without much difficulty, for they need not be accurate. But the motions of writing or drawing must be very closely controlled if the work produced is to be readable or understandable. It is also necessary that the control be carried out in three dimensions. since the machine must make a two-dimensional drawing and also lift the pen off the paper, Thus three directional components must be provided to move the writing arm of the automaton.

By far the most convenient method of satisfying these requirements is to use a set of three metal disc cams, which Maillardet adopted. The arm joints have the same play as those of the human arm, except that the wrist does not bend. There were no electric motors available when Maillardet built his automaton 150 years ago, but he constructed spring motors to operate his mechanism smoothly. The "brain" of the automaton is three groups of cams. Two groups produce horizontal and vertical motion on the drawing, while the third group takes the pen on and off the paper. Each disc in a group has a mate in each of the other groups with which it operates simultaneously. Indexing the cams is done by a large drum fitted with a single helical rib. The drum is rotated by the spring motor, and on its end are engraved the titles of the seven drawings and verses, so that the operator can set the drum for any desired drawing. Each drawing requires several sets of cams for its execution, and at the end of each revolution the cams stop with the followers resting on transverse bars. They remain undisturbed as the indexing drum automatically slides the next set of cams into alignment, so that the work can continue. The time spent in this operation is utilized to advantage, for the doll raises her pen, straightens up, and looks critically at the paper, as if deciding what to draw or write next.

The Jaquet-Droz automata in the Neuchatel Museum in Switzerland do not possess such an extensive repertoire as does the Maillardet automaton but their work is more delicate. However, the very delicacy of the work does not permit frequency of operation. In order to witness them in operation, the visitor must time his visit to the Museum for a Sunday. The Maillardet writer and draughtsman operates cheerfully every day and produces an average of 10,000 drawings and verses during the course of the year.

THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

Abstract of **Combustion Studies Using the Golay Photothermal Detector with an Infrared Monochromator.**[1]—J. T. AGNEW.[2] At the Franklin Institute Research and Development Laboratories, there is a research project in which certain techniques of infrared detection and spectroscopy have been developed.

In recent years, it has become obvious in combustion research that new methods must be developed in order to achieve an insight into the reaction kinetics of combustion processes. Two types of information needed are temperature, and composition of the reactants as a

[1] Contributed by the Heat Transfer Division and presented at the Semi-Annual Meeting, Milwaukee, Wis., May 30–June 5, 1948, of The American Society of Mechanical Engineers. Published in *Transactions of the ASME* for February, 1949.

[2] Research Engineer, The Franklin Institute Laboratories for Research and Development, Philadelphia 3. Penna.

function of time. Knowledge of these two parameters, together with pressure, would allow a more accurate determination of the actual conditions existing at a given instant in a combustion process, such as the explosion in the cylinder of an internal-combustion engine. A better and more thorough understanding of the actual conditions existing throughout the combustion process ultimately would lead to a more efficient conversion of the potential energy, stored in the fuel, to useful energy.

An experimental approach to the investigation of the parameters mentioned is the determination, as a function of time, of the spectral distribution of the radiant energy given off during the combustion. It would be desirable to determine this spectral distribution over as large a portion of the spectrum as possible, since many of the radiating components have an identifying band structure in various regions of the spectrum. A rather large region of the spectrum is covered by infrared detectors. Some of these detectors are even sensitive throughout the visible spectrum and partially into the ultraviolet region.

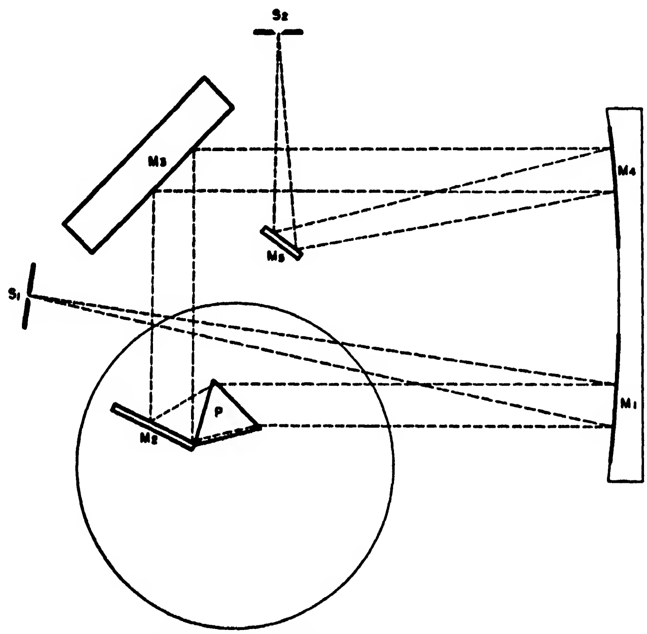

FIG. 1. Schematic diagram of monochromator.

Another factor in favor of using infrared detectors in the initial phase of radiant-energy investigations is that, in the range of temperatures expected to be encountered, perhaps 530 deg. K to 4000 deg. K, the peak radiant intensity from a black radiator, operating in this same temperature range, lies in the infrared region of the spectrum from approximately 0.72 to 5.4 microns. Finally, it is known that many polyatomic molecules have characteristic radiation bands in this same region of the spectrum, for example, the water vapor band at 2.7 microns and the carbon dioxide band at 4.4 microns.

The instrumentation for the first preliminary combustion studies consisted of: a combustion bomb equipped for spark ignition, an infrared monochromator which utilized a Golay photothermal detector as the sensitive element, an oscilloscope for presenting the information from the detector, and a 35-mm. camera for recording it.

The method used for operating the photothermal detector is as follows: A small volume of gas is contained in a chamber, either end of which is closed by a thin collodion membrane. A thin film of aluminum is evaporated on one membrane and a thin film of antimony on the other. The aluminum film acts as an absorber of radiant energy with the result that the enclosed gas is heated slightly and expands, causing a dilation of the antimony film. This film has the prop-

erties of an optical mirror and, in conjunction with a meniscus lens, causes the image of half of an illuminated lined grid to be focused in the plane of the other half, thus producing a light-modulating system. The light modulation caused by the dilating antimony mirror is detected by a light-sensitive photoconductive cell and amplified.

A detector of this type was used in conjunction with a special low-dispersion monochromator, the schematic diagram of which is shown in Fig. 1. The two small sections of the same large parabolic mirror, used, respectively, for collimating M_1, and focusing M_4, are essentially equivalent to two small off-axis parabolic mirrors. The radiation enters slit S_1 and strikes parabolic mirror M_1. From there parallel rays travel to the 60-deg. prism of sodium chloride. The prism and mirror M_2 are arranged according to the Wadsworth type of mounting, wherein the rays that pass through the prism at the angle of minimum deviation are reflected from plane mirror M_2, at an angle to the rays incident to the prism-mirror combination of twice the angle θ between the perpendicular to the prism base and a line parallel to the face of M_2.

Hence, for this monochromator, the total deviation of the rays which pass through the prism at minimum deviation is 90 deg. From M_2, parallel rays strike plane mirror M_3 and are again deviated through 90 deg. to mirror M_4, which would ordinarily focus them at S_1, but plane mirror M_5 is interposed in their path to deflect the rays through slit S_2 which is mounted on the pneumatic detector.

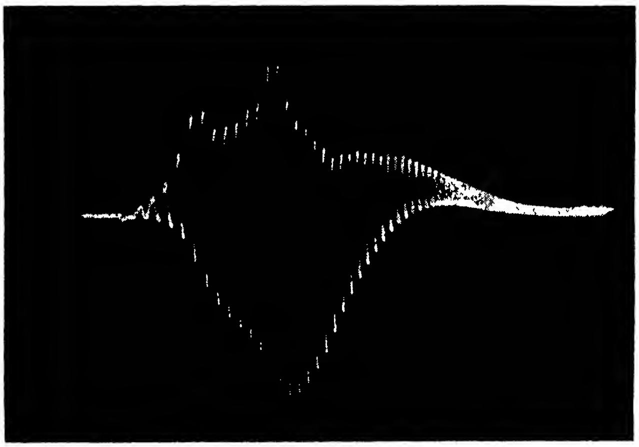

FIG. 2. Typical oscillograph.

The electronic signal obtained from the light-sensitive cell of the detector is amplified and fed to an oscilloscope and the tracing on the screen photographed.

The use of this low-dispersion monochromator with the pneumatic detector permits the determination of the spectral distribution of phenomena existing for very short intervals of time (of the order of a few milliseconds), or the determination of the spectral distribution, as a function of time, for phenomena existing for longer periods of time. A study of the latter type has been carried out and will be described.

A methane-air mixture containing approximately 8 per cent methane was ignited by means of a spark in the bomb. The combustion chamber was approximately 12 in. long and 6 in. in diameter.

After the bomb had been filled with the combustible mixture, the shutter was set in motion to intercept, at intervals, the radiation emitted from the burning gas through the sodium chloride window. The monochromator was adjusted to the desired wavelength and the spark ignited. The sweep circuit of the oscilloscope was set in motion simultaneously, and a short time-delay relay allowed the sweep to travel a short distance before the spark was ignited. With the ignition of the spark, the combustion process began, and the energy emitted from the reaction at a given wavelength, as chosen by the monochromator, was sent to the gas cell

of the pneumatic detector. The entire process was repeated for a series of wavelengths, 65 different ones being taken in the spectral region from 1 micron to 11 microns.

A typical oscilloscope trace is shown in Fig. 2. This trace was obtained by a simple time exposure during the combustion. The trace, as shown in Fig. 2, traveled from left to right, the first downward streak from the left being caused by the ignition of the spark. The maxima and minima of the trace tended to equalize around a centralized base line which itself tended to shift, depending upon the strength of the signal. The actual intensity, at a given time, was obtained by measuring the distance vertically downward from a maximum to the line joining the two minima on either side of the maximum being measured. It can readily be seen in Fig. 2 how this distance builds up and decreases with time.

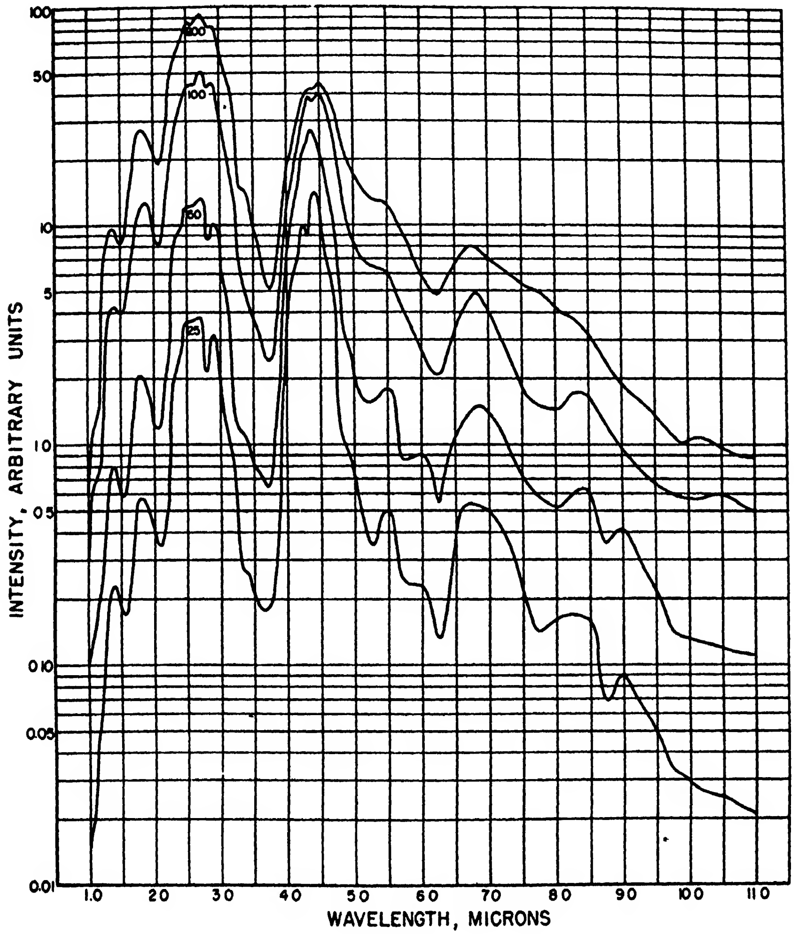

FIG. 3.

Figures 3 and 4 represent spectral intensity as a function of time for the radiation emitted during the combustion process. In Fig. 3, progressing upward, the successive curves are for times of 25, 50, 100, and 200 msec. The curves in Fig. 4 are, progressing downward, for times of 300, 400, and 500 msec. The curves are drawn with the intensity ordinate on a logarithmic scale. This was necessary because of the large range of intensities covered. The curves as presented are not corrected for the overlapping effect, due to the use of a fairly wide slit of 380 microns.

The complete analysis of the curves will be presented in a later paper. However, the

following remarks apply to the curves as presented: At 25 msec. there are well-defined emission peaks at 1.42 and 1.80 microns, two broad peaks covering the range 2.4–2.9 microns and 4.1–4.6 microns, and other peaks at 5.5, 6.0, and 6.75 microns. There are irregularities, almost completely masked, in the range 3.2–3.7 microns. Methane is known to have a band in this region so that the minimum in the vicinity of 3.3 microns in the early stages of the combustion may be absorption by unburned methane. The development of these peaks can be followed with time. The rather complicated structure of the 2.4–2.9 micron and 4.1–4.6 micron bands is probably somewhat due to self-absorption. The OH radical is known to have a fundamental frequency corresponding to a wavelength of 2.8 microns. A well-defined minimum at 2.8

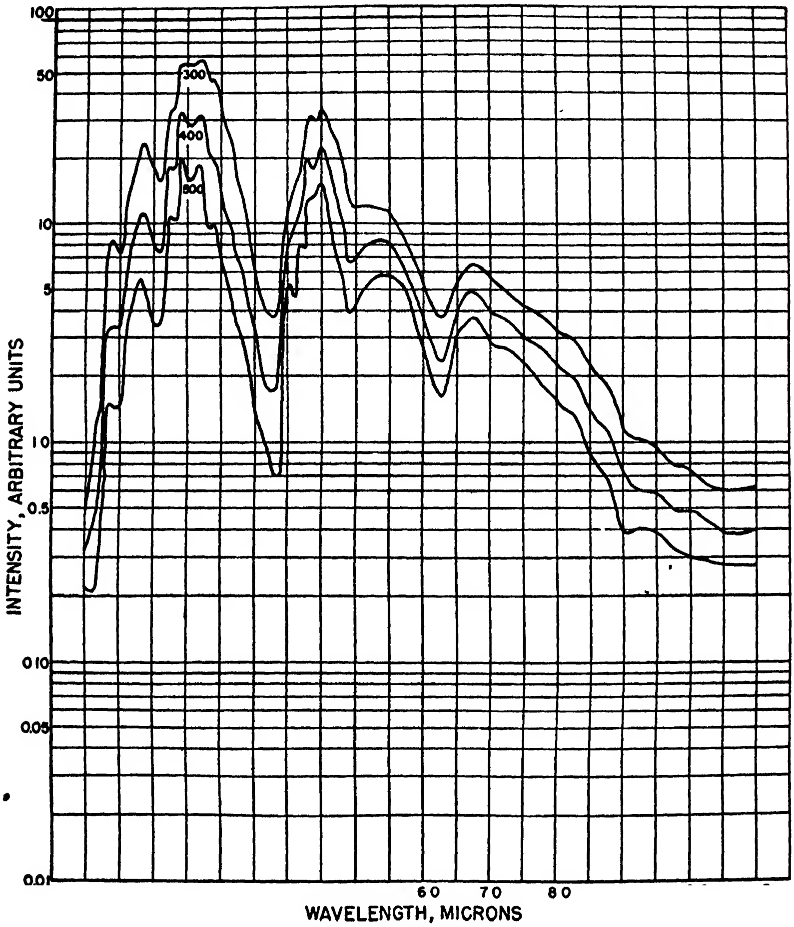

FIG. 4.

microns can be seen at 25 msec. This minimum grows less pronounced up to a time of 200 msec., but becomes slightly more pronounced at later times. The complete time analysis of the spectral distribution could not be satisfactorily reproduced in Figs. 3 and 4. Only those curves corresponding to the times indicated could be presented.

Further work will be carried out, of a more precise nature, with a large-aperture infrared spectrograph, utilizing a bank of ten photothermal detectors. This unit will be capable of determining the spectral intensity, as a function of time, simultaneously at ten different wavelengths. Further experiments will be directed toward a true determination of the temperature as a function of time, the incorporation of a pressure gage to determine pressure as a function of time, the effect of type and exposed area of surface, and other effects.

COMMITTEE ON SCIENCE AND THE ARTS

(*Abstract of Proceedings of Stated Meeting held Wednesday, June 8, 1949.*)

HALL OF THE COMMITTEE,
PHILADELPHIA, JUNE 8, 1949.

MR. W. LAURENCE LEPAGE in the Chair.

The following reports were presented for final action:

No. 3198: Work of Edgar C. Bain.

This report recommended the award of a John Price Wetherill Medal to Edgar Collins Bain, of Pittsburgh, Pennsylvania, "In recognition of his pioneer investigations into the structure of metallic alloys and particularly for his original work in the application of the isothermal method of studying the rates of transformation of austenite."

No. 3203: Work of William H. Millspaugh.

This report recommended the award of an Edward Longstreth Medal to William Hulse Millspaugh, of Sandusky, Ohio, "In consideration of his outstanding contributions to the art of paper making and to his introduction and promotion of the art of the centrifugal casting of rolls for Fourdrinier machines and other purposes."

No. 3208: Henderson Medal.

This report recommended the award of the George R. Henderson Medal to John Van Buren Duer, of Fairfield, Connecticut, "In consideration of his judgment and guidance in the coordination of engineering matters, including full technical responsibility in the electrification of the Pennsylvania Railroad, and in recognition of his contributions and resourcefulness in research developments in over-all railroad problems."

JOHN FRAZER,
Secretary to Committee.

MEMBERSHIP

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS, JUNE 15, 1949

ACTIVE FAMILY

Donald A. Bunce
Hugh R. Radbill
Richard T. Smith, M.D.
Harold A. Zintel, M.D.

ACTIVE

Philip M. Andress
Russell E. Benner
Henry S. Bourland, M.D.
Edward Brautigam, Jr.
Ralph T. K. Cornwell
Robert J. Donaldson
Raymond M. Fine, M.D.
John M. Foster
Rudolf Friesinger
Harris S. Gerber, M.D.
Reginald N. Gonzalez
Frederic O. Hess
W. Meredith Heyl, M.D.
Stewart Huston
Julian Johnson
Henry J. Karlavage
Paul Karnow
Rudolph M. Lombardi
Hugh B. McCauley, M.D.
Frank T. McGinnis, M.D.
Grace R. Nachod, M.D.
Henry N. Renton
Margaret N. Robins
Henry P. Royster, M.D.
Robert P. Schoenijahn
J. Shepard Smith, M.D.
James B. Taylor

ACTIVE NON-RESIDENT

James C. Cox, Jr.
Edgar D. Sibley

NECROLOGY

Walter I. Cooper '38
Benjamin Rice Faunce '46
George W. Mink, Jr. '42
Edward P. Simon '36
Caroline S. Sinkler '36

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

Choice of Workers in Research.—ELLICE McDONALD, Director. No matter how much research is organized, the unit is the worker. Houses are arranged habitations but the unit is the building block. Without careful choice of the unit no research can be successful. After all, it is the men who do the work.

There is no royal road to success in choosing men. Part of it is experience—in life as well as in science, but a good bit of it is a gift, a quality of estimating men and their capacity in a short time and a wise director is one who pays particular attention to the choice of his personnel. The power of organization is great and the effect of a few may be multiplied almost indefinitely and what one individual can accomplish may not be great, but if the units are not carefully chosen, the aggregation into an organization will suffer.

Of course, the choice is limited by the type of research organization. In an industrial research, there must be profits to keep the show going: in some, there are more immediate demands for profits from research than others. Some are so short-sighted as to require research to produce results as required—"Put in the thumb and pull out a plum." This is as unsatisfactory as is the long time research in universities which often continues on and on without a definite aim but goes by continuation and often becomes merely a multiplication of detail.

At one end is technology and at the other is abstract science. One looks upon the other as academic and the other scorns the one as merely practical. The classification of science as organized knowledge is confusing because it suggests an entirely artificial separation between pure science and technology. Civilization began in technology and pure science arose when technology made possible the leisure necessary for its pursuit: but today the applied scientist uses the same strict criteria of accuracy and logical evaluation as does the pure scientist. Actually there are all graduations and no definite division except at one end is the shop laboratory to control the quality of product and at the other end is the cloistered pure scientist remote from the world who may or may not produce a valuable result.

Should all results be valuable? Practical necessity may not be required for scientific advance and many inventions of apparatus devised for scientific advance have been subsequently transferred to practical use. As examples of such invention, Haldane cites the telescope, the cinematograph, barometer, gasometer, galvanometer and cream separator. Most of these have to do with measurement and are really of the nature of scientific aids to extend man's senses and powers

of observation. But after all, most of us are inclined to what Benjamin Franklin called "useful knowledge." Still it is a fallacy to suppose that scientific results come only in response to material needs.

In general, there is some futility in planned research toward a definite end: but there is more futility in research without planning. The great mass of academic research is of that character. This does not mean that the first is technology or applied science and the second pure science or fundamental science so-called. (The purity claimed by some scientists reminds one of Mark Twain's statement, made once with his inimitable drawl, "To the pure, a-all things are i-impure.")

Still in research, as in any other modern phase of effort, it is better to start with a plan and that plan must influence the choice of workers who must carry it out. The field of scientific effort is so great that it takes more than one mind to encompass it. So comes specialization.

When the scope of the research is known and the areas of science well defined, then begins the scrutiny of those who must carry it out. The ideal is the exceptional man but is he to be recognized? The celebrated German chemist, Wilhelm Ostwald, was once asked by a Japanese student of his, "How are you likely to predict what members of a class are likely to distinguish themselves in science?" (It was asked because the Japanese Government sent several hundred young men annually to foreign universities and wanted to get the largest possible return for the financial and human investment.) To clarify his own mind, Ostwald wrote his "Groesse Maenner" in which he examined the great in science, only to reach the conclusion that, not only do we fail to recognize budding geniuses, but that we and our educational institutions do our best to treat them as white crows.

While it is difficult to pick genius in the bud, that is no reason why there should not be some careful screening in the choice of young research workers. The first requirement is that they should want to work. A widow, once in the civil war, brought her son to President Lincoln, looking for a job for him and saying that he wanted to work. Lincoln sent him with a note to Stanton, Secretary of War, "This young man says he wants to work. This is so rare a desire that nothing should be put in its way."

The research worker must be a *worker*. Every student should realize that enduring results are only obtainable through a series of small but continuous daily efforts. Knowledge, whether in science or art or in any human occupation, can only be achieved by daily toil and effort. Time is very vindictive and takes a cruel revenge on everything accomplished without its cooperation. The motto in the library of the Biochemical Research Foundation is "There is no substitute for time."

There must be consideration whereby the man must be driven to act so as to offset Epstean's law that man tends always to satisfy his needs and desires with the least possible exertion. This may be a tendency

in general, but not in particular. But no doubt there must be some compelling impulse. Thomas Alva Edison's wall motto must have been of the nature of a continual spur and reminder to him, "There is no expedient to which a man will not go to avoid the real labor of thinking."

In research, a man must be imbued with the ardor of science. He must have a consuming desire to find out about things and to make them work better. As Willstaetter has said, "Our destiny is not to create but to reveal."

Through many years the question has been asked as to what impels a scientific research worker upon his continued course. Some have answered that insatiable curiosity is the answer; others that man is a born explorer. The spirit of adventure has existed within him since time began. He has ever been trying to extend the frontier, to go over the next hill to see the vision, to find something he did not know and understand.

The adventurous spirit of man (and research is a high adventure) with a burning desire to know things will drive him forward in research now that other frontiers are conquered. His weapons are stronger equipment, new instruments to extend his senses and measurements to a degree not considered possible a short while ago.

Some have said that the research man works for reputation or acclaim. This may undoubtedly enter into it, but it is only a part. The desire for accomplishment and the sense of achievement mean more to the real scientist. An inward feeling of work well done weighs heavier than the gold of the medals. The greater one's accomplishments, the more intensely the smallness of one's ability is realized. It is through this feeling of inadequacy and insignificance that knowledge and achievements can continually progress.

The real research worker, however his impulse may be described, is driven by an inward force toward an unseen goal and he does not understand this impulse himself. It is more than insatiable curiosity: it is more than the exploration urge: it is best described in the words of Balzac, when speaking of a medical friend and a painter friend, in their desires for discovery and accomplishment. "These passions which are unintelligible to the crowds are perfectly explained by that thirst for the ideal which characterizes creative minds." The creative ideal may influence many types of effort other than research, but it is, without doubt, the compelling impulse in research accomplishment, dimly as it may be seen by those chiefly involved.

The main thing in choice of a research worker is that he wants to work. If he has had some experience in work so much the better, like those who have worked their way through college or have been brought up in a family with the atmosphere of work. But above all, beware of the gifted amateur, the lily-finger boy who plays at research. As Kipling said to a graduating class of Middlesex Hospital Medical School,

"You have been and always will be exposed to the gifted amateur, the gentleman who knows by intuition everything that it has taken you years to learn." To do research, no job should be considered too small or beneath contempt. Dr. William Osler once said to some of his students, "No one can be expected to be a good doctor who is afraid to get his hands in feces."

The real research worker has a different mode of thought from what might be called normal or usual. Many years ago, there was a famous neurologist and psychiatrist, for he was both, named Charcot, who worked at the Salpetriere in Paris. He had three great pupils, Freud, Jung and Janet. The first two returned to their own country and developed systems of psychiatry suitable to their own people. Pierre Janet remained in France and developed with French logic a system based on Charcot's original findings. In general, Janet divided modes of thought into two deviations from the normal or customary.

He called these "hysteric" and "obsessional." The phrase "hysteric" has little to do with hysteria, the disease, and the phrase "obsessional" has little to do with the obsessions of insanity. The hysteric mode of thought is one which is easily influenced and easily diverted, all things to all men, suggestible and going on to the easily hypnotized or the "somnambule." The obsessionist is the one with the fixed idea, resolution in the face of difficulties and even in the face of reason. A typical example of the obsessionist is the Scottish Calvinistic "meenister" who pursues one idea, as did the late President Wilson. It is no wonder that he could not understand Senator Borah who was the diverse type.

Hervey Allen in a recent book has given a good description of an obsessionist with his qualities and the defects of his qualities: "Most people are diffuse or diverse in their interests, fortunately. What in the end attracts permanent attention and fixes a reputation in memory is a man with a single end in view, one which he pursues relentlessly. This lends a precision, and eventually an economy to his actions, in comparison with which the diffuse activities of more normal contemporaries seem trivial and merely time-passing. That a certain insane tenseness frequently accompanies the man of one purpose seems only to cast a more focused and strange light upon his doings. It accentuates him by contrast. And if, as frequently happens, the man with one idea has no fear of consequences in the attainment of his object, a 'hero' is in the making. Such a man can be completely rational about one thing alone. But he can often be superrational about that, and to be superrational is a characteristic of genius."

It may be that the research worker is not so extreme, but the good one is tarred with this brush. He must pursue his aim relentlessly although that aim may be changed with changing conditions, that is, with greater knowledge and vision the result of his own experiments

or those of others. He must think about his work at all times and this will remain in his subconscious mind where prevision and the lead to discoveries are often made.

An example is of August Kekulé whose name should be honored forever for his method of writing down the structural formulae of organic compounds so that anyone with chemical knowledge can see how the atoms of a molecule are linked and can infer many of the properties of the substance. His inspiration gave him foresight or prevision. He had been talking chemistry with a friend and rode home on the top of a bus. "I sank," he said, "into a reverie. The atoms flitted about my eyes. I had seen them in movement, these little beings, but I had never succeeded in interpreting the manner of their movement. That day I saw how two small ones often joined into a pair: how a larger one took hold of two smaller, and a still larger clasped three or even four of the small ones and how all span round in a whirling round dance. I saw how the larger ones formed a row and only at the end of the chain smaller ones trailed along. The cry of the conductor 'Chapham Road' woke me from my reverie, but I occupied part of the night in putting at least sketches of these dream-products on paper. Thus originated the structure theory."

The idea of ring structure of benzene revolutionized organic chemistry and Kekulé, during his residence at Ghent, while he was writing his textbook, describes his prevision. "But it did not go well: my spirit was with other things. I turned in my chair to the fireplace and sank into a half-sleep. Again the atoms flitted before my eyes. Long rows, variously, more closely united: all in movement, wriggling and turning like snakes. And, see, what was that? One of the snakes seized his own tail and the image whirled scornfully before my eyes. As though from a flash of lightning I awoke; this time again I occupied the rest of the night in working out the consequence of this hypothesis.

"Let us learn to dream, gentlemen, then perhaps we will find the truth";—but almost immediately he gave a warning against the publication of dreams until they had been put to the test of waking intelligence and experimentation.

This may seem an extreme case but the Bible says "If there be no vision, the people perish." Still even so matter-of-fact a man as Lord Kelvin has said, "I think about the problem all the time and I gather all the information I can get about it and then, when I am perfectly saturated with the subject and think about it all the time, at some point my mind takes a mortal leap past all the facts." Call this reverie, prevision or what you will, but it is the research imagination which is necessary to everyone in science. More than the daily absorption in the task is required. It requires the projection of one's mind beyond the facts in that intellectual and imaginative realm which is called the research spirit. Occupation and routine are useless alone. Constant

and persistent efforts do not alone suffice, but there must be the divine curiosity to search out new facts in regard to life and the environment in which life exists.

Innumerable trifles collected in an orderly mass although contemptible individually may, by vision and imagination, contribute largely to knowledge. Research requires more than patience, dexterity and exactness, necessary as these are, now since the present criteria are more quantitative than qualitative. An army of excellent but very ordinary people are at present carrying on research into the *minutiae* of unimportance, but these are required to prepare for the exceptional man by whom real advance comes. Where would the world be without such exceptional men as Pasteur, Newton, Franklin, Faraday and other great men whose vision illumines the whole subject so that specialization could take it up and further elaborate it. Advance comes through the exceptional man.

The first condition in an applicant for a research position is that he should be reasonably young. Balzac has said "Integrity and brains in a man under 30 are commodities which can be mortgaged. After that age there is no counting on a man." If he has not begun to find his niche by that time, it will be difficult for him to do it later. In general, research men choose themselves because they seek places where they can follow their innate and overwhelming curiosity with regard to the structure and processes of the world.

Not all men have the research capacity, eager as some of them may be to do this character of work. Desire to do research in a person who is unable to do it is a pathetic thing. The eagerness outstrips the capacity. The accomplishment often fails the desire.

The longevity of the research desire is only to be maintained by a constant struggle against the diffusion of the specialization of its energy. If the struggle is not made, the research desire succumbs after a brief flowering into the common processes of diffused thought. Other people unconsciously fight a man's concentration whenever it diminishes his use to them. Inside of himself, it is fought by the indolence of partial achievement and the incidence of new and extraneous interests which produce a diffusion of energy. Resting upon laurels, enjoying the rewards of partial achievement are fatal to research effort. Motive must overcome the scattering of life's energy into the little parts of less and less usable forms. Once the research desire, the intense curiosity as to the arrangement and order of existing things, is lost, it is rarely recovered. Nothing is more pathetic than the burnt out scientist. The disillusionment of a goal attained should never affect the research worker. There are always new worlds to conquer.

For this reason, the research desire should be sedulously cultivated. The scientist must continue to be prodigiously interested. Research

is strenuous because it involves no routine. Each day is a new opportunity and perhaps a new miracle. For this reason, it is often fatiguing.

My experience in research as such leads me to believe that the lone worker with research ideas has a span of about 3 years in which his ideas are prolific and vital. At the end of that time, it becomes the elaboration of past detail and the continuance of repetition in *minutiae*. Not all fall into this exact category but, in general, I believe this to be true. In our laboratories, immediately a man comes to us we attempt to enlarge his vision and extend his horizon, by interesting him in other phases of work, in other sciences, in other methods of attack upon scientific problems. This prevents the dampening of the stimulation to which the lone worker is subject. This reeducation, so to speak, is not a conscious thing, but the man is caught up into the spirit of the diffusion of knowledge which is present in our laboratory and it is pleasant often to see the flowering of his vision. In research, there is an imaginative impulse and the greatest research worker is he whose intuitions most nearly happen.

The quality which makes a man a real research worker is an independent attribute or in Mendelian parlance a "unit character." This quality is innate or inborn and, if it is not there, no amount of training will supply the lack. Achievement of a high sort is always a combination of several fundamental faculties in no matter what phase of human endeavor. There are differences in degree of attainment and the number with exceptional achievement is small. Take such a phase of human effort as fast-running. It does not require as many fundamentals for outstanding success as hurdling or broad-jumping. The first must be a sprinter, but also must be able to jump; the broad-jumper must sprint to gather momentum and in order to increase that momentum, he must get a forward push into it. These extra faculties are required besides sprinting so that there are greater differences in quality in them than in sprinters where the effort is simpler. So in research which is more complicated as to faculties, because of its very complexity, there are few great. Indeed, the proportion of research workers in general is very small in relation to the total population.

It may be possible some time that research ability may be foreseen and plotted as are certain of the physical characteristics at present. The distribution of many simple characteristics of human capacity follows what is known as the normal probability curve. The wide range over which this normal distribution is found indicates that the law applies to practically all types of inequality, not only for physical characteristics, such as height, weight and head size, but also for mental characteristics as memory, speed and accuracy. Talent of every type and especially of the higher grades is native and cannot be acquired by training or education, but if it is there, it may be developed.

It may seem far-fetched that the normal or probability curve which

is shaped like a World War II helmet with a hump in the middle and a skewness at each side, should be applied to anything as abstrusely intellectual as research capacity, but if it were possible to measure the fundamental faculties necessary to any type of undertaking, then it would be possible to predict the pattern of distribution, the poor, the the average, the exceptional which would be the marked skewness to the right and so a numerical gradation would be obtained.

What qualities are to be sought for? The desire-to-work motive bulks large: perseverance, resolution, and patience are requisites. Good health is a help to realize the enduring results, obtainable only through a series of small but continuous daily efforts. Knowledge, whether in science, art or in any human occupation, can only be achieved by daily toil and effort.

The amount of previous training and the character of the work the man has done before is important. If there are any publications, these will throw great light, but these should be considered as a background of earlier work, probably immature, and should be judged as to the possibility of improvement in future work.

The applicant's master is of great importance and this implies the atmosphere from which he has come. There have been scientists, all of whose pupils seem to have been imbued with their spirit and to have received part of their capacity. The late Emil Fischer was apparently one of these; every one of his pupils seems to speak of him in the same term, "the great master," and his pupils throughout the world do him credit. The character of the previous work should be weighed. Is he an observationist or an experimenter? Has he been working alone or in a group? And if in a group, what has been his role in it?

Group cooperation and coordination are so important in present-day research that it is well to get some idea of how a new man will function in such a group. For some, it seems impossible to give and take, which is required in group research. Often a new man must be educated in this so that he finds that by taking his part in a group of workers, exchanging his ideas with others, he is not depreciated, but gains from the ideas of others and of other branches of science more then he gives. For some this type of group research consists in having all work on his particular ideas and more or less under his domination. These have no place in the modern scheme of research. There are some who appreciate teamwork only when captain of the team.

After all is said, the director's opinion of a man is the deciding factor in his addition to a research organization. A wise director is careful of the choice of his personnel, for upon that depends its success. The homely old rule still holds—character stands for three-quarters and training for a quarter. Real adjustment and education can be done after induction into the organization and this is always necessary. Inborn capacity counts more, training and environment less.

Good men are those who are stable on the job, are industrious, research productive and are satisfied and happy in their work. However, a potentially good man may turn out poorly if misplaced. If he fails on a job which requires more than his particular abilities, the error is not one of selection but of placement. But it may be taken as a fundamental principle that satisfactory placement presupposes the selection of the proper material.

There are two ways of improving the personnel of an organization: one is getting the best, the other is getting rid of the worst. The desirable qualities are judgment, common sense, modesty, originality and precision, a formidable list and to that must be added an unquenchable curiosity.

BOOK REVIEWS

UNRESTING CELLS, by R. W. Gerard. 439 pages, 14 × 21 cm., drawings and illustrations. New York, Harper and Bros., 1949. Price, $4.00.

This is a re-issue of the first printing which came out in 1940. The author attempts to bring to the intelligent layman some appreciation of the fundamental problems and the basic concepts of certain phases of biology. Since the emphasis throughout is upon the chemical and physical mechanisms governing biological reactions, careful reading is required, particularly in those sections dealing with cell substances and enzymes. This is made as easy as possible for the reader by the excellent organization and the careful selection of the material presented.

With regard to a very few points, the author is on shaky ground. For example, it is doubtful if any living material could traverse interplanetary space without being killed by the radiation that would be encountered. It is even more doubtful that bacteriolytic antibodies act as protein-digesting enzymes, or that the genic material in the chromosomes is protein in nature. Few other criticisms can be raised against the material here presented. Over-simplification of complicated concepts has been avoided and the amount of basic information that is provided is truly astonishing. Suggestions for further reading are not provided, nor is any simple bibliography furnished. The indexing, however, is excellent.

LOUIS DESPAIN SMITH

PATENT LAW, by Chester H. Biesterfeld. Second ed., 267 pages, 15 × 24 cm. New York, John Wiley and Sons, Inc., 1949. Price, $4.00,

For its size, the volume under review is the most comprehensive known to the writer. The conciseness and the novel arrangement or grouping of the subject matter are due, at least in part, to the fact that the author did not deliberately attempt to write a book. The net result is quite salutary.

In most of its chapters, the book deals with various topics of patent law in a substantially standard manner, except for the brevity above noted. In certain other chapters, the author makes exceptionally good presentations of certain other topics. For example:

In Chapter IX the non-patentability of natural products and of "new uses" of old devices is clearly explained and this chapter should be valuable to a practicing lawyer who is always having trouble convincing clients why their pet brain storm cannot be patented.

In Chapter XXI, the necessity of making searches, independently of the Patent Office, and how such searches are made is treated for the first time in any book on patent law known to this reviewer. This too, should be of considerable help to a practicing patent lawyer in convincing clients as to why a search should be made and why searches to be worth making, cost as much as they do.

In its sub-title, the book is said to be intended for "chemists and engineers." This may be true of the more general chapters of the book such as Chapter XIII dealing with the application for patent, Chapter XVII dealing with Licenses (except for the part relating to price fixing which even lawyers find abstruse) and Chapter XVIII dealing with shoprights which, under certain conditions, employers may automatically acquire in the inventions of employees. But, and with all due respect to chemists and engineers, it is felt that the remaining parts of the book, and the book as a whole, belong in the library of a practicing lawyer rather than on the book shelf of a chemist or engineer.

Unlike those who deal in the exact sciences, the author, like all other authors on patent law, attempts to make tangible that which is intangible and to make objective that which is wholly subjective. Reduced to its lowest denominator, the main question is "*What is invention*" and a more subjective concept can hardly be imagined. The veritable plethora of cases deciding this point is, at best, good for the particular facts from which it arose and it is seldom that the facts in any two cases are exactly identical. This is only by way of pointing out difficulties attending a study of this question and not by way of detracting from the merit of chapters II and III which constitute a well documented presentation.

LOUIS NECHO

FIBRE SCIENCE, edited by J. M. Preston. 341 pages, 14 × 22 cm., illustrations. Manchester, The Textile Institute, 1949. Price, 30/.

The art of converting fibers into yarns and yarns into fabrics is as old as human civilization. The high degree of skill, ingenuity and efficiency reached in the textile industry has been attained mostly without assistance from the sciences and scientists. In recent years, more and more scientific research has been done on the structure and behavior of textile fibers, which is one of the principal subjects with which the textile industry is concerned. Modern textile scientists have applied all of the findings of atomic physics, the advances in electrical instrumentation, the knowledge of physical chemistry and all the other scientific tools to unlock the secrets of textiles and the behavior of textile fibers. The present work combines the efforts of fourteen of the leading scientists in this industry, who have written on the subject about which each is the accepted authority.

For the most part, comparatively simple mathematical equations are used and most of the solutions of those problems encountered in the text are given in the form of curves of actual values. Chemical formulas are given frequently in conjunction with their diagrammatic representation together with curves showing their behavior. Many figures and photographs are used to explain clearly what the scientist sees in his laboratory and what the textile engineer or chemist should expect to see in practice.

At the same time, none of the tools of modern science is neglected. Studies made by the use of X-ray diffraction, microscopic examination and chemical analysis are thoroughly explained. Consideration is given to the various theories of fiber structure and a good picture is presented of the chain molecules and their orientation into fibers. The macromolecular structure of both the natural and synthetic fibers is shown to consist of macromolecules of great length relative to their diameter formed from very large assemblies of atoms held together by principal valency bonds. The chemical significance of the cross linkages on the lattice structure is also explained.

After presenting the fundamental concepts of fiber structure in the early chapters of this book, the later chapters discuss the chemistry of various ingredients for synthetic fibers. The characteristics of various fiber-forming polymers are given and the methods of evaluating them are described. Finally, the actual effect of the fiber properties in their relation to dyeing and finishing is described in detail. This work should give textile engineers, chemists and scientists a thorough understanding of the modern trend in fiber science.

FRANK R. SIMPSON

PRINCIPLES OF MECHANICS, by John L. Synge and Byron A. Griffith. Second ed., 530 pages, 15 × 23 cm., illustrations. New York, McGraw-Hill Book Co., 1949. Price, $5.00.

It is seldom that teachers of mathematics write textbooks which seem to be suited primarily for engineering students. But here in this second edition of a book intended for advanced undergraduate students, the authors, both professors of mathematics, present the principles of mechanics as an orderly, self-contained subject, to be studied and learned as a deductive science. In one respect the field of mechanics can be thought of as being one branch of applied mathematics treating of a combination of physical fact and mathematical know-how.

The main theme of the text is the subject of equilibrium, both static and dynamic. How this equilibrium is established and the mechanics of analysis and computation are somewhat secondary in nature.

The book is divided, somewhat artificially from the physical point of view, into two parts—plane mechanics and mechanics in space—a division which allows the use of simpler mathematics in the plane theory. Part I is devoted to plane statics and dynamics. A first chapter introduces the foundations based upon physical ideas and a logical structure of the science. This is followed by chapters on methods and applications of plane statics and kinematics, methods and applications of plane dynamics and impulsive motion.

While very little of the three dimensional theory was introduced into the first part, Part II is devoted almost wholly to the subject of mechanics in space, with almost complete dependency upon the use of space vectors. One chapter is devoted to products of vectors, vectors

having been introduced early in Part I. Armed with this working knowledge, there are developed results valid for the general force systems encountered in statics in space. The subjects of kinematics, kinetic energy and momentum and methods and applications of dynamics in space, are then considered. Two final chapters devoted to Lagrange's equations and the special theory of relativity conclude the text, except for an appendix devoted to the theory of dimensions.

While this edition is essentially the same as the previous one, several principal revisions were made as concerns the treatments devoted to: the motion of a particle in an electromagnetic field; principal axes of inertia; Foucault's pendulum; the spinning projectile; and the gyro compass.

Transitions between mathematical and physical concepts for specific problems are many and necessary. Each subject considered required its own approach, not only between the mathematical and the physical, but also as to whether the treatment should be based upon fundamental concepts or upon the body of general fact developed up to that point. The result of this open-minded, logical presentation is a book which is extremely understandable and clear in argument.

The typography is well suited. Important passages are italicized, vector equations are written in bold face and the diagrams are clear and unambiguous. Examples are worked out in the text proper, and the many more exercises appearing at the ends of the chapters are sufficient for class room use in a course of one year's duration.

S. CHARP

THEORY OF LIMIT DESIGN, by J. A. Van den Broek. 144 pages, 14 × 22 cm., drawings. New York, John Wiley & Sons, Inc., 1947. Price, $3.50.

Limit Design is not the usual structural engineering book. Most books on the strength of materials present the subject so that the reader assumes everything is exact. The various beam formulas, column formulas, stability formulas and so forth are all based upon assumptions, and, while most authors mention these assumptions, they do not emphasize the dependence of the final results upon them as often as Van den Broek does. Van den Broek writes in a conversational, chatty fashion—sort of lets the audience behind the scene—that may be distracting and even annoying to some readers. He does have a sound argument, and he is constantly justifying it.

Most structures are designed on the bases of the theory of elasticity, and that failure occurs when the material, at some point, yields. Limit design proposes to permit yielding of the material until all, or almost all, the material yields, thereby utilizing more of the material. It uses a ductile stress distribution instead of an elastic stress distribution, an allowable deformation instead of an allowable stress. Most competent engineers realize the limitation of the former basis of design and that all the material may not be utilized. But they also realize that the loads upon which the design is based are not always accurately determinable. Even though most static structures, as bridges and buildings, carry comfortable margins of safety, this extra factor— that the material can yield and yet not rupture —is pleasantly reassuring. Using limit design this extra factor would be eliminated. However, there is no question that Van den Broek has a sound argument and it should be accorded consideration. And, in fact, in some components of airplane structures, this theory, although not known as limit design, has been used.

The *Theory of Limit Design* is a short book. The introductory chapter is the usual one in books on the strength of materials in which such properties of metals as stress, strain, modulus of elasticity, yield stress and ductility are explained. Following are chapters on limit design of simple structures, of redundant beams, of trusses (including columns), connection details, and a short, concluding chapter on the evaluation of limit design. In the chapters on simple structures, beams and trusses, the designs of each by the theory of limit design are compared with the designs by the theory of elasticity. In each case it is shown that by using the theory of limit design, a lighter structure invariably results. In some of our present design practices—as in assuming even distribution of load throughout a rivet pattern under tension—we un-

knowingly apply the theory of limit design. An appendix to the book contains several problems with answers.

As a handbook of design, in the sense that an ordinary strength of materials book contains the solution for many structural problems, the *Theory of Limit Design* is not very useful. But as a statement of the principles of a new approach to structural engineering design, it is an interesting and stimulating book.

E. W. HAMMER, JR.

BOOK NOTES

PRINCIPLES OF STRUCTURAL GEOLOGY, by C. M. Nevin. Fourth ed., 410 pages, 15 × 24 cm., illustrations. New York, John Wiley & Sons, Inc., 1949. Price, $6.00.

In revising this standard text on structural geology, the author has eliminated specific references to source material, as he found that they were little used by the students for whom the book was written. However, material for further study is indicated in the form of selected references at the end of each chapter.

A new section of laboratory exercises has been added with both sample and practice problems. This continues to remain a useful introduction to the deformations of the earth presenting the various ideas and viewpoints.

THE NATURE OF PATENTABLE INVENTION, ITS ATTRIBUTES AND DEFINITIONS, by John E. R. Hayes. Second ed., 187 pages, 15 × 24 cm. Cambridge, Mass., Addison-Wesley Press, Inc., 1948. Price, $5.00.

Rearranged and in part rewritten, this study on invention is made available in a second edition. The discussion of the attributes of invention as a means to a definition of invention is of interest. Many additional cases have been cited in support of the author's thesis.

ROUTE SURVEYING, by George W. Pickels and Carroll C. Wiley. Third ed., 434 pages, 11 × 17 cm., illustrations, tables. New York, John Wiley & Sons, Inc., 1949. Price, $4.75.

This practical book on route surveying, which may serve either as a textbook or a reference book, has been extensively revised to increase its value. Primary emphasis is on railroad surveys, but highway surveys are also treated, with a brief mention of canal and drainage, pipe-line and transmission-line surveys.

Of particular interest to railroad engineers will be the full revision of chapter six on stringlining railroad curves, which presents the basic principles involved as well as detailing the mechanical procedures. Although logarithmic tables have been omitted as of little value, the five-place tables of the natural trigonometric functions have been replaced with seven-place tables and other new tables added.

INTRODUCTION TO PHYSICS, by Harley Howe. Second ed., 599 pages, 15 × 23 cm., illustrations. New York, McGraw-Hill Book Co., Inc., 1948. Price, $4.50.

An elementary college textbook in physics which requires only elementary algebra and plane geometry. It attempts to be "practical" by referring frequently to the everyday experience of the reader. Chapter summaries and numerous problems are among the features of the book.

THE MATHEMATICAL BASIS OF THE ARTS, by Joseph Schillinger. 696 pages, 17 × 24 cm., illustrations, tables. New York, Philosophical Library, 1948. Price, $12.00.

Over a period of twenty-five years Mr. Schillinger did much research on principles of mathematical logic underlying varying forms of art structures. This volume presents his "scientific theory of the arts" in its three main branches: the semantics of esthetic expression, the theory of regularity and coordination, and the technology of art production. He foresees eventually the use of the scientific method in art production and the possibility of having music or painting or poetry designed and executed just as engines or bridges are.

CURRENT TOPICS

Chemical Scrubwoman (*Du Pont Magazine*, Vol. 43, No. 3).—"Tranox" cleaning composition has found many applications, but it was developed as the result of a specific problem. Here is how that came about:

Just prior to D-Day, a large railroad maintenance shop was finding it hard to clean the finished surfaces of passenger cars used by the armed forces. These cars had to be cleaned while in service, and reconditioned and refinished periodically. Because of the time and work involved in removing soilage, schedules were quite often delayed.

The Du Pont Company was asked to assist in the development of some method or product that would speed the clean-up job. As a result, this particular maintenance shop has been using "Tranox" for over two years on cars covered with grime, carbon, rust, oils and grease. It is reported that car surfaces can be cleaned 25 to 50 per cent faster, and that cars are smoother after being cleaned with this agent, making possible a better bonding of new coats of enamel.

Since it is specially designed for heavy-duty work, "brushing" on two or three coats of "Tranox," mixed with water, is enough to take off most soils. However, extra attention may be required in those places where foreign substances tend to collect, such as around window frames, around bolt heads, and on car ends. After the final spot has been touched up and the cleaner has been allowed the required time to work, either warm or cold water from a pressure hose is used for rinsing.

People in the antique business have found the new acidic cleanser good for taking rust off iron grille work, lawn ornaments and old iron hitching posts before refinishing them. The old method of cleaning grilles was to take a steel brush, use plenty of elbow grease and scrub the rust out of the intricate designs. Now antique restorers simply put "Tranox" on with a brush and, after a half hour, rinse it off with a hose. If the amount of rust on the grille is excessive, it may need more than one application. The final result is that the iron is left rust-free and ready to be restored to its original beauty.

Metals cleaned with the new composition will not show "burn marks." Nor does the cleaner leave a film or streak, even though the equipment is washed out of doors and inspected in sunlight.

Indoors, "Tranox" can be used to clean vitreous enamel, acid-resistant porcelain, and glass or lacquered surfaces. It is also utilized in cleaning sanitary equipment because it quickly removes stains and odors. Since "Tranox" is a concentrated paste, the hazard of spilling liquid acid is eliminated.

One note of caution, however—anyone employing the new compound should wear rubber gloves, eye protectors, and, in some cases, special clothing. Since the cleaner contains oxalic acid, it should not be allowed to have porlonged contact with one's skin.

Thirty Miles Up Measured By New Air-Borne Device (*Science News Letter*, May 14, 1949).—Distances above the earth up to 30 miles can be measured with high accuracy by a new instrument, revealed by General Electric engineers. It is a new type of hypsometer, an instrument that measures altitudes by determining the boiling point of a liquid and from it calculating the atmospheric pressure.

The fact that the boiling point of water decreases as the atmospheric pressure decreases is well known. It is also well known that the atmospheric pressure depends largely upon the altitude. This new hypsometer was developed particularly for use in free balloons which are sent high above the earth, reporting automatically by radio the weather conditions encountered. It looks like a radio tube, and was designed to replace the presently used bellows-type devices which expand and contract with changes in the air pressure.

The instrument contains a small vacuum flask which holds about five thimbles-full of liquid. Water is sometimes used, but a liquid with a lower freezing point, such as carbon disulfide, is usually chosen. Inserted in the open end of the liquid chamber is a thermistor, a delicate device for measuring temperature. It is a device in which the electrical resistance changes as the temperature changes.

The steam from the boiling liquid causes the temperature changes. It is the changes in the electrical resistance of the thermistor that are transmitted to the ground station by the balloon's radio. The greater accuracy of this instrument is due to the fact that it is the temperature of the steam itself that is used as an index, not that of the boiling water.

Indestructible Phenoplast (*Science Illustrated*, Vol. 4, No. 6),—On its way from a modern factory in New Jersey is a new chemical product designed to ease the burden of the housewife, boat builder, carpenter or any person who looks at, touches or sits on wood, plaster or metal.

Called Phenoplast, this chemical is a phenolic resin—an almost indestructible plastic. It is a clear, colorless, liquid finish which, when applied to a surface, works its way so intimately into it and coats the surface so thoroughly that hardly anything can damage it. Applied to a desk or table, it successfully resists demolition from even the metal wheels of a professional roller skater. Used as a glue, it joins two pieces of wood so perfectly that even Charles Atlas, a well-known strong man, cannot separate them.

One of Phenoplast's most important applications today is in boat building. With a Phenoplast coat, a vessel becomes waterproof, sunproof, resists fouling, needs no varnish. A. J. Lockrey, the chemist who developed Phenoplast, claims his product is so waterproof that an ordinary 60-ft. Phenoplast-covered fishing craft can carry two more tons of fish than a non-Phenoplasted boat.

Although plastics exist which do what Phenoplast does, to be effective they must be baked, or pressure-molded into the surface. Phenoplast's secret is a small bottle of catalyst which comes with each can. You simply add the catalyst to Phenoplast before using, spread, wait four hours for it to harden. A second coat must be applied before the coating reaches full effectiveness. Because of its high gloss, a Phenoplast surface can be cleaned with only a damp cloth.

Since it is so new, both Lockrey and Charles Roman, president of the Phenoplast Corporation of New York City, have not yet explored all the possibilities of their new products. One thing they are now investigating is mixing Phenoplast with pigment to create a permanent color coat to walls and furniture. "Permanent" is used advisedly. For, once Phenoplast covers a surface, only hard work and tools can remove it.

Dispersion of Large Cities.—The threat of an atomic war makes it imperative that we break up our large cities into smaller ones as rapidly as we can, Dr. William F. Ogburn, professor of sociology at the University of Chicago, told a class in contemporary trends at the University of Wisconsin recently.

Professor Ogburn has made an intensive study of the problems of dispersing large cities. Since the first atomic bomb was dropped, he has urged that some program of dispersion be adopted.

"There has been a revolution in modern warfare," he said, "a revolution that has brought the destructiveness of war to the civilian population."

The very nature of the atomic bomb makes it particularly effective against large groups of civilians as are found in cities, he continued. At the same time, the cost of the bomb and the problem of delivering it to a target limit its use pretty much to large cities.

"An atomic bomb would probably not be used on a city of less than 50,000 people unless the city were of special military importance, Professor Ogburn said. "The problem, then, is to disperse our large cities in order to protect their civilian population.

"The difficulty with an immediate dispersion is the tremendous cost and resistance to the change that would have to be overcome," he continued. "We at the University of Chicago did some figuring and concluded that rapid dispersion would cost something like 500 billion dollars!"

A more practical approach to the problem, according to the sociologist, is a gradual dispersion over a period of 25 to 150 years. But he emphasized that such a program should begin at once and proceed as rapidly as social and economic conditions permit.

"Even if we avoid war within the next three or four years, it would seem that we will face the danger of war again in 25 or 30 years," Professor Ogburn continued. "By that time Russia will have become highly industrialized and she will certainly seek to readjust her zone of influence."

The plan Professor Ogburn suggested would gradually break up the larger cities into units of 50,000 people or less by locating new industry outside of the cities and by moving old industries as their present plants become obsolete.

Although the trend is for new industries to locate outside the large cities, there is much resistance to dispersion, Professor Ogburn pointed out.

"Those who own real estate resist such a movement because it would mean losing property value," he said. "Big corporations, like a telephone company, resist it because their plants would become obsolete and they would have to build new equipment outside the city. Mayors resist it because it would decrease their cities' income from taxation.

"One of the other groups opposing the plan, curiously enough, is the idealists, who don't want to see any preparation for war, because they want to prevent all wars."

"We in America tend to wait until a war is upon us, then act. But if we are to be prepared for atomic warfare, we must begin at once to speed up the trend toward dispersion."

Air Shipment of Flowers Under Study.—Tests to determine how flowers hold up in air shipment under varying flight conditions have been inaugurated in the laboratories of the Lockheed Aircraft Corporation, Burbank, Calif.,

under the joint auspices of the U. S. Department of Agriculture, the Air Cargo Institute of California, the Southern California Floral Association, and the Lockheed Corporation.

The Floral Association will provide flowers of different varieties of sweet peas, carnations, daisies, stocks, roses, orchids, and gardenias for use in making the tests.

The first of the tests will simulate flight in cargo aircraft, which is unpressurized, unrefrigerated, and not humidified. Other tests are designed to show the effects of shipment in pressurized aircraft where temperature and humidity can also be controlled. The effects of high altitudes will be studied in two of the tests.

B. A. Rose is in charge of the mechanical research for Lockheed which is providing facilities for the test program. W. T. Pentzer and W. R. Barger, storage and shipping specialists of the Bureau of Plant Industry, Soils, and Agricultural Engineering with headquaters at Fresno, Calif., will inspect and judge the flowers to determine any change in color, freshness, firmness, texture, and fragrance and make other records. They will make an official report on their findings.

Directory of Translators.—Solution of a long-felt need in connection with the recurrent problems of locating persons with special language skills for translating scientific and technical articles is forecast in the announcement of the establishment of a Directory of Translators by the Science-Technology Group of Special Libraries Association. The Directory is operated as a free service to bring together clients having translation problems and language specialists competent in various subject fields in science and technology. The Directory has been housed at the Southwestern Research Institute, San Antonio, Texas, under the management of Wayne Kalenich, Librarian.

Individuals or firms needing qualified translators are invited to address inquiries to Mr. Kalenich, and translators are urged to place their qualifications on file with the Directory. Twenty-three different languages are already included among the specialists listed there, and it is hoped soon to expand the Directory to include translators for all fields of knowledge. While no data concerning interpreters has been collected as yet, such information is solicited.

Mr. Kalenich also conducts the Special Libraries Association Translations Pool, which records the existence of technical translations within private files in the U. S., and the conditions of their loan or purchase. This Pool was created in 1945 to prevent or at least reduce unnecessary duplication in the translation of scientific articles. About 3000 translations have already been indexed in the Pool. Persons contemplating the translation of a technical article are urged to make a preliminary check with Mr. Kalenich to be sure that such a translation is not already available.

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THE SUBSTANCE OF MANAGEMENT *

BY

DALE PURVES [1]

Many years ago, a young engineer went to see Charles Day for advice. He was, as might be expected, nervously diffident about approaching a man of Mr. Day's stature in the community. Mr. Day not only dispelled the young man's first reluctance, but also gave him, from a profound store of experience and wisdom, an appreciation of broad and fundamental values which served and guided him in every subsequent step of his career, including this evening which we are spending together.

It is most happy indeed that these yearly gatherings have been made possible to preserve the wisdom and ideals of a man who, himself, could have made such a great contribution if he were still among us today.

I feel again tonight some of the same reluctances of that evening many years ago; for our discussion is, so to speak, a personal accounting on my part regarding the use which I have been able to make of Mr. Day's example and precepts. I can only report to you that the precepts and example given so long ago have proven more extensive and inspiring than any young engineer had any right to expect.

It is but fair to forewarn this distinguished group that this lecture will not, and indeed cannot, live up to the broad scope of its title. I had originally more correctly called it "Some Observations on the Substance of Management," but the experts said that this was altogether too general, and advised the shorter description as possibly more in-

* Presented under the Charles Day Lecture Foundation of The Franklin Institute of the State of Pennsylvania, given in the Hall of The Franklin Institute at the Stated Meeting held May 18, 1949.

[1] Formerly Vice President and General Manager, La Consolidada, S. A., Mexico; now Member, Edward N. Hay and Associates, Management Consultants, Philadelphia, Pa.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

triguing. Actually, I hope to touch upon the subject in terms of a sub-title: "Some Comments on the Education of an Engineer."

Many years ago, I was for a time a thorough reader of the "Atlantic Monthly." I was living abroad at the moment and the "Atlantic" not only helped to maintain the proper intellectual standards, but also happened to be about the only magazine on sale locally. One article and its repercussions have always stuck in my mind—it was titled "An Engineer Looks at Medicine." He not only looked at it; he took it apart critically and dogmatically, especially as regarded homeopathy. Among the letter-writers who responded was one who said, according to my recollection, "Dear Sirs—I, too, as a doctor could look at engineering, but I know my limitations.—Very truly yours, etc."

Those were the days when engineers, with general consent and approval, took over a lot of territory. Besides the recognized fields of engineering, we had Efficiency Engineers, Human Engineers, and others who barely avoided invading the professional fields of law and medicine. Many of these so-called engineering activities have since gone to their long home. But not all—I have just read an article titled, "Nutritional Engineering." The subject matter was interesting and encouraging, but did not to my mind establish dietetics as an engineering rather than a medical field—I may be wrong, and if anyone wishes to explore this point further, the article appears in the JOURNAL of this institute, Vol. 238, No. 2, August, 1944.

This brings us to the subject which we are to consider together this evening; to wit, some part at least, of the substance of management as seen by one engineer; and the same engineer's tentative comments on management requisites of moment. In the field of management, engineering has a very proper place; for an engineer, by definition, is among other things, "a skillful manager"; and management, by definition, is among other things, "the judicious use of means to accomplish anything." But to my mind, an engineer is not *ipso facto* a skillful manager; he becomes skillful only by *judiciously* using means to accomplish the objectives of management. And the objectives of management, in this industrial stronghold of an ancient mechanical society, are not merely the accomplishment of a narrow "anything," but rather the broad and very important accomplishment of a great many fundamental objectives demanded by a complex society. Mr. Morris L. Cooke in the first Charles Day Lecture five years ago, and Mr. William Meinel in last year's lecture, dealt profoundly with this same general theme.

To my way of thinking, the real substance or stuff of management is the creative ability of managers to sense and to respond successfully and continuously to the challenges which confront them in the broad area affected by their management. At first blush, this may appear to be a deceptively vague and obvious generalization. It takes for granted, as it must, technical proficiency. We have spent twenty or forty years becoming technically proficient, without giving too much

thought to the question—"For what purpose?" Our creative and successful response to some challenges has been extraordinary—to other challenges less so, as the economic and social upheaval culminating in two devastating holocausts will attest. Sometimes the successful response to our challenge has, in effect, diminished our ability to respond to the next, and very different, challenge. This is not due to enervation or indifference, but rather to the historically recorded fact that the achievement of great success turns the path away from greater successes. Thousands of years ago, the shark was, as he continues to be, such a successful fish that he swam out to sea with great speed and eclat. He left behind a small, unsuccessful fish with puny flippers who climbed out of the water and started the evolution towards higher forms, including ourselves.

But let us not despair; creativeness and vigor burn brightly with us, although perhaps not in a self-replenishing fashion.

To give point and definition to this theme, I intend to draw upon a few experiences in Mexico, where I have spent the last three years in managerial work. Mexico is a beautiful and vivid country, which has a peculiar way of attaching itself to one. It also, in its past and present history, presents dramatically some very illuminating points on cause and effect. In Mexico, causes produce effects with little delay, and with even less benefit of the "shock absorbers" which soften and sometimes disguise the impact in a highly organized society like ours.

MANAGERIAL INSIGHT

At the turn of this century, Mexico was in the midst of a period unique in its history since the conquest. It was passing through thirty years of uninterrupted peace. Foreign capital was welcomed; basic economic development strode forward, railroads were built and run on time, the arts flourished in their particular way. In superficial material terms, it was a golden age in Mexico, and would have been so regarded in any other country. The man and group responsible for this happy state of affairs were the president, General Diaz, and a coterie of gifted administrators called *los científicos*—the scientific ones. These men were early and worthy exemplars and students of "scientific management." Whatever else may be said of Diaz and his *científicos*, their administrative and technical abilities have been generally acknowledged.

Because he was a dictator and because he and his *científicos* failed to sense the essence of the internal Mexican challenge, the Diaz regime bred the Mexican revolution which not only ended that regime in 1910 but plunged an orderly country into thirty years of fermentation, which is a tactful way of referring to Mexico's travail.

The revolutionaries who upset the Diaz regime had in mind three purposes which were not unreasonable, and about which something useful might have been worked out:—

1. They wanted an effective ballot which would make it impossible for any group to perpetuate itself against the wishes of the people.
2. They wanted the Mexican government to stop being a "mother to foreign interests and a stepmother to Mexico."
3. They wanted the benefits of Mexico's progress to go to the many instead of the few.

Diaz and his *científicos*, apparently secure in their position, at first made the efforts of the revolutionaries appear ludicrous; but within a year, the revolutionary cloud, at first no larger than a man's hand, had covered the country and Diaz with his associates were completely and irrevocably liquidated. We may well inquire if this development had to be inevitable. I think not: for Diaz himself was once a revolutionary. It was not necessary for him to lose touch with his countrymen and their aspirations. His success, of a kind that we can appreciate, carried him out to sea; and although he competently exercised his superb talents, the area and objectives were unacceptable to the millions of Mexicans who groped for something else.

My point here is that the managers, if you please, failed to sense the challenge of the moment in all its importance and implications. To them, the impossible and fantastic happened. The essential management ingredient of insight was lacking in them.

While we do not often think of management insight, it happens to be one of the factors which is functional to management, observable in action, and predictable in the individual manager. The engineers, in their millions of words on "scientific management" have generally missed this one; those that recognized and understood it have been men like Charles Day.

MANAGERIAL ELAN

The revolution had, and continues to have, a terrific impact on management in Mexico. It is perhaps obvious that management prestige suffered, and that revolutionary objectives, especially as regards the worker, were strongly and firmly established. Since returning to this country, I have encountered great interest and incredulous wonderment regarding the conditions under which management in Mexico must strive towards its goal.

Managers up here would consider these conditions novel and somewhat restrictive, and rightly so. Under the revolutionary Federal Labor Law of 1932, which covers everything imaginable touching upon labor, including the alcoholic content of intoxicating liquors, the following conditions, among others, are imposed:

1. The employer must pay to any worker released a dismissal compensation of three months plus twenty days salary for every year of service, except in an exceedingly limited group of discharges for cause.

2. The employer can shut his plant down only with government approval, regardless of the duration of the shut-down. A legal shut-down does not terminate the economic liability of the employer to his employees.

   In our company, we suspended taking orders for one of our departments because of high costs. For a period of four months the entire department work force drew full salaries and reported at the plant to read newspapers and so forth. Finally work was resumed with a new and somewhat better wage scale.

   The general effect of this feature of the law is to give the employer full initiative in expanding his activities, and to the government and unions almost complete initiative as regards the curtailment or suspension of activities.

3. In the event of a legal strike (and for practical purposes all strikes have a way of being legal) the employer is liable for payment of all salaries for the duration of the strike.

   Our company underwent a three-week strike over the renewal of our labor contract. The strike was launched amidst fire-works, music by strolling serenaders, and fair quantities of *pulque* [2] as the red banner was stretched across the yard entrances. The workers established their 24-hour vigil; because if the company could succeed in removing a shipment from the plant, the status of the strike would have automatically been altered. Access to the plant except by the Union safety committee was prohibited. There was a general air of "cops and robbers." When the strike ended, again with fire-works and music, the Company not only paid strikers' wages for the three weeks, at the new scale of course, but also a lump, unitemized sum for the Union strike expenses (telegrams, fire-works, music, *pulque* and miscellaneous).

Aside from the basic provisions of the Federal Labor Law, collective labor contracts can and do add additional restrictions. In the voluminous contract between our company and the Miners' Union, many burdensome clauses appeared, of which the two following are fairly simple and typical:

1. Time lost on account of accident is paid at full earning including all production premiums, starting from the moment of accident and continuing to the moment of return. Thus, a worker loses nothing financially as a result of a lost time accident. In some instances, this might be considered as equitable. But it is possible, and in fact often happens, that workers "arrange" for convenient and not too serious mishaps having previously established high earnings as a basis for accident compensation. And it is unusually difficult for management to police malingering.

[2] Fermented cactus juice; hence, cactus in one of its lower forms.

2. The Union has the right to complain of management, and demand the removal of management representatives, directors excluded. I know of a case where, in another company, the union demanded from the general manager the removal of the medical director. Against advice, the manager acceded. The precedent having been established, a few months later the union demanded, and obtained, the removal of the manager himself.

These five examples will suffice to suggest that managers in Mexico must have a fair amount of resiliency. In fact, management in Mexico or anywhere else must have a *vital impulse towards its job*. Some people call this impulse drive; but I prefer the connotation of the word elan, which has happily become part of the English language. "Drive" implies vigorous pressure, and this is good as far as it goes, which is not far enough. It is apt to be construed in its more physical sense.

To be sure, managers abundantly need physical vigor, and in this country especially, the constant doer commands admiration. He commands my admiration as long as he pushes his problem forward, and not over. By carefully nursing an egg along at the proper heat for three weeks, you can hatch a chicken if all goes well. If you want to speed up the process at higher temperatures, you can get a hard-boiled egg in about twenty minutes. Sometimes drive boils eggs; elan hatches them, if possible. Artemus Ward tells the story of an amateur pyrotechnist whose elan finally produced a display of fireworks which was stupendous. His drive also resulted in blowing all his fingers off.

Can elan be effective under the restrictive conditions which I have described? Yes, for management can adapt itself to the conditions under which it must operate, developing the ingenuity and fore-sightedness required. In the case of lost time accidents, the record of some of the company's departments was deplorable, especially in departments which presented no special hazards. But in other departments, some with dangerous exposure, the record was excellent and altogether comparable with U. S. experience. The difference?—management effectiveness. Again, the practical effect of the management discipline clause was nil, and for the same reason. The most acceptable managers were those who were strict, fair and competent and possessed elan; the least acceptable and most under fire were those who lacked management impulse, conviction and competence.

Elan is both physical and emotional. Like insight, it is functionally essential to management; it is observable; and it is predictable in the individual. As a fundamental element in management, it has perhaps not received its qualitative and quantitative due from the engineers.

SOCIAL SKILLS

Some of you must be wondering by this time how managers in Mexico solve problems, even when they are generously endowed with

some unusual qualifications. How, for example, do strikes get settled when there appears to be no particular inducement for the worker to get back to work? The answer is, of course, that people everywhere are, in the last analysis, people, and respond to devices which recognize this. A high degree of social skill is required; for besides the difficulties outlined above, the workers' leaders are in many cases revolutionary characters. Table-thumping becomes a luxury beyond the margin of diminishing returns.

In our strike meetings, progress usually took place between four and seven o'clock in the morning, when resistance was low. We also used some kind of pill which restored vigor and acuity when these lagged. And we talked and bargained, and kept on talking and bargaining. But above all, we assembled our facts, threw emotions out of the window, and used every social skill involved in face-to-face dealing. I will always remember vividly the skill of Mexicans to interpret a gesture and to catch a meaning. Up here, we take for granted that there are several ways of saying the same thing: in Mexico, there is only one way of saying something. When you say it some other way, it means something else, and sometimes the difference is major.

Consequently, the development of social skill and sensitivity to others becomes an art which is useful wherever people deal with each other. Flat-footed statements like "it stands to reason" are quickly discarded. You size up the other fellow, and persuade, cajole, charge, retire, pull out stops, push them in again, and remember all the time that you want to settle for 15 per cent increase in a two-year contract, which we did.

I am not implying that social skill is a substitute for facts and figures. Rather, I am making the very important point that without social skill, facts and figures go to their long statistical rest.

Mr. Meinel last year touched importantly on this point, and suggested how managers might usefully improve themselves in this area. One need not be born with high social skills; they can be developed, given a reasonable base. But the acquisition of high skills is not easy; one does not learn how to play a piano from handbooks, and I suggest that the human being is a fairly complex instrument.

EMOTIONAL STABLIITY

Emotional stability, or at least emotional control, should accompany social skills. One might easily and rightly imagine that the atmosphere described above encourages all manner of conspirational and emotional situations. First of all, the art of veiled and unveiled insult, along with other forms of irritations, is effectively and purposefully cultivated. Fantastic edifices of misconstruction are built on basically forthright declarations. Fifth columns exist like wheels within wheels; it is sometimes very difficult to know just where or why they are headed, and a

certain amount of traffic control is essential. Mexico is a great place for meetings under the old eucalyptus tree at midnight.

All these goings-on require a sense of humor; in fact, Mexicans possess this sense to a high degree, and it is expected of managers. Humor is essential to keep everything on a game level; otherwise the relations become seriously dangerous. Inherently the situations are serious enough without obstructing the flow of effective human·dealing with emotional blocks and distorted sense of values.

While the demands upon social skills and emotional stability might be somewhat more urgent in Mexico than here, we should not ignore the growing fundamental need for these same qualities in managers wherever they exercise their responsibilities today. There is nothing vague or hazy about the importance of these essential qualifications—without them, other managerial skills are frustrated.

THE ART OF MANAGEMENT

By this time, you will feel that the substance of management relates closely to what is often called, in an off-handed way, the art of management. Perhaps so, but some interesting things can be said of art. Art is both skill and the thing for which the skill is used. Furthermore, Art with a capital A has been widely recognized as the most accurate index of the rise, vigor and decline of civilized societies. We are perhaps justified in considering that some of the creative urge which expressed itself in painting, sculpture, architecture and music in other years, now expresses itself in management. I say this because in our highly industrialized society, Art appears in many industrial forms not previously encountered.

As regards the decline of Art in recorded history, the skills were not lost, except in one or two cases (Mexico being one) where the destruction of skills was planned and deliberately executed. But Art can lose, and has lost in times past, its substance. It can cease to have meaning and relevance. It can lose its insight and vital impulse, and deteriorate into meaningless forms which lose touch with people and their aspirations.

So it is with management. We have gained rather than lost ground in skills and techniques. It is not quite clear, as regards both Art and Management, that insight and impulse have kept the same pace. There has been a good bit of honest groping in this respect. Sometimes when I hear a bit of modern cacophony or look at a "mobile," I wonder if other people feel as bewildered as I do with the high degree of skill and the rather complete lack of meaning, at least to me.

Our driving concern is with the substance: this has always been so. And of particular concern is the substance of management, for modern management is certainly one of our greatest contributions and means of self-expression, from which hundreds of millions of people take meaning.

GRAPHICAL ANALYSIS OF IMPACT OF BARS STRESSED ABOVE THE ELASTIC RANGE*

BY

KALMAN J. DEJUHASZ[1]

Part II[2]

II. IMPACT IN THE STRESS RANGE ABOVE THE ELASTIC LIMIT

As it was discussed before, the materials under discussion here have a stress-strain relationship such, that the elastic modulus $dp/d\epsilon = E_0$ is constant below a certain stress value which is called the elastic limit; at this stress the value of E_0 undergoes a sudden decrease to a new E_1 value, which at increasing stress values decreases gradually still further. It has been found however, that on unloading the initial E_0 elastic modulus is valid, and that, furthermore, once the material is stressed to a p_i value above the elastic limit, thereby its new elastic limit is raised to the p_i value, that is, on the reapplication of stress the material will follow the E_0 line up till that p_i value is attained.

In accordance with these characteristics, also the velocity of propagation a, and the wave factor E/a, undergo first a sudden, then a gradual decrease in the stress range above the elastic limit. The change of the velocity of propagation with stress is illustrated in Fig. 15. The derivation of the $E = f(p)$; $a = f(p)$; $E/a = f(p)$ relationships from the basic $\epsilon = f(p)$ curve has been already given (Fig. 4), and is repeated here for annealed copper, at larger scale, Fig. 16. By integration of the a/E curve according to stress, the $\int dv = \int (a/E)dp$ curve is obtained, which has the characteristic that for any p value its slope is equal to the wave factor E/a for that p value. The integration can be performed either by computation or by graphical construction, in the known manner.

With the aid of the $\int dv$ curve in the v–p diagram the change of state in the bar can be followed in a manner similar to that explained in the preceding examples for elastic impact. The differences of the procedures may be listed as follows:

1. In the stress range below the elastic limit, that is, for $p_i < p_e$ the velocity of propagation a_i; and the wave factor L_i/a_i are constants:

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[2] Part I appeared in JOURNAL OF THE FRANKLIN INSTITUTE, Vol. 248, No. 1, July, 1949, p. 15.

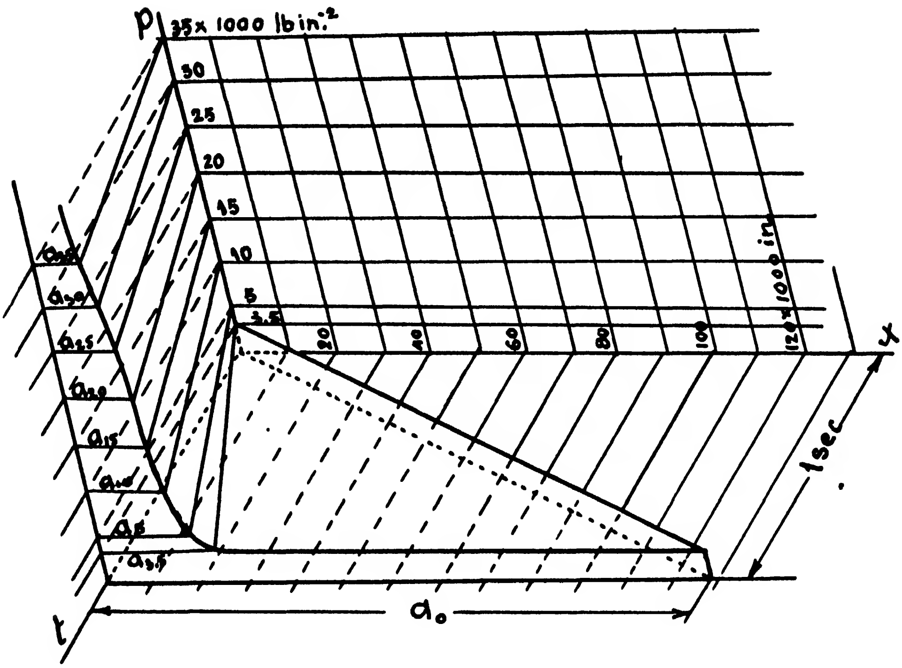


FIG. 15(*a*). Stereogram of stress for instantaneous change of stress, showing variation of velocity of propagation as a function of stress (for annealed copper, according to Figs. 4 and 16).

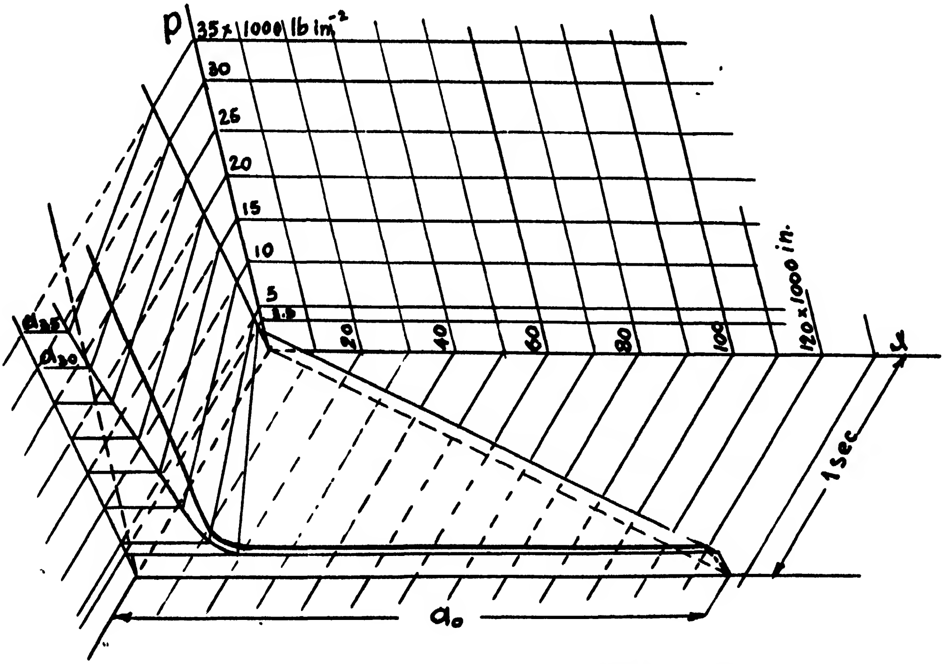


FIG. 15(*b*). Stereogram of stress for gradual change of stress ($p = p't$), showing variation of velocity of propagation as a function of stress. Gradient of stress is shown for the $t = 1$ time (for annealed copper, according to Figs. 4 and 16).

for $p_i < p_e$:

$$a_i = a_1 = \tan \varphi_1; \qquad \left(\frac{E}{a}\right)_i = \frac{E_1}{a_1} = \tan \alpha_1.$$

2. At the elastic limit, $p_i = p_e$, both the velocity of propagation, and the wave factor, undergo a sudden decrease to new values: for $p_i = p_e$:

$$a_i = a_2 = \tan \varphi_2; \qquad \left(\frac{E}{a}\right)_i = \frac{E_2}{a_2} = \tan \alpha_2.$$

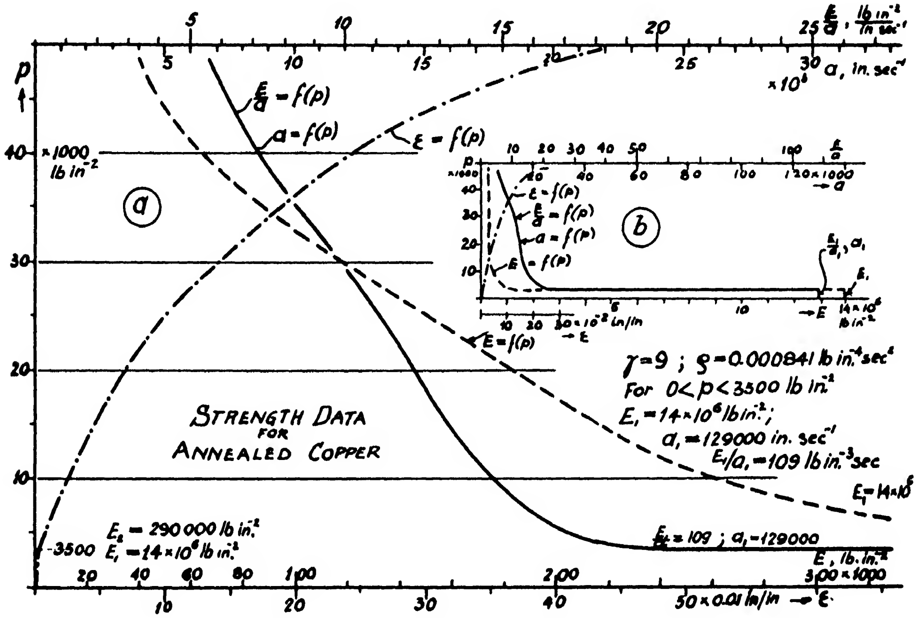


FIG. 16. The significant elastic and plastic relationships as derived from the stress-strain curve (for annealed copper, according to White and Griffis, ref. 34).

3. Above the elastic limit, $p_i > p_e$, the velocity of propagation, and the wave factor, change gradually, in a manner corresponding to the change of the square root elastic modulus for $p_i > p_e$:

$$a_i = \sqrt{\frac{E_i}{\rho}} = \tan \varphi_i; \qquad \left(\frac{E}{a}\right)_i = \sqrt{E_i \rho} = \tan \alpha_i.$$

The nature of these relationships for annealed copper is illustrated in Fig. 16.

4. The velocity of propagation and the wave factor, are, for any stress value, proportional:

$$a_i = \sqrt{\frac{E_i}{\rho}}; \qquad \frac{E_i}{a_i} = \sqrt{E\rho};$$

hence

$$\frac{E_i/a_i}{a_i} = \rho.$$

(The density was assumed to be constant.) Therefore, tan φ_i is proportional to tan α_1, and by suitable choice of the t, x, v, and p scales can be made equal (which facilitates the drafting work in the graphical construction).

5. It is to be noted, that if in the t–x diagram two adjacent domains are separated by a tan φ_i disturbance line, then the corresponding states in the p–v diagram are situated on a corresponding tan α_i directrix.

6. An interesting corollary of the variable velocity of propagation is that a reflection of waves is possible not only at the boundaries of the body but within the body itself. Such internal reflections will be treated in some of the following examples dealing with infinite bars.

The application of these principles shall be illustrated by a number of examples. First some examples shall be worked out in which the modulus of stiffness in the plastic range is assumed to be constant. In this case the operations will be carried out with two values a_1 and a_2, and also two corresponding values E_1/a_1 and E_2/a_2. In succeeding examples cases will be treated in which the velocity of propagation, and also the wave factor change as functions of stress.

Problem 11. Impact Slightly Above Elastic Limit, in Compression, of Infinite Bar (Fig. 17)

A simple problem will be analyzed, in which the impact stress is only slightly above the elastic limit, and in which the stress-strain relation above the elastic limit can be assumed to be linear. For the sake of clarity a definite material, annealed copper, is assumed, the properties of which are given in Fig. 16.

The impact at velocity v_3, is assumed to last $T = 1$ second; its intensity is 4000 lb. in.^{2} (elastic limit is 3500 lb. in.^{2}) corresponding to an impact velocity $v_3 = .63$ in. sec.$^{-1}$.

At the onset of impact two waves start: one with state 2 corresponding to the elastic limit, and propagated with velocity a_1, and the other with state 3, corresponding to the velocity of impact v_3, propagated with the slower velocity a_2 valid for the plastic range. The t–x domains which these states characterize are given in Fig. 17(b) and (b'). The bar length in domain 2 undergoes only elastic deformation, the bar length in domain 3 undergoes also a plastic deformation. At the termination of the impact period T the unloading wave is initiated, having state 4, which is propagated at velocity a_1 and overtakes the wave 3 (which is propagated at the slower velocity a_2) at the instant t_1, at which a length L_D of the bar has been plastically deformed. This wave is

reflected back toward the bar end as state 5, where it arrives at instant t_2 whereupon the bar resumes its initial state 6 (= state 1).

The displacement of the cross sections, and the location diagram are given by Fig. 17(b) and (b'), and the diagram of state by Fig. 17(c),

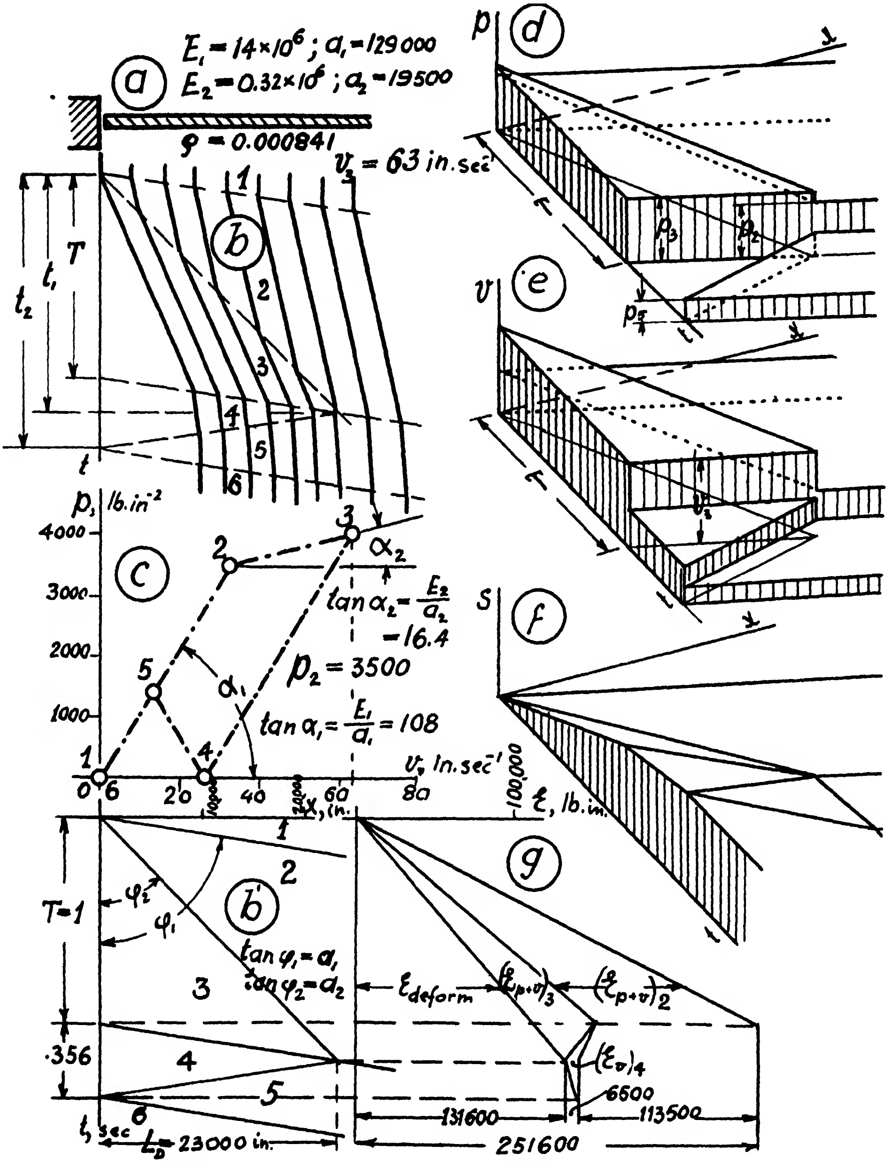


FIG. 17. Impact slightly above elastic limit, in compression, bar of infinite length.

(a) Schematic arrangement.
(b) Displacement of cross sections.
(b') Location diagram.
(c) Diagram of state.
(d) Stereogram of stress.
(e) Stereogram of velocity.
(f) Stereogram of displacement.
(g) History of energy relations.

which can be readily followed on the basis of the information given in the foregoing examples for elastic impact. From Fig. 17(b') and (c) the stereograms of stress and velocity, Fig. 17(d) and (e) can be readily constructed, and from these the stereogram of displacement, Fig. 17(f), derived. The energy relations are given in Fig. 17(g).

The calculation will be followed through, both by general algebra and numerical values.

Elastic Modulus: $E_1 = 14 \times 10^6$ lb. in.$^{-2}$. Elastic Limit: $p_2 = 3500$ lb. in.$^{-2}$.
$\rho = 0.000841$ lb. in.$^{-4}$ sec.^{2}.
Stiffness Modulus Above Elastic Limit: $E_2 = 0.32 \times 10^6$ lb. in.$^{-2}$.

$a_1 = 129{,}000$ in. sec.$^{-1}$; $E_1/a_1 = 108$; Duration of Impact: $T = 1$ sec.
$a_2 = 19{,}500$ in. sec.$^{-1}$; $E_2/a_2 = 16.4$.

For the instant t_1:

$$(t_1 - T)a_1 = t_1 a_2,$$

whence

$$t_1 = \frac{a_1 T}{a_1 - a_2} = 1.178 \text{ sec.}; \qquad t_1 - 1 = \frac{a_2}{a_1 - a_2} = 0.178 \text{ sec.}$$

and similarly

$$t_2 = \frac{a_1 + a_2}{a_1 - a_2} T = 1.356 \text{ sec.}; \qquad t_2 - 1 = \frac{2a_2}{a_1 - a_2} = 0.356 \text{ sec.}$$

Elastic Limit: $p_2 = 3500$ lb. in.$^{-2}$; the corresponding velocity is:

$$v_2 = p_2 \frac{a_1}{E_1} = 32.4 \text{ in. sec.}^{-1}.$$

Impact Stress: $p_3 = 4000$ lb. in.$^{-2}$; for the corresponding velocity it can be written:

$$\frac{p_3 - p_2}{v_3 - v_2} = \frac{E_2}{a_2},$$

whence $v_3 = 62.9$ in. sec.$^{-1}$. For the unloading velocity corresponding to state 4 ($p_4 = 0$) it can be written

$$\frac{p_3 - p_4}{v_3 - v_4} = \frac{E_1}{a_1},$$

whence $v_4 = 25.8$ in. sec.$^{-1}$. For state 5:

$$\frac{p_5 - p_4}{v_4 - v_5} = \frac{E_1}{a_1}, \quad \text{and} \quad \frac{p_5 - p_1}{v_5 - v_1} = \frac{E_1}{a_1},$$

whence

$$v_5 = \frac{v_4}{2} = 12.9 \text{ in. sec.}^{-1} \quad \text{and} \quad p_5 = 1390 \text{ lb. in.}^{-2}.$$

The energy relations can be expressed, in general terms:
Input energy

$$\mathcal{E}_1 = p_3 v_3 T = p_3 v_3.$$

This energy is distributed and applied as follows:

Energy at state p_2, v_2, for a bar length a_1:

$$\mathcal{E}_2 = p_2 v_2.$$

Energy at state p_5, v_5, for a bar length $(t_2 - T)a_1$.

$$\mathcal{E}_3 = (t_2 - 1)p_5 v_5.$$

Deformation energy for a bar length $(t_1 - 1)a_1$:

$$\mathcal{E}_4 = (t_1 - 1)a_1 \frac{p_3^2 - p_1^2}{2}\left(\frac{1}{E_2} - \frac{1}{E_1}\right).$$

From the conservation of energy it follows:

$$\mathcal{E}_1 = \mathcal{E}_2 + \mathcal{E}_3 + \mathcal{E}_4,$$

$$p_3 v_3 = p_2 v_2 + (t_2 - 1)p_5 v_5 + (t_1 - 1)a_1 \frac{p_3^2 - p_2^2}{2}\left(\frac{1}{E_2} - \frac{1}{E_1}\right)$$

$$= p_2 v_2 + \frac{2a_2}{a_1 - a_2}\frac{E_1}{a_1} v_5^2 + \frac{a_1 a_2}{a_1 - a_2}\tfrac{1}{2}(p_3^2 - p_2^2)\left(\frac{1}{E_2} - \frac{1}{E_1}\right)$$

Expressing every term in terms of p_2, v_2 and p_3, v_3, that is,

$$\frac{a_1}{a_2} = \frac{E_1/a_1}{E_2/a_2} = \frac{p_2}{v_2}\frac{v_3 - v_2}{p_3 - p_2}; \qquad \frac{E_1}{a_1} = \frac{p_2}{v_2}; \qquad \frac{E_2}{a_2} = \frac{p_3 - p_2}{v_3 - v_2};$$

$$\frac{a_1}{E_2} = \frac{a_1 a_2}{a_2 E_2} = \frac{p_2}{v_2}\frac{(v_3 - v_2)^2}{(p_3 - p_2)^2}$$

and substituting the above equation leads to identify. With the numerical values used in this Example:

Input Energy:

$$\mathcal{E}_1 = p_3 v_3 = 4000 \times 62.9 = 251{,}600 \text{ lb. in.}$$

Energy at state p_2, v_2:

$$\mathcal{E}_2 = p_2 v_2 = 3500 \times 32.4 = 113{,}500 \text{ lb. in.}$$

Energy at state p_5, v_5:

$$\mathcal{E}_3 = (t_2 - 1)p_5 v_5 = 0.356 \times 1390 \times 12.9 = 6380 \text{ lb. in.}$$

Permanent strain of the $L_D = (t_1 - 1)a_1 = 23{,}000$ in. length is:

$$e = L_D(p_3 - p_2)\left(\frac{1}{E_2} - \frac{1}{E_1}\right) = 35.2 \text{ in.}$$

Deformation energy:

$$\mathcal{E}_4 = (t_1 - 1)a_1 \frac{p_3^2 - p_2^2}{2}\left(\frac{1}{E_2} - \frac{1}{E_1}\right) = 131,500 \text{ lb. in.}$$

which satisfies the equation of energy within computational error.

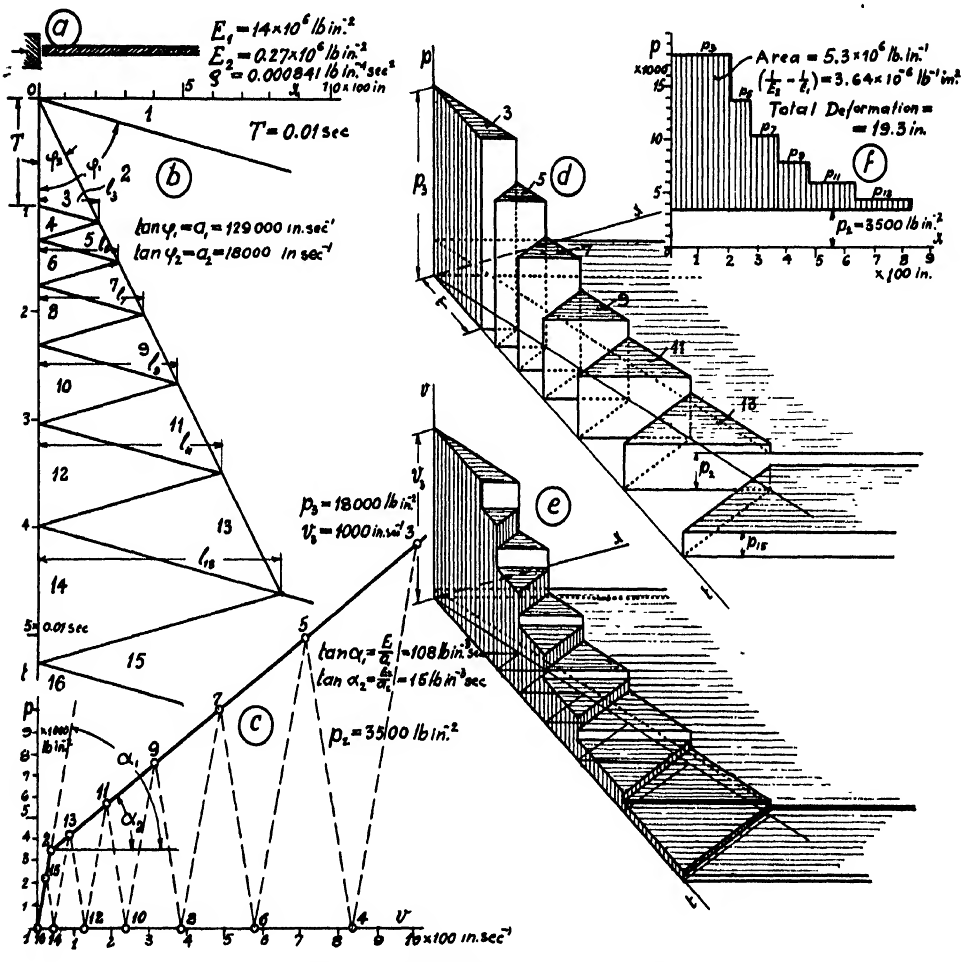


FIG. 18. Impact considerably above elastic limit, in compression, bar of infinite length.

(a) Schematic arrangement.
(b) Location diagram.
(c) Diagram of state.
(d) Stereogram of stress.
(e) Stereogram of velocity.
(f) Deformation diagram.

Problem 12. Impact Stress Considerably Above Elastic Limit, in Compression, of Infinite Bar (Fig. 18)

An impact at higher velocity, $v_3 = 1000$ in. sec.$^{-1}$, is analyzed in Fig. 18, making again the simplifying assumption, that the stiffness factor E_2 above the elastic limit is constant. The material is assumed

to be annealed copper, as in the previous example. The impact stress is 18,000 lb. in.$^{-2}$, at which the actual stiffness factor is 0.20×10^6 lb. in.$^{-2}$ (see Fig. 16); at the elastic limit, $p_2 = 3500$ lb. in.$^{-2}$, the stiffness factor is about 0.34×10^6 lb. in.$^{-2}$. Therefore the analysis will be performed with a mean value $E_2 = 0.27 \times 10^6$ lb. in.$^{-2}$, corresponding to values $a_2 = 18{,}000$ in. sec.$^{-1}$, and $E_2/a_2 = 15$ lb. in.$^{-3}$ sec.

The location diagram and the diagram of state are given in Fig. 18(b) and (c), from which the stereograms of stress, Fig. 18(d), and of velocity, Fig. 18(e), can be constructed. These give a very clear mental picture of the changes of pressure and velocity during and after the impact period. It is seen that after the termination of impact (at the end of the period when the stress p_3 is reduced to zero) repeated internal reflections take place at stresses of ever decreasing intensity (p_3, p_5, p_7, . . .) affecting an ever increasing length of the bar, and taking more and more time, until the stress decreases to a value p_{15} below the elastic limit. In the p-stereogram these changes appear as flat-topped peaks p_3, p_5, p_7 . . . arising out of a large plateau of p_2 value, and a foothill of p_{15}, above the t–x plane. Thus, while the impact itself lasts only $T = 0.01$ sec., the disturbance wave is of much longer duration, in this case 0.053 sec. After it passes, it leaves in its wake a portion of bar, 850 in. long, permanently strained. Two adjoining elastic waves remain in the bar: one of intensity p_2 and 0.04 sec. duration, and the adjoining one of intensity p_{15} of 0.013 sec. duration which are traveling along the bar with the velocity of propagation $a_1 = 129{,}000$ in. sec.$^{-1}$.

Interest attaches to the total amount of permanent deformation suffered by the portion of the bar which was subjected to stresses above the yield point. Referring to the t–x diagram Fig. 18(b) it is seen that the first length $l_3 = 200$ in. of the bar is subjected to stress p_3, then the first length $l_5 = 260$ in. to p_5 stress, then again the first length $l_7 = 370$ in. to stress p_7, and so on, until finally the first length $l_{13} = 850$ in., to stress p_{13}, beyond which distance only elastic, but not plastic deformation will take place. In calculating the total permanent deformation, it is assumed (with more or less experimental verification) that once the bar has been subjected to a stress p_i beyond the elastic limit, then the elastic limit has been raised thereby to p_i, and only stresses above will produce a further yield of the bar. Thus, with the notations of Fig. 18 the total permanent deformation will be:

$$D = \left(\frac{1}{E_2} - \frac{1}{E_1}\right)[(p_3 - p_2)l_3 + (p_5 - p_2)(l_5 - l_3) + (p_7 - p_2)(l_7 - l_5) \\ + (p_9 - p_2)(l_9 - l_7) + (p_{11} - p_2)(l_{11} - l_9) + (p_{13} - p_2)(l_{13} - l_{11})],$$

which is equal to:

$$D = \left(\frac{1}{E_2} - \frac{1}{E_1}\right)[(p_3 - p_5)l_3 + (p_5 - p_7)l_5 + (p_7 - p_9)l_7 \\ + (p_9 - l_{11})l_9 + (p_{11} - p_{13})l_{11} + (p_{13} - p_2)l_{13}.$$

The graphical significance of the expression within the square parenthesis is the boundary silhouette of the p-stereogram, projected against the p–x plane, integrated above the p_2 line, as shown in Fig. 18(f). The area multiplied by the $(1/E_2 - 1/E_1)$ factor gives the total permanent deformation. In the present example:

$$\text{Area} \times \left(\frac{1}{E_2} - \frac{1}{E_1}\right) = 5.3 \times 10^6 \times 3.64 \times 10^{-6} = 19.3 \text{ in.}$$

Problem 13. Impact Above Elastic Limit, in Compression, of Finite Bar, with Other End Free (Fig. 19)

The numerical data, the location diagram, and the diagram of state are given in Fig. 19(a), (b), and (c). The assumed length of the bar is 64.5 in.; it is subjected at its one end to a sustained impact velocity of 150 in. sec.$^{-1}$. The wave 2 corresponding to the elastic limit of 3500 lb. in.$^{-2}$, and a velocity of v_2 = 32.3 in. sec.$^{-1}$, arrives at the free end at the t = 0.0005 sec. instant; the stress is relieved there, and the wave 4 (v_4 = 64.6 sec.$^{-1}$, p_4 = 0) returns towards the impact end. The wave 3 corresponding to the impact velocity (v_3 = 150 in. sec.$^{-1}$, p_3 = 5275 lb. in.$^{-2}$) in the meantime proceeds towards the free end with the velocity a_2 of propagation, and meets the returning wave 4 at the 0.000875 sec. instant whereby the plastic wave 6, and elastic wave 5, are originated, traveling towards the two ends of the bar. The further progress of the stress-velocity phenomenon can be followed in the v–p diagram, Fig. 19(c), and the domains for which successive states 7, 8, 9 . . . are valid can be determined in the t–x diagram. In the v–p diagram changes of state below the elastic limit of 3500 lb. in.$^{-2}$, and also changes of state at decreasing stress take place along $\tan \alpha_1 = E_1/a_1$ directrices; changes of state above the elastic limit and at increasing stress take place along $\tan \alpha_2 = E_2/a_2$ directrices. It can be observed that whenever in the t–x diagram two adjacent domains are *separated* by a $\tan \varphi_1$, or a $\tan \varphi_2$ line, then in the v–p diagram the corresponding states are *connected* by a $\tan \alpha_1$, or a $\tan \alpha_2$ directrix, respectively.

The stereograms of stress and of velocity may be constructed from the data of Fig. 19(b) and (c) without difficulty.

Particular interest attaches to the history of velocity at the two ends of the bar. At the T = 0.00235 sec. instant the impact end of the bar attains a velocity higher than the impact velocity, v_{16} = 162 in. sec.$^{-1}$, and the bar separates from the hammer, the impact ends. The free end of the bar has gradually attained and surpassed the impact velocity, in steps of v_4, v_8, v_{13}, v_{17}, etc. The mean velocity of the bar is of the order of 170 in. sec.$^{-1}$, and there remains in the bar a residual elastic energy which is continually interchanged from stress energy to velocity energy and *vice versa*.

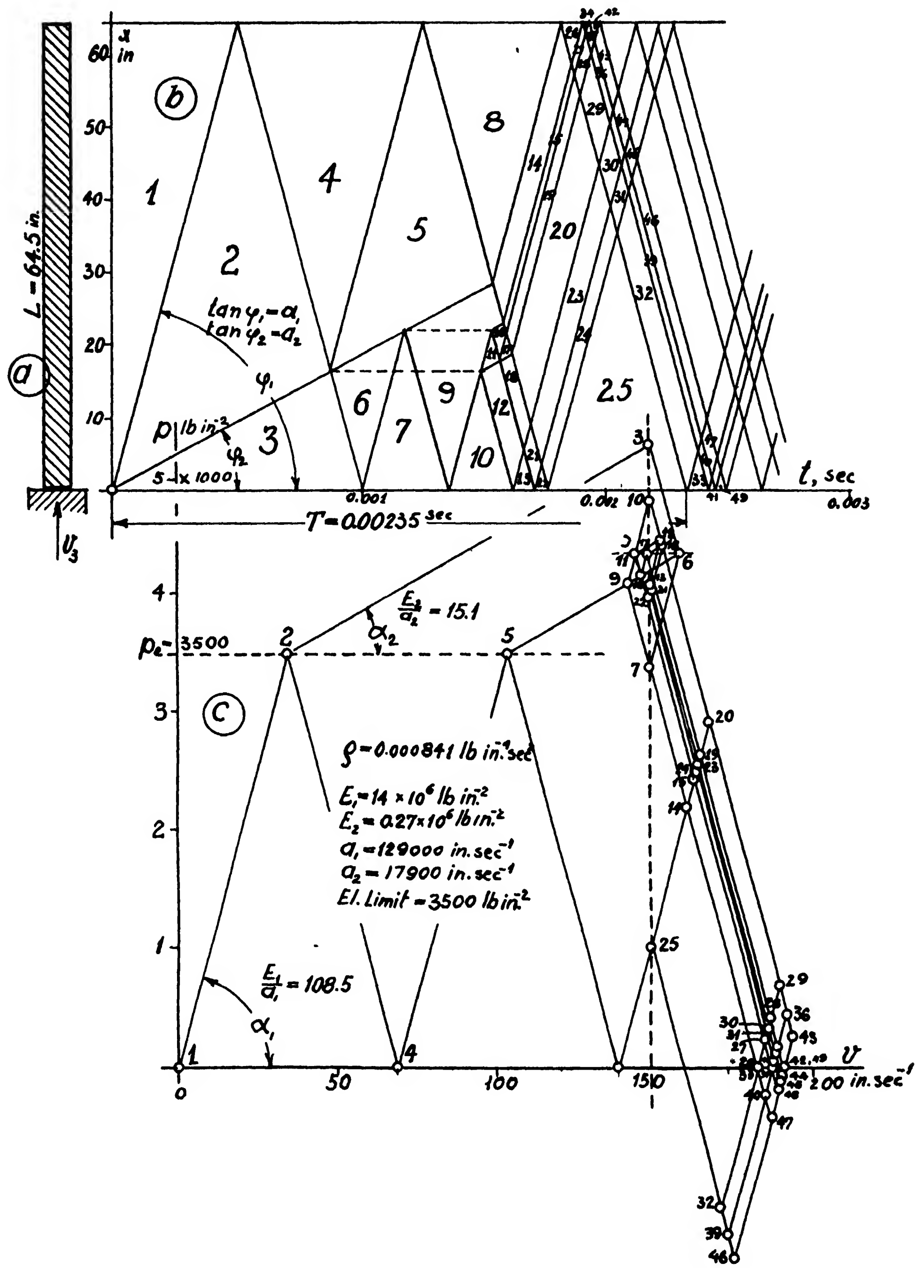


FIG. 19. Impact above elastic limit, in compression, bar of finite length, with free end.

(a) Schematic arrangement.
(b) Location diagram.
(c) Diagram of state.

The energy balance can be set up as follows:

Input energy

$$= v_3p_3 \times 0.001 + v_7p_7 \times 0.0075 + v_{12}p_{12} \times 0.0006,$$
$$= 150(5275 \times 0.001 + 3500 \times 0.0075 + 2100 \times 0.0006),$$
$$= 1375 \text{ lb. in.}$$

Deformation energy

$$= \frac{1}{2}\left(\frac{1}{E_2} - \frac{1}{E_1}\right)[(p_3^2 - p_2^2)L_3 + (p_6^2 - p_2^2)(L_6 - L_3) + (p_9^2 - p_2^2)(L_9 - L_6)],$$
$$= \tfrac{1}{2} \times 3.629[15.75 \times 16.15 + 7.15 \times 4.85 + 2.95 \times 6.45],$$
$$= 558 \text{ lb. in.}$$

$$\text{Velocity energy} = \frac{170^2}{2} \times 64.5 \times 0.000841 = 785 \text{ lb. in.}$$

The difference: $1375 - (558 + 785) = 32$ lb. in. is the residual elastic energy in the bar.

Problem 14. Impact Above Elastic Limit, in Compression, of Finite Bar, with Other End Fixed (Fig. 20)

The numerical data, the location diagram and the diagram of state are given in Fig. 20(*a*), (*b*), and (*c*). The assumed length of the bar is 15.75 in. whereby the returning wave 5 arrives at the impact end at 0.001 sec., at which instant the impact is assumed to be terminated, $T = 0.001$ sec. In the v–p diagram changes of state below the elastic limit of 3500 lb. in.$^{-2}$, and also changes of state at decreasing stress, take place along the E_1/a_1 directrices; changes of state above the elastic limit and at increasing stress take place along E_2/a_2 directrices. Similarly, in the t–x diagram the tan φ_1, resp. tan φ_2 disturbance lines are used for changes along the E_1/a_1, resp. E_2/a_2 directrices. Thus the domains 1, 2, 3 . . . in the t–x diagram are obtained, for which the correspondingly numbered states in the v–p diagram apply. It is to be noted that if in the t–x diagram two adjacent domains are *separated* by a tan φ_1, disturbance line, then in the v–p diagram the corresponding states are *connected* by a tan α_1 directrix; similar relation is valid between the tan φ_2 disturbance lines and the tan α_2 directrices.

The final result is that the l_7 portion of the bar is stressed and correspondingly deformed by $p_7 = 6261$ lb. in.$^{-2}$, and the l_5 portion of the bar is stressed and deformed by $p_5 = 5123$ lb. in.$^{-2}$. The total deformation energy is 488 in. lb., while the input energy is 499 in. lb. The difference remains in the bar in the form of elastic energy, which is continuously interchanged from potential to kinetic energy and *vice*

versa, as would be seen in the stereograms of stress and velocity if these were constructed.

After the impact is over, and assuming that the hammer stays in its final position, the bar is elastically clamped between the hammer and

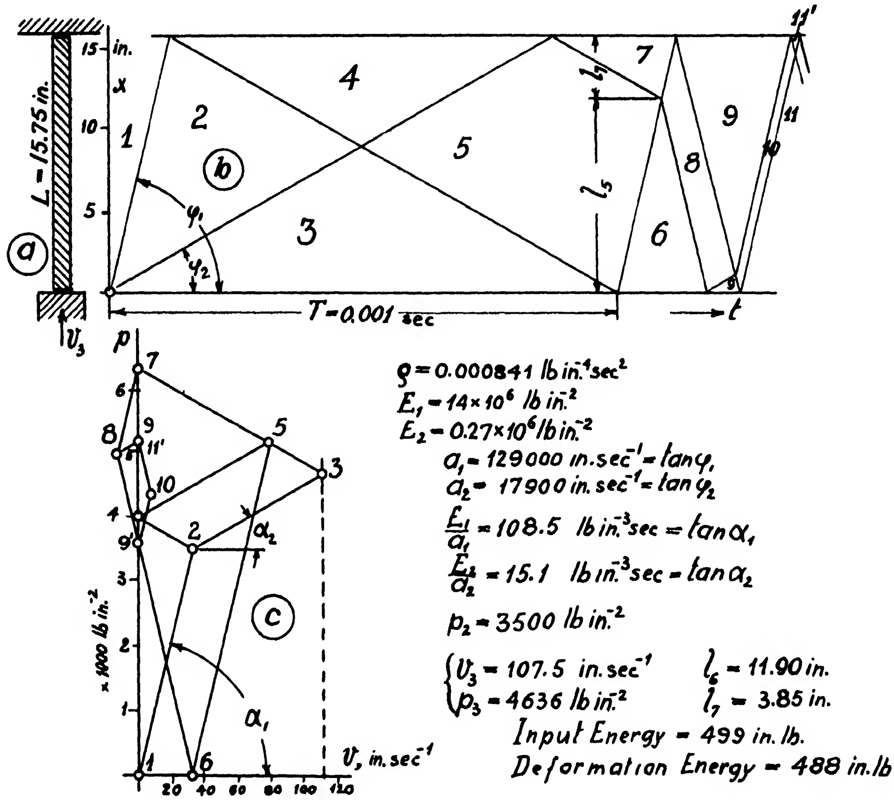


FIG. 20. Impact above elastic limit, in compression, bar of finite length, with fixed end.

(*a*) Schematic arrangement.
(*b*) Location diagram.
(*c*) Diagram of state.

the end mass, exerting a pressure alternating between 3500 and 6500 lb. in.$^{-2}$ (points 9, 9′, 11, 11′, etc., in the v–p diagram), the mid-portion of the bar vibrating between the two fixed ends with a velocity of about ± 14 in. sec.$^{-1}$ in both directions.

Problem 15. Impact Above Elastic Limit, in Compression, of Infinite Bar, Gradually Changing Elastic Properties (Fig. 21)

The velocity of propagation $a_1 = \tan \varphi_1$, and also the wave factor $E_1/a_1 = \tan \alpha_1$ are constant within the range of 0 stress up to the elastic limit. Henceforth they change abruptly to a new $\tan \varphi_2$ and $\tan \alpha_2$ value, and from the elastic limit onward they gradually change to $\tan \varphi_a$, $\tan \varphi_b$, $\tan \varphi_3$, and $\tan \alpha_a$, $\tan \alpha_b$, $\tan \alpha_3$. Assuming an impact at v_3

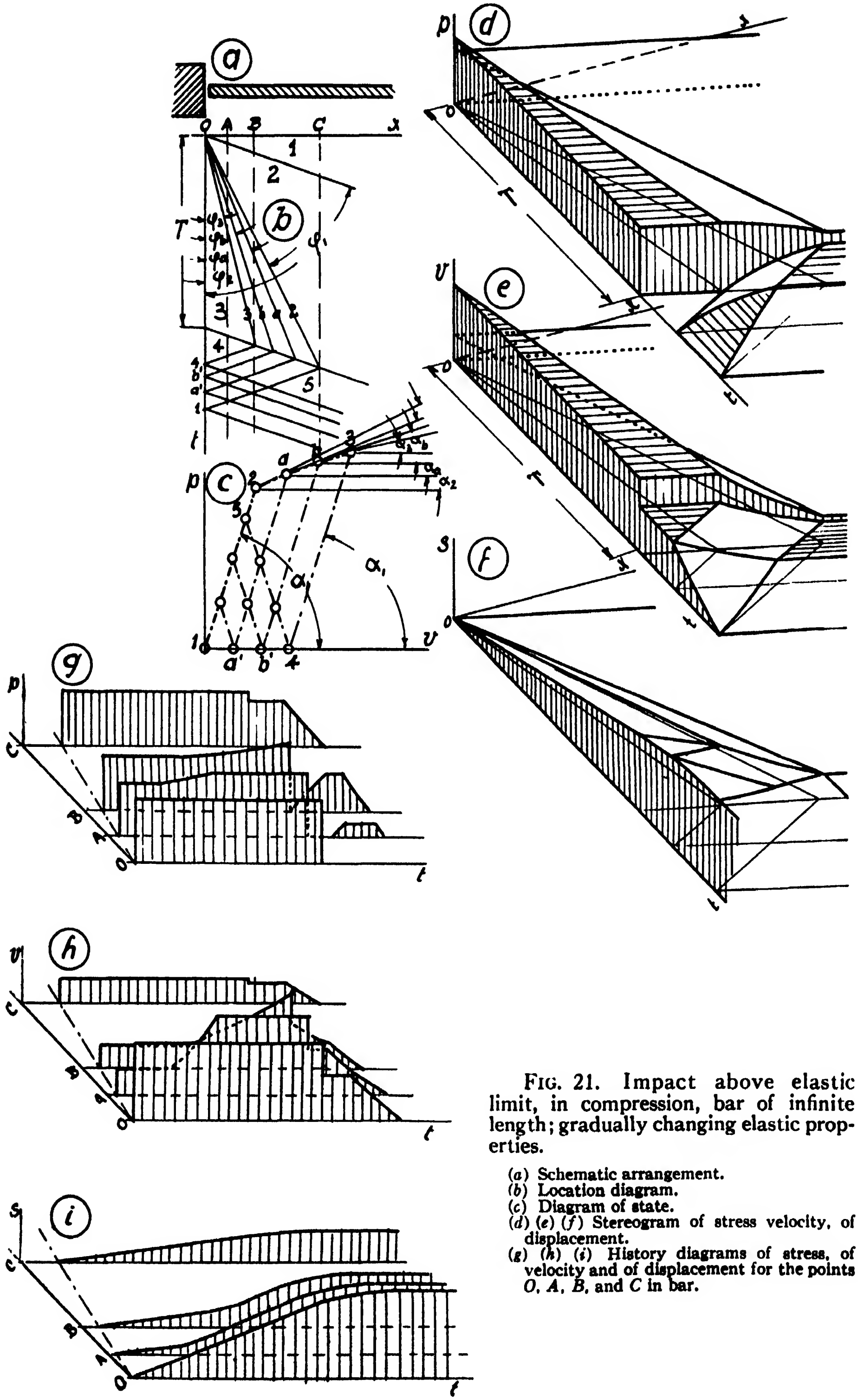


FIG. 21. Impact above elastic limit, in compression, bar of infinite length; gradually changing elastic properties.

(a) Schematic arrangement.
(b) Location diagram.
(c) Diagram of state.
(d) (e) (f) Stereogram of stress velocity, of displacement.
(g) (h) (i) History diagrams of stress, of velocity and of displacement for the points O, A, B, and C in bar.

velocity lasting T time the t–x and v–p diagrams are given in Fig. 21(b) and (c). Constructing the stereograms Fig. 21(d) and (e) it is seen that the end portion of the impact is represented by gradually sloping surfaces and with plateaus of constant states corresponding to points 2, 3, 4, and 5 in the v–p diagram.

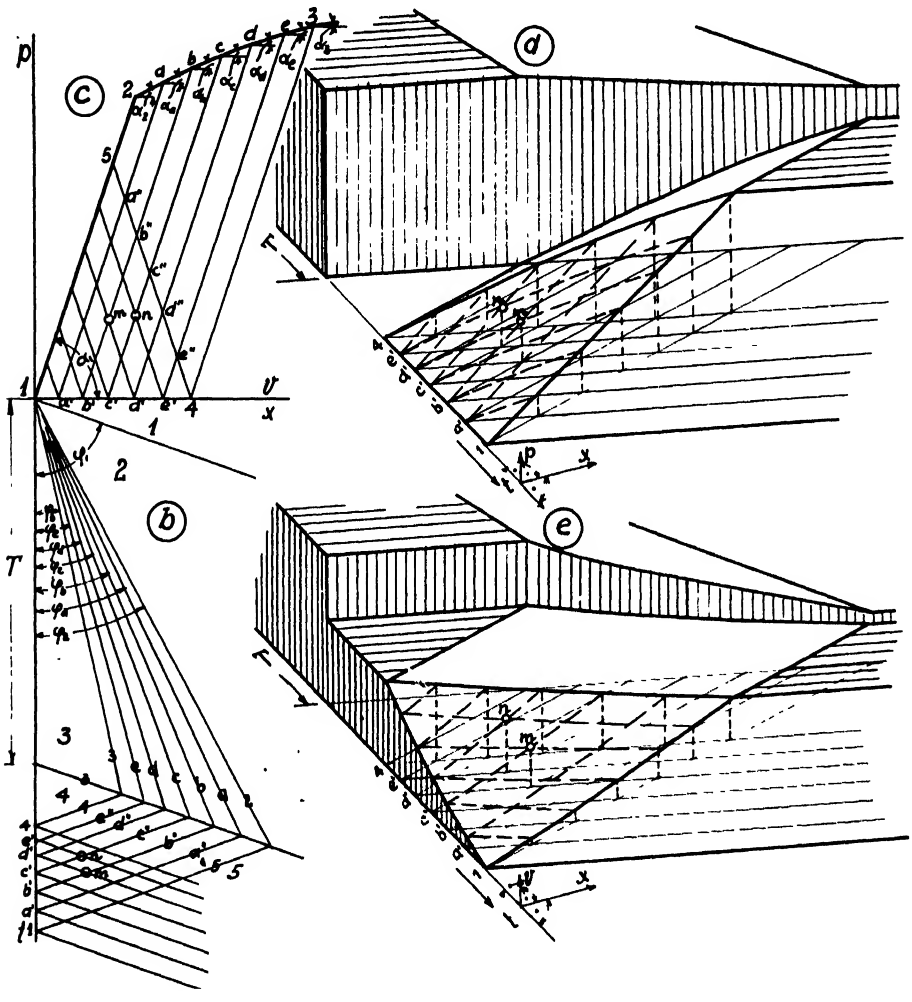


FIG. 22. End of impact period for case of Fig. 21.

(b) Location of diagram.
(c) Diagram of state.
(d) Stereogram of stress.
(e) Stereogram of velocity.

In order to show the phenomenon with greater clarity, Fig. 21(g) and (h) show the history diagrams of stress and velocity for 4 cross sections: 0, A, B, C; from Fig. 21(h), by integration the history of displacement, Fig. 21(i) can be derived, which in their entirety form the stereogram of displacement, Fig. 21(f).

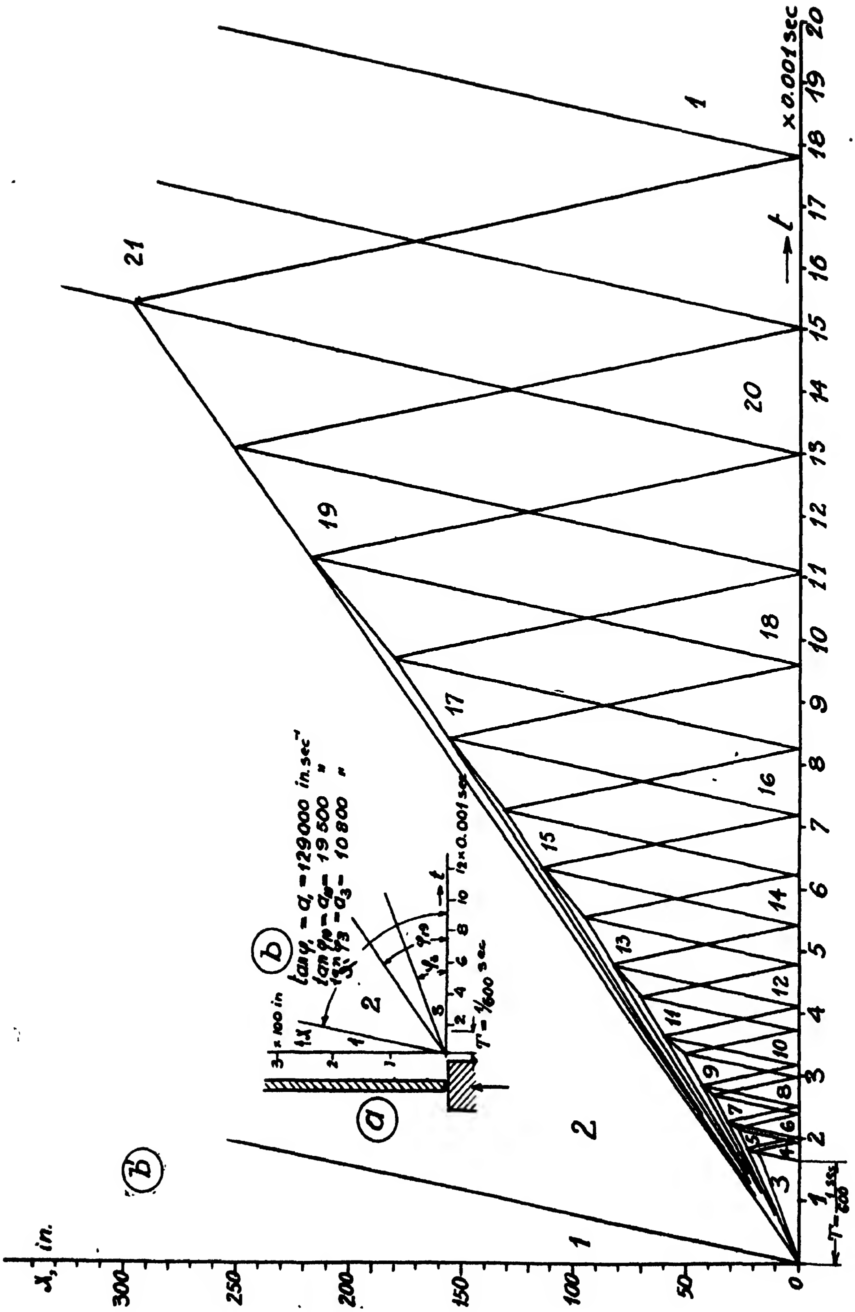
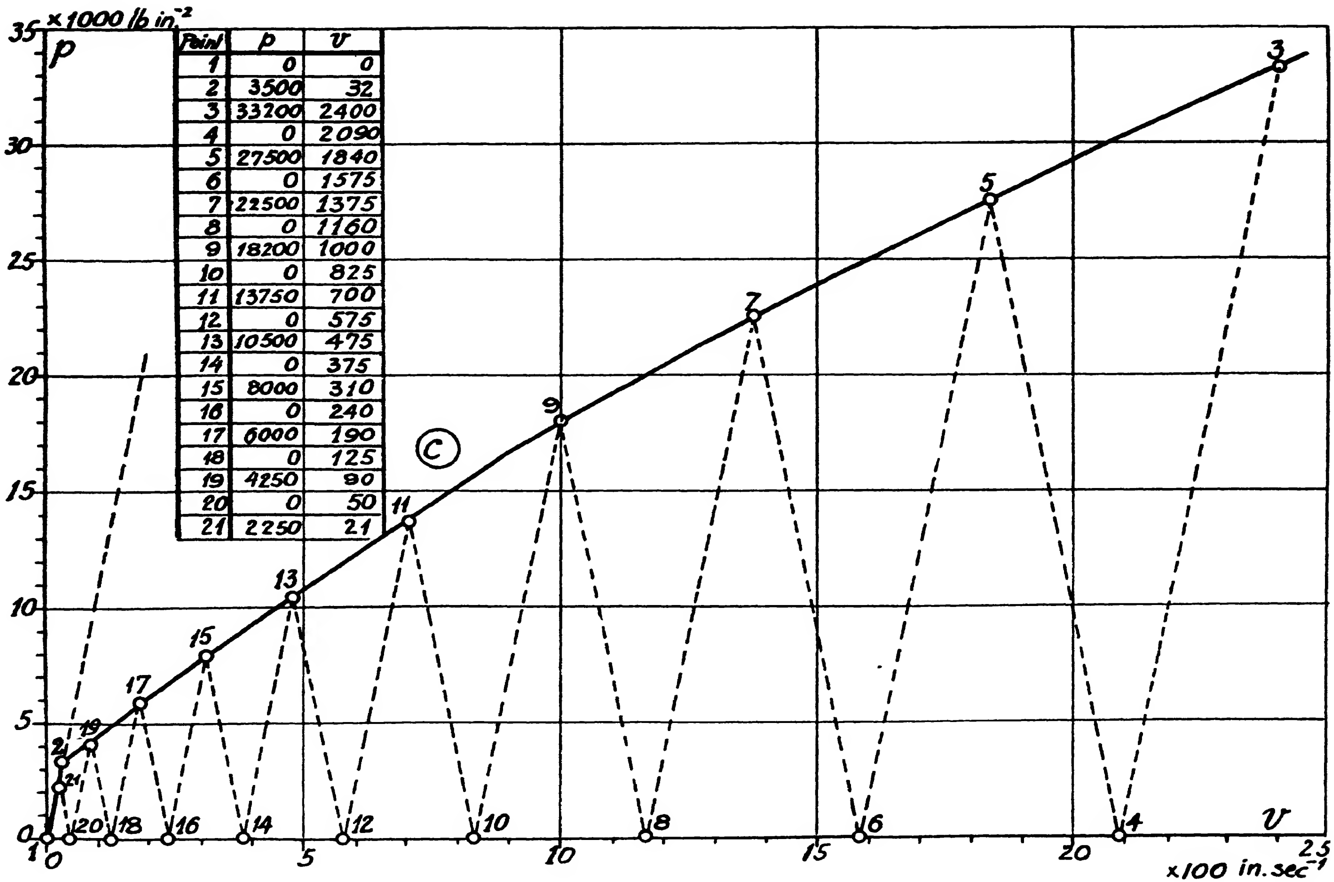


FIG. 23(*a*, *b*, *b'*). Impact above elastic limit, in compression, at high stress, bar of infinite length; gradually changing elastic properties.

(*a*) Schematic arrangement.
(*b*) Location diagram.
(*b'*) Location diagram at enlarged scale.



Point	p	v
1	0	0
2	3500	32
3	33200	2400
4	0	2090
5	27500	1840
6	0	1575
7	22500	1375
8	0	1160
9	18200	1000
10	0	825
11	13750	700
12	0	575
13	10500	475
14	0	375
15	8000	310
16	0	240
17	6000	190
18	0	125
19	4250	90
20	0	50
21	2250	21

FIG. 23(*c*). Impact above elastic limit, in compression, at high stress, bar of infinite length; gradually changing elastic properties.

(*c*) Diagram of state.

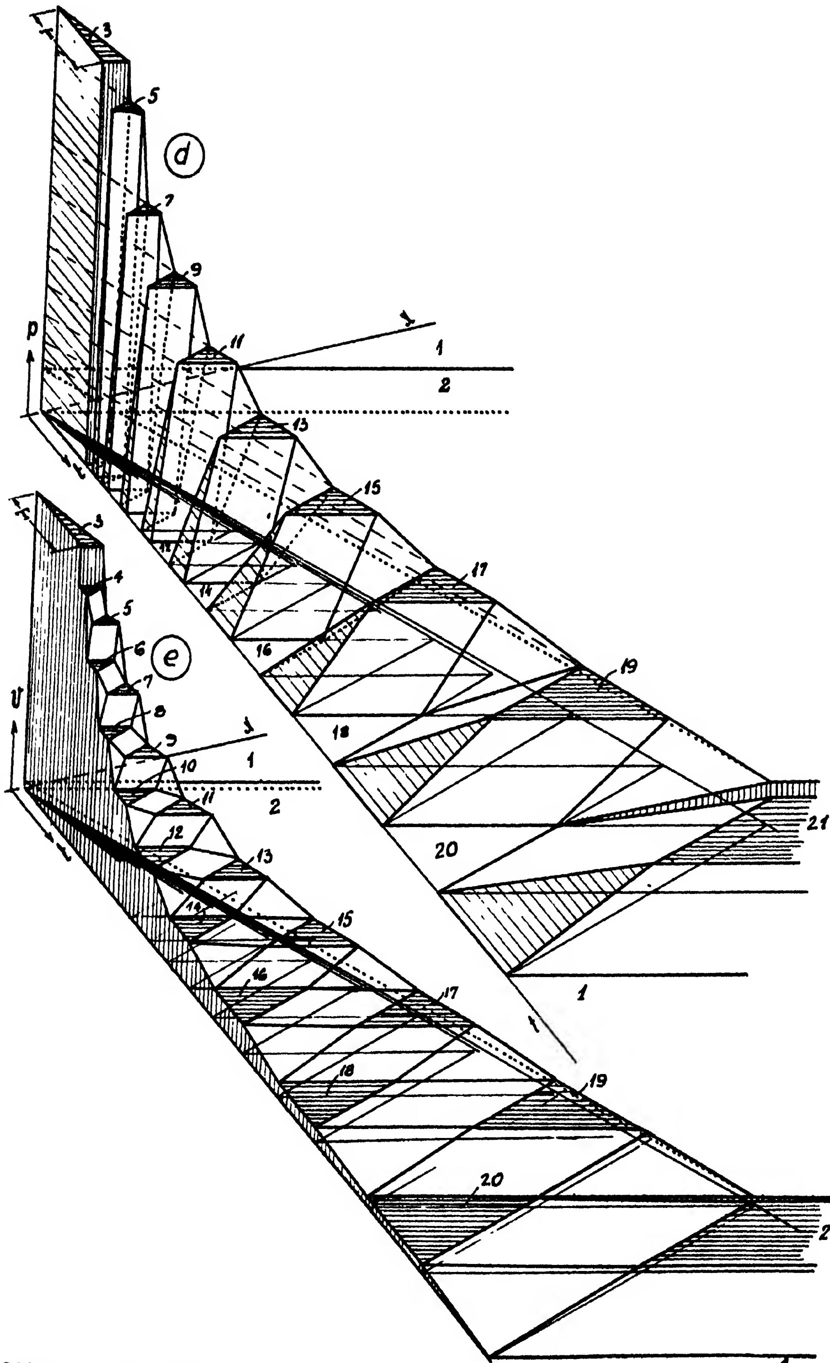


FIG. 23(*d*, *e*). Impact above elastic limit, in compression, at high stress, bar of infinite length; gradually changing elastic properties.

(*d*) Stereogram of stress.
(*e*) Stereogram of velocity.

The construction of the sloping portions of the stereogram is of interest, because this condition did not arise in the examples of elastic impact given before. Therefore in Fig. 22 the t–x and v–p diagrams of the previous Fig. 21 are given at a larger scale, and also the end portion of the stereograms of stress and velocity. For two points, m and n the graphical analysis is carried through. Basically, the procedure is quite simple, to assign a point in the v–p diagram (which point is located at the intersection of two directrices tan $(+\alpha)$ and tan $(-\alpha)$), to a point in the t–x diagram (which point is located at the intersection of two disturbance lines, tan $(+\varphi)$ and tan $(-\varphi)$). The procedure is fundamentally the same as in the case of elastic impact, but it requires more attention.

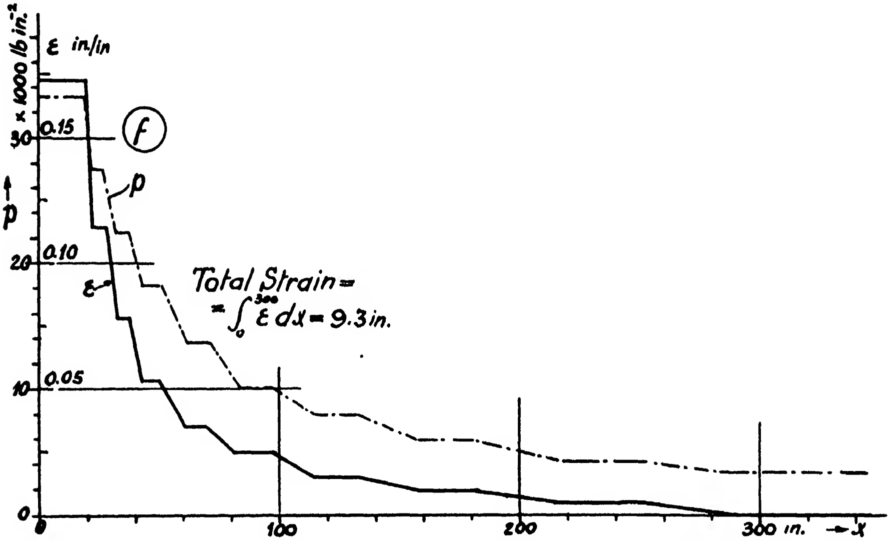


FIG. 23(f). Impact above elastic limit, in compression, at high stress, bar of infinite length; gradually changing elastic properties.

(f) Deformation diagram.

Problem 16. Impact Above Elastic Limit, in Compression, at High Stress, of Infinite Bar, Gradually Changing Elastic Properties (Fig. 23)

The compression at impact for $T = 1/600$ sec. at 2400 in. sec.$^{-1}$, of an annealed copper bar, will be graphically analyzed. This problem has been treated by White and Griffis mathematically before, and the present treatment furnishes a good comparison of the two methods.

The t–x diagram and v–p diagrams are based on the stress strain diagram of annealed copper, Fig. 16. The procedure can be readily followed on the basis of the former examples. The stereograms of stress and velocity are given in Fig. 23(d) and (e). It is of interest to determine the total permanent deformation undergone by the bar. In a manner similar to that of a previous example (Fig. 18(f)) also in this

case the silhouette of the stress stereogram is taken on the p–x coordinate plane, Fig. 23(f). From this the corresponding strain diagram ϵ–x is determined on the basis of the $\epsilon = f(p)$ curve. The area of the ϵ–x line above the elastic limit gives the total elongation $\int_0^{800} \epsilon dx = 9.3$ in. (Fig. 23(f)).

The data obtained by graphical analysis agree with the mathematically determined results of the previous authors within the limits of computational and constructional error.

CONCLUSION

The graphical method of analysis of impact in the elastic and plastic range has been explained, and numerous applications have been given. The examples treated are believed sufficient as a guidance in the application of the method to other similar phenomena.

APPENDIX

Determination of the Velocity Function from the Stress-Strain Function

It was shown that the relation between velocity and stress, $v = f(p)$ and the relation between velocity and strain, $v = f(\epsilon)$, are interrelated with the stress-strain function, $\epsilon = f(p)$, by the following equations:

$$\frac{dv}{dp} = \sqrt{\frac{d\epsilon}{dp}\frac{1}{\rho}}\left(= \frac{1}{a\rho}\right),$$

and

$$\frac{dv}{d\epsilon} = \frac{dv}{dp}\frac{dp}{d\epsilon} = \sqrt{\frac{dp}{d\epsilon}\frac{1}{\rho}}\,(=a).$$

If the stress-strain function is given in analytical form, then the velocity functions can be derived analytically. If the stress-strain function is given graphically by a curve then the velocity functions can be determined by means of graphical construction. Examples will be given of both procedures.

Analytical Method

Assume that the stress-strain function is represented in the range of $0 < p < p_1$ and $0 < \epsilon < \epsilon_1$ by a straight line:

$$\epsilon = Ap \quad \text{(elastic range)},$$

and beyond that, that is, for $p_1 < p$, $\epsilon_1 < \epsilon$ values, by a quadratic parabola:

$$\epsilon = Bp^2 \quad \text{(plastic range)},$$

which two lines join at the p_1, ϵ_1 point for which:

$$Ap_1 = Bp_1^2; \qquad \text{i.e., } p_1 = \frac{A}{B}; \qquad \epsilon_1 = \frac{A^2}{B} \quad \text{(elastic limit)}.$$

The velocity functions can be expressed in the elastic range:

$$\frac{dv}{dp} = \sqrt{\frac{d\epsilon}{dp}\frac{1}{\rho}} = \sqrt{\frac{A}{\rho}} \quad \text{and} \quad \frac{dv}{d\epsilon} = \sqrt{\frac{1}{A\rho}};$$

$$v = \sqrt{\frac{A}{\rho}} \cdot p, \quad \text{and} \quad v = \sqrt{\frac{1}{A\rho}} \cdot \epsilon,$$

and in the plastic range:

$$\frac{d\epsilon}{dp} = 2Bp, \qquad \frac{dv}{dp} = \sqrt{\frac{d\epsilon}{dp}\frac{1}{\rho}} = \sqrt{\frac{2B}{\rho}} \cdot p^{\frac{1}{2}},$$

and

$$\frac{dv}{d\epsilon} = \sqrt{\frac{dp}{d\epsilon}\frac{1}{\rho}} = \sqrt{\frac{1}{2B\rho}} \cdot p^{-\frac{1}{2}} = \sqrt[4]{\frac{1}{4B\rho^2}} \cdot \epsilon^{-\frac{1}{4}},$$

whence

$$v = \frac{2}{3}\sqrt{\frac{2B}{\rho}} \cdot p^{\frac{3}{2}} = \sqrt{\frac{8}{9}\frac{B}{\rho}}\sqrt{p^3},$$

and

$$v = \sqrt[4]{\frac{1}{4B\rho^2}} \cdot \frac{4}{3}\epsilon^{\frac{3}{4}} = \sqrt{\frac{8}{9\rho\sqrt{B}}}\sqrt[4]{\epsilon^3}.$$

Numerical Example

For annealed copper the stress-strain diagram can be approximated by:

in the elastic range

$$\epsilon = Ap = \frac{1}{E_0}p, \qquad A = \frac{1}{14 \times 10^6} = 0.0715 \times 10^{-6},$$

and in the plastic range

$$\epsilon = Bp^2, \qquad B = 1.5 \times 10^{-10}.$$

(This gives for the elastic limit: $p_1 = A/B = 475$ lb. in.$^{-2}$ instead of the actual 3500 lb. in.$^{-2}$. For a closer approximation a better fitted curve could be obtained by a parabola of higher order. But for the purpose of illustration the present assumption of a quadratic parabola appears to be adequate.)

The velocity functions will be for the elastic range:

$$v = \sqrt{\frac{0.0715 \times 10^{-6}}{0.000841}} \cdot p = 0.92 \times 10^{-2}p$$

and

$$v = \sqrt{\frac{1}{0.0715 \times 10^{-6} \times 0.000841}} = 129{,}000\epsilon,$$

and for the plastic range:

$$v = \sqrt{\frac{9}{8} \cdot \frac{1.5 \times 10^{-10}}{0.000841}}\sqrt{p^3} = 4 \times 10^{-4}\sqrt{p^3}$$

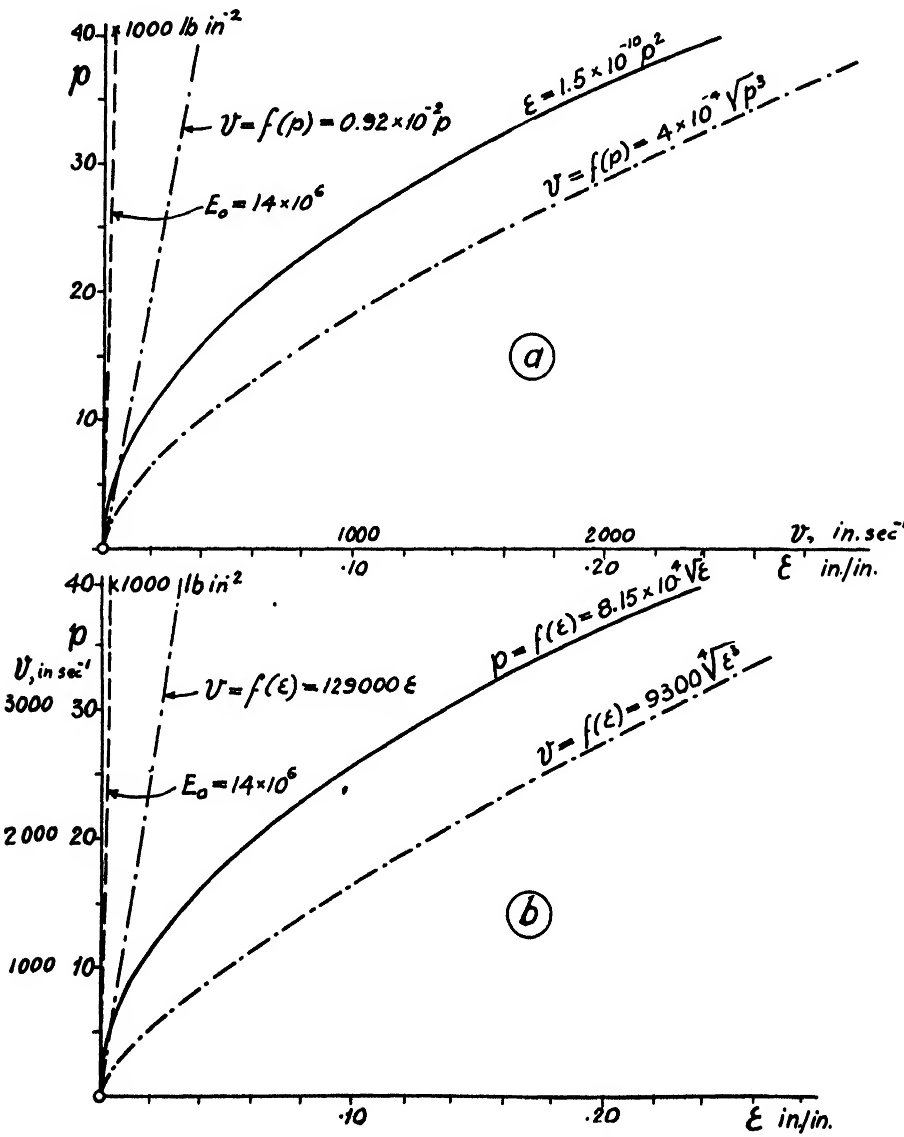


FIG. 24. Determination of the velocity function from the stress-strain function. (Functions given by formulas.)

(a) $v = f(p)$ derived from $\epsilon = f(p)$.
(b) $v = f(\epsilon)$ derived from $p = f(\epsilon)$.

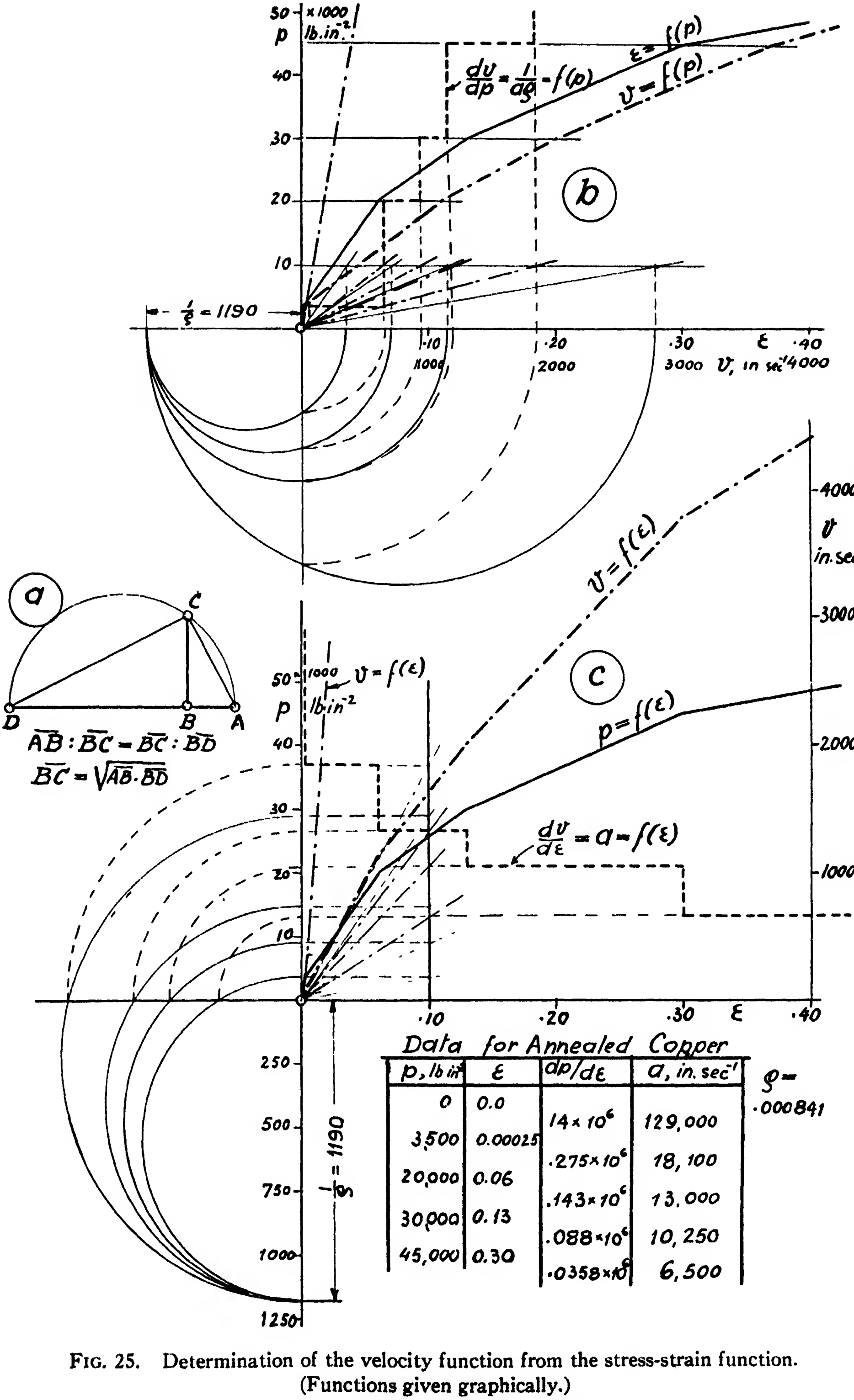


FIG. 25. Determination of the velocity function from the stress-strain function. (Functions given graphically.)

(a) Method of extracting square root.
(b) Relations as functions of stress.
(c) Relations as functions of strain.

and

$$v = \sqrt{\frac{8}{9} \cdot \frac{1}{0.000841\sqrt{1.5 \times 10^{-10}}}} \sqrt[4]{\epsilon^3} = 9300\sqrt[4]{\epsilon^3}.$$

These functions are represented in Fig. 24.

Graphical Method

This is based on the known methods of extracting the square root, and integration by graphical procedures.

Extracting the square root is illustrated in Fig. 25(a). From similar triangles it can be written:

$$\overline{AB}:\overline{BC} = \overline{BC}:\overline{BD},$$

whence

$$\overline{BC} = \sqrt{\overline{AB} \cdot \overline{BD}}.$$

If, then $\overline{AB} = \dfrac{d\epsilon}{dp}$, and $\overline{BD} = \dfrac{1}{\rho}$, then $\overline{BC} = \sqrt{\dfrac{d\epsilon}{dp} \cdot \dfrac{1}{\rho}} = \dfrac{dv}{dp}$, or if $\overline{AB} = \dfrac{dp}{d\epsilon}$, and $\overline{BD} = \dfrac{1}{\rho}$, then

$$\overline{BC} = \sqrt{\frac{dp}{d\epsilon} \cdot \frac{1}{\rho}} = \frac{dv}{d\epsilon}.$$

The application of this method to the determination of the v-functions is shown in Fig. 25. The stress-strain diagram of annealed copper is represented by a broken line formed by the tangents of the experimentally obtained characteristics, each tangent being valid for a definite $p_i - p_{i-1}$ interval. Drawing, from the origin of the coordinate system, lines parallel with each of these tangents, the intersects with a $\Delta p = 10{,}000$ lb. in. line will be proportional to the $d\epsilon/dp$ differential quotients. Applying the above explained procedure, the values of dv/dp are obtained for each stress interval; integrating these latter according to pressure, the $v = f(p)$ function is obtained.

Applying a similar procedure to the $p = f(\epsilon)$ curve, the $\sqrt{(dp/d\epsilon)(1/\rho)} = f(\epsilon)$ curve is determined, which, by integration yields the $v = f(\epsilon)$ function.

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Riccati, Daniel Bernoulli, Boussinesq, Navier, Poisson, Hodgkinson, Lamé and Clapeyron, Morin, Young, Homersham Cox, de Saint Vénant, Hamburger, Neumann, Hertz, L. Boltzmann, W. Voigt, V. Hausmanninger, A. Ritter, Dunn, Hatt, C. Ramsauer, W. Hoeniger.

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Small copper cylinders compressed at high speeds are used to measure the pressure from underwater explosion. Apparatus is described for the dynamic calibration of these cylinders; results compared with results of other investigators. Propulsive system is a compressed-air operated piston thrown against the copper cylinder, which is supported against an anvil of large mass; velocity of piston can be varied by varying the air pressure. Energy absorbed by copper cylinder is measured by determining the velocity of the piston just before and after impact. Energy absorption depends on mass of piston used. Bibilography of 9 titles.

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Discussion in *Journal of Applied Mechanics*, Sept. 1943, Vol. 10, No. 3, pp. A-178–179, with 4 figures, by K. J. DeJuhasz and A. Yorgiadis shows application of graphical analysis to the stress and velocity variations in a long rod having three different cross sections.

E. H. LEE, "The Impact of a Mass Striking a Beam," *Ibid.*, December 1940, Vol. 7, No. 4, pp. A-129 to A-138, with 11 figures.

An approximate theoretical solution of the equation of horizontal impact of a mass on a beam, making the assumption that the duration of contact is small in comparison with the fundamental mode of vibration of the beam.

H. C. MANN, "High Velocity Tension Impact Tests," *Proc. Am. Soc. Testing Mats.*, Vol. 36, 1936, Part II.

R. C. MANSLEY, "The Transient State and the Analysis of Shock Excitation," *Engineering*, Vol. 156, No. 4058, October 22, 1943, pp. 321–322, and No. 4060, November 5, 1943, pp. 361–363, with 8 figures.

Recurrent transients, such as the shock loading of crank-connecting rod mechanism under the action of explosions are discussed. Analysis by Fourier series.

LOUIS A. PIPES, "Theory of Longitudinal Impact," *J. App. Phys.*, August 1942, pp. 503–511, with 13 figures.

By making use of the analogy existing between the differential equations describing the longitudinal displacements and velocities of a uniform rod, and the charge and current along a uniform electrical transmission line, the known results of electrical circuit theory are translated into mechanical terms and the method of operational calculus applied to the former. The problem considered is that of determining the subsequent behavior of a long uniform thin rod that is struck on one end by a massive body moving in the direction of the length of the rod. Various states of mobility of the other end of the rod are assumed.

——, "Analysis of Longitudinal Motions of Trains by Electric Analog," *Ibid.*, Vol. 13, No. 12, December 1942, pp. 780–786, with 3 figures.

The relative motion of a train of n uniform cars and a locomotive is considered. If slack and rolling resistance are neglected the system may be idealized to that on $n+1$ masses connected by springs. To generalize the analysis it is assumed that identical shock absorbers of the dashpot and viscous friction type are placed between the units. By considering the electrical analog of this dynamical system its natural frequencies are determined. It is found that if the shock absorbers have a certain critical strength no free oscillation between the cars is possible. Expressions are obtained for the velocities of several cars when the locomotive exerts an oscillatory tractive effort. By the use of operational calculus the response of the system when the tractive effort is impulsive is also determined.

WILLIAM PRAGER, "The Stress-Strain Laws of the Mathematical Theory of Plasticity—A Survey of Recent Progress," *J. App. Mech.*, Paper No. 48-APM-14 (1948), 8 pages, 0 figures, with bibliography of 50 titles.

Typical stress-strain laws of flow and deformation types are discussed with particular reference to the conditions of continuity and uniqueness which these laws must fulfil if they are to make sense physically. Alternative forms of some of these laws are presented, and conditions are discussed under which different laws yield identical results. Methods of integration are indicated and the use of variational principles is stressed.

C. RAMSAUER, "Experimentelle und theoretische Grundlagen des elastischen und mechanischen Stosses" (Experimental and Theoretical Foundation of the Elastic and Mechanical Impact), *Annalen der Physik.*, Vol. 30, 1900, pp. 417–494.

Discussion of the history of impact problems, listing the work of previous investigators. Experiments on bars of various lengths and cross sectional areas.

M. RANKINE, "On the Problem of Impact," *The Engineer*, Vol. 58, February 15, 1857, pp. 133.

J. E. SEARS, "On the Impact of Bars with Rounded Ends," *Trans. Cambridge Philosophical Society*, Vol. 21, 1909–1911.

K. H. SWAINGER, "Strain Energy in Greatly Deformed Elastic or Inelastic Anisotropic Engineering Metals," JOUR. FRANKLIN INSTITUTE, Vol. 245, No. 6, June 1948, pp. 501–516, with 7 figures and bibliography of 20 titles.

The equilibrating stresses on an orthogonal element in a strained body induce normal strains defined as relative displacement per unit length of deformed body. The work done by the forces on the faces of an orthogonal element of unit volume in the loaded body during a differential increase of deformation is found. Integration gives the total work done on the element from the initial unstrained state up to the current state of deformation.

D. L. THORNTON, "Mechanics Applied to Vibrations and Balancing," John Wiley & Sons, Inc., 1940, New York, pp. 204–296.

Chapter IV deals with the propagation of stress in elastic materials, including phenomena in pile driving, vibrations in light wire, water hammer, spherical waves, elastic properties of earth, earthquakes, impact, transmission of energy by pressure waves, fuel systems in internal combustion engines, velocity of sound in various solids.

S. TIMOSHENKO, "Theory of Elasticity," McGraw-Hill Book Co., New York, 1934.

Chapter 12, pp. 381–403, "The Propagation of Waves in Elastic Solid Mediums," gives a summary of impact problems and their mathematical treatment. A history of these problems, as treated by several investigators, namely, Young, Cauchy, Poisson, de Saint Vénant, Boussinesq, and others, is briefly reviewed.

THEODORE VON KÁRMÁN, H. F. BOHNENBLUST, AND D. H. HYERS, "The Propagation of Plastic Waves in Tension Specimens of Finite Length," National Defense Research Committee. Report No. A-103. O.S.R.D. 946, October 1942, 38 pages with 15 figures.

The fundamental differential equations of wave propagation in elastic-plastic materials are given and methods of integration of these equations are discussed. The physical process is represented geometrically in two planes, the Lagrangean plane and the velocity plane. Problems of impact in specimens of finite length showing reflection of plastic waves. A graphical method is explained and applied to the case of an impirical stress-strain relation. The Hugoniot relations are written in Lagrangean form. The distribution of the residual strain along the specimen can be determined.

THEODORE VON KÁRMÁN AND POL. DUWEZ, "The Propagation of Plastic Deformation in Solids," Paper presented at the Sixth International Congress for Applied Mechanics, Paris, 1946. (To be published in the Proc. of the Congress, not yet published.) 21 pp. with 13 figures, bibliography of 6 titles.

Longitudinal impact at end of bar extending to infinity; bar of finite length; experiments on the propagation of plastic strains in a long specimen subjected to tension impact; experimental technique; results of high velocity impact tests; relation between velocity of impact and amplitude of plastic wave front; shape of plastic wave and velocity of propagation of the plastic front; force-time relation at fixed end of specimen subjected to tension impact; practical applications.

MERIT P. WHITE, "The Force Produced by Impact of a Cylindrical Body," National Defense Research Committee. Report No. A-157. March 1943, 19 pp. with 9 figures.

Two kinds of behavior are possible in compression impact: (1) the material of moving specimen is brought to rest by the stress waves propagated from the impact zone; this occurs in low-velocity impact. (2) The material is not stopped but flows away from the impact zone parallel to the target plane; this occurs at impact velocities above the "critical velocity." For very large velocities the force approaches the force produced by a fluid stream of the same density, velocity and area. This analysis has a bearing on the development of very high velocity.

MERIT P. WHITE AND LEVAN GRIFFIS, "Wave Propagation in a Uniform Bar whose Stress-Strain Curve is Concave Upward," National Defense Research Committee. Report No. A-152, February 1943. 14 pp. with 6 figures.

The plastic wave analysis recently developed for materials whose engineering stress-strain curves are concave downward is extended to materials for which the stress-strain curves are partly or entirely concave upward, but always with a positive slope. In this case a steady wave form is possibly one having a very sudden rise. This is identical to the shock wave occurring in gases. Expressions are developed for relating stress and strain, particle velocity, and velocity of wave propagation.

——, "The Permanent Strain in a Uniform Bar Due to Longitudinal Impact," *J. App. Mech.*, Vol. 14, No. 4, December 1947, pp. A-337 to A-343, with 15 figures.

A method is presented for predicting the final strained state of a long uniform bar or wire of ductile material that is subjected to a longitudinal impact of finite duration during which the impact stress is constant and exceeds the yield strength. The final strains are greatest immediately adjacent to the impact point and are constant over a finite length of the specimen. Bibliography of 7 titles.

——, "The Propagation of Plasticity in Uni-Axial Compression," *Ibid.*, Paper No. 48-APM-17, 5 pp. with 5 figures.

Four different kinds of behavior are analyzed: in the concave downward range of the stress-strain curve; in the inflexion point vicinity; in the concave-upward range of stress-strain curve; at supersonic rate of compression. Static and dynamic stress-strain relation is also discussed. Bibliography of 8 titles.

C. R. WIKANDER, "Draft-Gear Action in Long Trains," *Trans. A.S.M.E.*, Vol. 57, August 1935, pp. 317–334, with 45 figures.

A study of coupler pressures, car speeds and accelerations in a long train, produced by external forces; based on the similarity to an elastic bar of the same length of the train, the same mass per unit length and the same yield when subjected to longitudinal compressive or tensile forces. Diagrams of pressure and speed are derived for the various periods through which the waves are passing. Formulas are derived for the maximum bar pressures and tensions, for the locations at which they occur and for the maximum accelerations and retardations of the particles. Formulas are applied to freight trains of known characteristics.

——, "Draft-Gear Action in Train Service," *Ibid.*, Vol. 66, No. 8, November 1944, pp. 691–696, with 7 figures.

In order to investigate the most desirable characteristics of a draft gear by mathematical analysis, a study has been made of the mechanics of an elastic bar, which is subjected to external forces that correspond to those acting in trains under various service conditions. Numerical examples are given showing the application to assumed test trains.

——, "Dynamics of an Elastic Bar," *J. App. Mech.*, Vol. 12, No. 2, June 1945, pp. A-101 to A-106, with 3 figures.

The dynamics of an elastic bar, solid and with free slack, forms a convenient basis for the study of train dynamics, owing to the similitude of the individual cars in the long train to the particles in an elastic bar. It is assumed that the stress and strain in the solid parts of the bar are proportional. Behavior of elastic bars corresponding to the starting, the braking and the change in grade of the track of a long train without or with free slack.

A NOTE ON ANALYSIS OF BEAM-COLUMNS UNDER ARBITRARY LATERAL LOAD AND END RESTRAINT

BY

R. H. SCANLAN[1]

While reviewing some basic relationships and subtleties of Euler column theory, the author was led to consider a general and novel method of analysis applicable to beam columns. He is indebted to M. Yachter for early suggestions on the method, which is presented here. The method applies to long, constant-section beam columns having elastically restrained ends and arbitrary lateral loads.

The beam-column problem for elastic members has been successfully solved for many years by numerous engineering investigators using approximate or specialized means. The present discussion is essentially mathematical, with suggestions for computation appended. The intent herein is to present a general solution rather than another particular one.

I. THE ELASTICALLY RESTRAINED COLUMN OF CONSTANT STIFFNESS

Consider a long column of length l, with P-load as shown in Fig. 1, with elastic end restraint moments $m_1 = c_1\theta_1$ and $m_2 = c_2\theta_2$ proportional

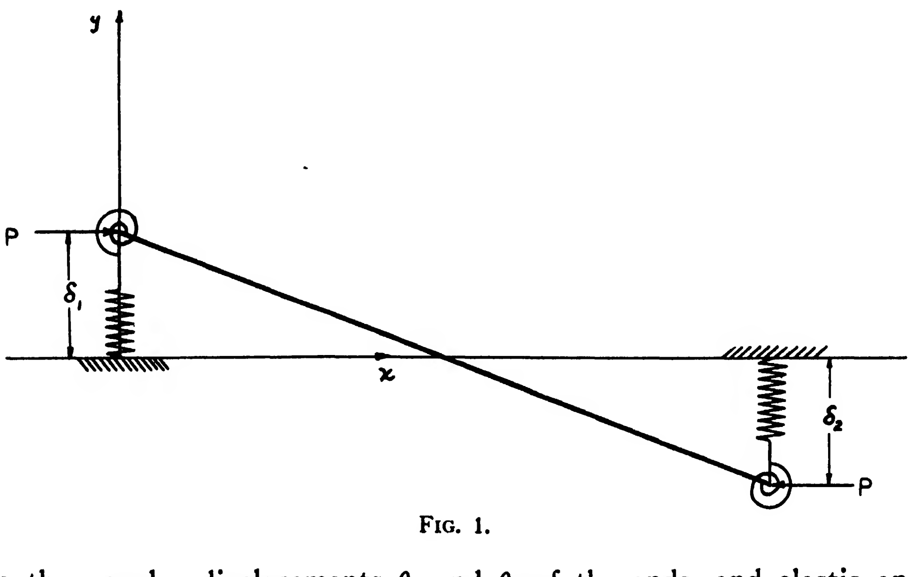

FIG. 1.

to the angular displacements θ_1 and θ_2 of the ends, and elastic and restraint forces $V = \delta_1 k_1 = \delta_2 k_2$ proportional to the lateral displace-

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ments δ_1 and δ_2 of the ends. If $\alpha^2 = P/EI$, the differential equation of the column is

$$y'' + \alpha^2 y = \alpha^2 \left(\delta_1 - \frac{Vx}{P} + m_1 \right), \tag{1}$$

with the end conditions

$$\begin{aligned} y(0) &= \delta_1; & y'(0) &= \theta_1; \\ y(l) &= \delta_2; & y'(l) &= \theta_2. \end{aligned}$$

Let the usual critical load be given by

$$P_{CR} = \frac{c\pi^2 EI}{l^2};$$

then let $\gamma = \alpha l = \sqrt{c}\pi$, $S_1 = \dfrac{EI}{lc_1}$, $S_2 = \dfrac{EI}{lc_2}$, $S_L = \dfrac{EI}{l^3}\left(\dfrac{1}{k_1} + \dfrac{1}{k_2}\right)$. The critical column condition can then be shown to be equivalent to the transcendental equation

$$\begin{aligned} D(S_L, S_1, S_2, \gamma) = \gamma(\gamma^2 S_L - 1)[(\gamma^2 S_1 S_2 - 1)\sin\gamma - \gamma(S_1 + S_2)\cos\gamma] \\ - [\gamma(S_1 + S_2)\sin\gamma + 2(1 - \cos\gamma)] = 0. \end{aligned} \tag{2}$$

This result is well known (**2, 3, 5**),[2] except for the slight generalization due to inclusion of transverse linear springs. This equation furnishes a useful relation between column fixity coefficient c and end fixity coefficients c_1, c_2.

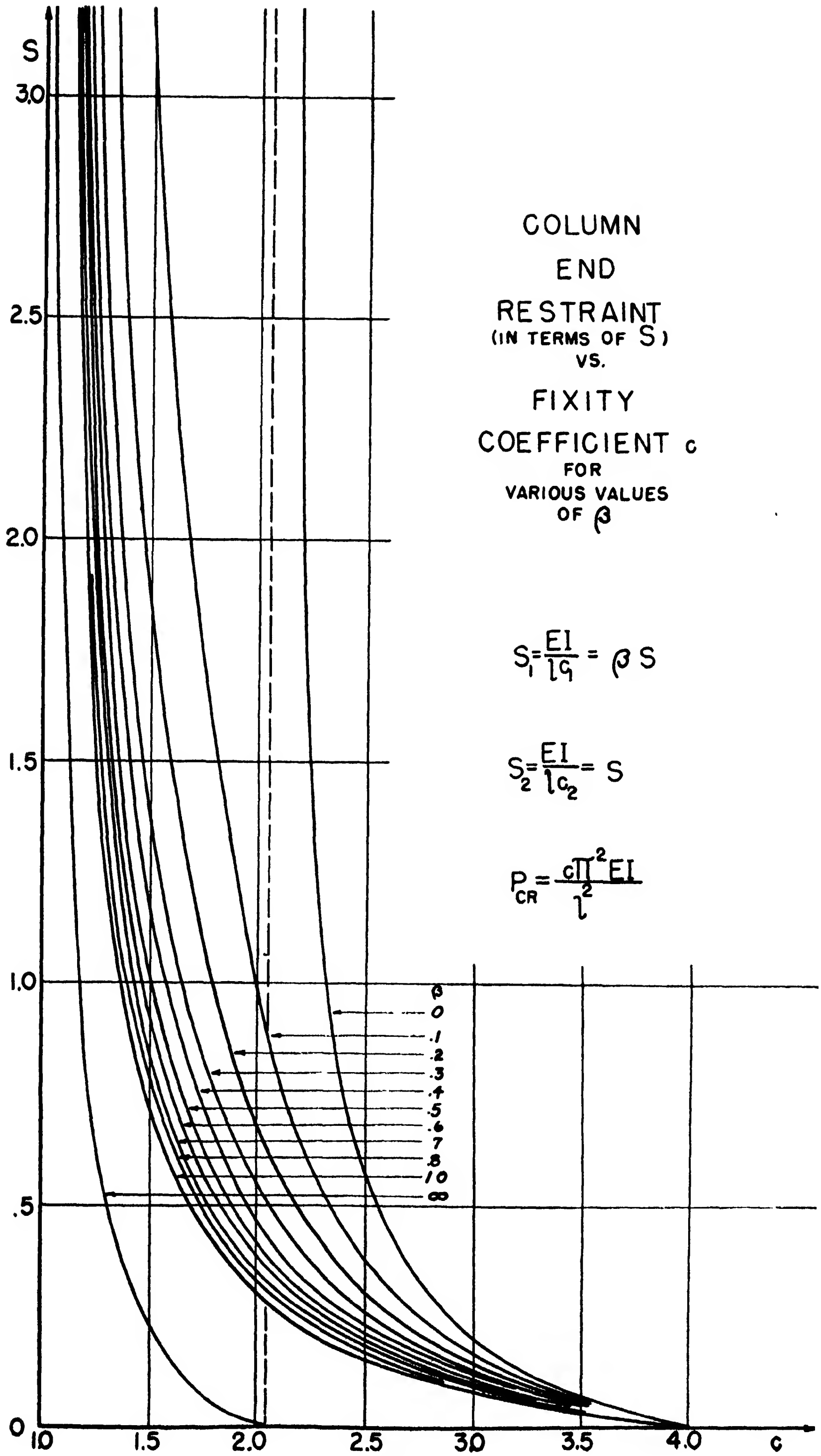
Figure 2 is a graphical presentation of Eq. 2 for the case $S_L = 0$ (that is, no permissible lateral displacement of the ends). Following is a list of characteristics and conventions of Fig. 2:

1. Basic convention—
$S_1 \leqq S_2$; $\beta = S_1/S_2$ for $S_2 \neq 0$; $S_1 = \beta S_2 = \beta S$.
2. Exceptional case—*where basic convention must be reversed* (that is, $S_1 > S_2$): when one column end is pinned ($S_1 = \infty$), the other being elastically restrained, not pinned or fixed ($\beta = \infty$).
3. All curves for finite β end at $c = 4$, $S = 0$.
4. The curve for $\beta = \infty$ ends at $c = 2.044$, $S = 0$.
5. The curve for $\beta = 0$ is asymptotic to $c = 2.044$ at $S = \infty$.
6. All curves for $\beta \neq 0$ are asymptotic to $c = 1$ at $S = \infty$.

II. THE ELASTICALLY RESTRAINED BEAM-COLUMN OF CONSTANT STIFFNESS

Figure 3 represents a beam-column of constant EI and elastically restrained ends. The ends are assumed free to rotate but not to translate laterally. If m_1 and m_2 are the contributions to the end moments due to elastic restraints under *both* column and beam loadings, the total

[2] The boldface numbers in parentheses refer to the references appended to this paper.



S
3.0
2.5
2.0
1.5
1.0
.5
0
1.0
1.5
2.0
2.5
3.0
3.5
4.0
c
COLUMN END RESTRAINT
(IN TERMS OF S)
VS.
FIXITY COEFFICIENT c
FOR VARIOUS VALUES OF β
$S_1 = \frac{EI}{lc_1} = \beta S$
$S_2 = \frac{EI}{lc_2} = S$
$P_{CR} = \frac{c\pi^2 EI}{l^2}$
β
0
.1
.2
.3
.4
.5
.6
.7
.8
10
∞

FIG. 2.

moment $M(x)$ in the bar is

$$M(x) = -Py + m(x) + m_1 - (m_1 + m_2)x/l, \tag{3}$$

and the differential equation for the deflection y is

$$EIy'' + Py = m_1 - (m_1 + m_2)x/l + m(x) = EIf(x), \tag{4}$$

where x is the spanwise coordinate and $m(x)$ is the bending moment which would exist on the beam-column if $P = 0$ and the bar were simply supported. Note that $m(x)$ may include applied end moments. The equation may be written simply

$$y'' + \alpha^2 y = f(x). \tag{5}$$

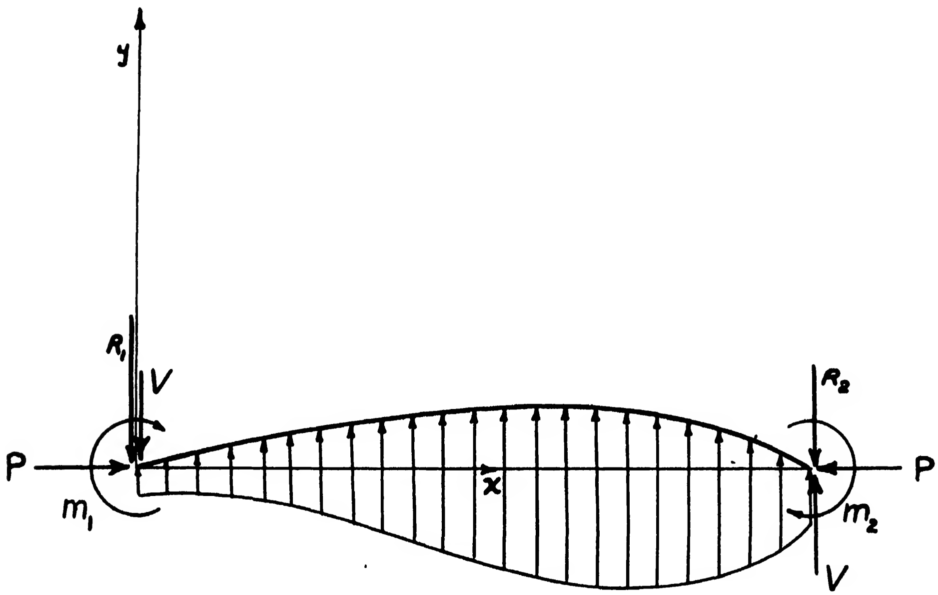


FIG. 3.

The boundary conditions are

$$\begin{aligned} y(0) &= y(l) = 0, \\ y'(0) &= \theta_1 = M(0)/c_1 = [m(0) + m_1]/c_1, \\ y'(l) &= \theta_2 = -M(l)/c_2 = [m_2 - m(l)]/c_2. \end{aligned}$$

The solution of (4) (for example, by the technique of the Green's Function (4)) is

$$y(x) = \int_0^x \frac{\sin\alpha(x-l)}{\alpha \sin \alpha l} \sin \alpha\xi f(\xi)d\xi + \int_x^l \frac{\sin\alpha(\xi - l)}{\alpha \sin \alpha l} \sin \alpha \times f(\xi)d\xi, \tag{6}$$

with use of the first two (deflection) boundary conditions. Use of the

second two (slope conditions) results in the following simultaneous equations for m_1 and m_2:

$$(-S_1\gamma^2 \sin\gamma - \sin\gamma + \gamma\cos\gamma)m_1 + (\gamma - \sin\gamma)m_2$$
$$= m(0)\gamma^2 S_1 \sin\gamma + \gamma^2 \int_0^1 \sin\gamma(1-\zeta)m(\zeta l)d\zeta,$$

$$(\gamma - \sin\gamma)m_1 + (-S_2\gamma^2 \sin\gamma - \sin\gamma + \gamma\cos\gamma)m_2$$
$$= -m(l)\gamma^2 S_2 \sin\gamma - \gamma^2 \int_0^1 \sin\gamma\zeta m(\zeta l)d\zeta. \quad (7)$$

The solutions of (7) are

$$m_1 = \frac{-\gamma}{D(S_1, S_2, \gamma)}\Big\{(S_2\gamma^2\cos\gamma + \cos\gamma + \gamma\sin\gamma - 1)\int_0^1 \sin\gamma\zeta m(\zeta l)d\zeta$$
$$+(-S_2\gamma^2\sin\gamma - \sin\gamma + \gamma\cos\gamma)\int_0^1 \cos\gamma\zeta m(\zeta l)d\zeta$$
$$+m(0)S_1(-S_2\gamma^2\sin\gamma - \sin\gamma + \gamma\cos\gamma) + m(l)S_2(\gamma - \sin\gamma)\Big\} \quad (8)$$

$$m_2 = \frac{-\gamma}{D(S_1, S_2, \gamma)}\Big\{(S_1\gamma^2 + 1 - \cos\gamma)\int_0^1 \sin\gamma\zeta m(\zeta l)d\zeta$$
$$+(\sin\gamma - \gamma)\int_0^1 \cos\gamma\zeta m(\zeta l)d\zeta$$
$$+m(0)S_1(\sin\gamma - \gamma) + m(l)S_2(S_1\gamma^2\sin\gamma + \sin\gamma - \gamma\cos\gamma)\Big\},$$

where $D(S_1, S_2, \gamma)$ is the transcendental instability expression, $D(S_L, S_1, S_2, \gamma)$, for the case $S_L = 0$, namely, no lateral deflection at the ends.

Equation 6 may be rewritten, letting $x = \eta l$:

$$y(\eta l) = \frac{\gamma}{P\sin\gamma}\Big\{\sin\gamma(\eta - 1)\int_0^\eta \sin\gamma\zeta[EIf(\zeta l)d\zeta$$
$$+ \sin\gamma\eta\int_\eta^1 \sin\gamma(\zeta - 1)[EIf(\zeta l)]d\zeta\Big\}. \quad (9)$$

This form is useful for computation as will presently be seen. Explanation should be given of the *apparent* discontinuity in $y(\eta l)$ at $\gamma = \pi$ since there is a $\sin\gamma$ in the denominator: Expression of $f(\zeta l)$ by use of (8) reveals a $\sin\gamma$ in the numerator which cancels that of the denominator; the actual denominator is then seen to contain the factor $D(S_1, S_2, \gamma)$, the previously established instability expression.

Thus no actual discontinuity in y exists at $\gamma = \pi$ unless the bar ends are pinned. This fact is not evident from the form (6). The total moment on the bar is finally given by

$$M(x) = M(\eta l) = -Py(\eta l) + EIf(\eta l). \quad (10)$$

III. PRACTICAL CALCULATION

Converting integrals to sums and taking, say, ten intervals, Eq. 9 may be rewritten approximately as

$$- Py(\eta l) = \frac{0.1\gamma}{\sin\gamma}\left\{\sin\gamma(1-\eta)\sum_{\zeta=0.05}^{\zeta=\eta}\sin\gamma\zeta[EIf(\zeta l)] + \sin\gamma\eta\sum_{\zeta=\eta}^{\zeta=0.95}\sin\gamma(1-\zeta)[EIf(\zeta l)]\right\} \quad (11)$$

for use in practical calculation.

Table I is a sample of a series of tables developed by the author to solve the beam-column problem for general loading conditions by practical calculation. The complete set of tables (not reproduced here) is calculated for $P/(P_{CR})_{\text{PIN}} = 0.1, 0.2, \ldots 0.8, 0.9, 1.2, 1.5, \ldots 3.9$, that is, covering the entire range of ratios of actual column load to critical load for the same column when pin-ended:

$$0 \leqq P/(P_{CR})_{\text{PIN}} \leqq 4.$$

Thus there were developed 19 tables in all. These tables are constructed to perform all the work implied in evaluating (11) with the exception of calculating $EIf(\zeta l)$ and summation. Such a set of tables can easily be constructed in any engineering department, following Eq. 11. Table I illustrates a property which aids in such construction, namely the symmetry of the tables about both diagonals.

A complete routine calculation procedure using such tables is outlined below, stepwise. Assumed given are EI, l, c_1, c_2, P and the lateral loading.

1. Calculate

$$(P_{CR})_{\text{PIN}} = \frac{\pi^2 EI}{l^2}.$$

2. Calculate

$$S_1 = \frac{EI}{lc_1}, \qquad S = S_2 = \frac{EI}{lc_2}, \qquad \beta = S_1/S_2.$$

NOTE: Choose $S_1 \leqq S_2$ above *except* when one column end is pinned ($S_1 = \infty$) and the other is elastically restrained ($S_2 \neq 0, \infty$). Then take $S_1 > S_2$ and $\beta = \infty$. (See text describing Fig. 2.)

3. Enter Fig. 2 with β and S as parameters and determine c, the column fixity coefficient.

4. Calculate

$$P_{CR} = c(P_{CR})_{\text{PIN}}.$$

5. Calculate

$$r = P/(P_{CR})_{\text{PIN}},$$
$$r' = P/P_{CR},$$

and

$$\gamma = \pi\sqrt{r}.$$

TABLE I.—$P/(P_{CR})_{PIN} = 1.2$.

η	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95	$\gamma^2 \sin \gamma\eta$
0.05	0.02542	−0.04283	−0.10607	−0.15686	−0.18926	−0.19946	−0.18627	−0.15124	−0.09848	−0.03416	2.02795
0.15	−0.04283	−0.12348	−0.30576	−0.45219	−0.54559	−0.57501	−0.53699	−0.43600	−0.28390	−0.09848	5.84616
0.25	−0.10607	−0.30576	−0.46957	−0.69445	−0.83788	−0.88306	−0.82468	−0.66958	−0.43600	−0.15124	8.97816
0.35	−0.15686	−0.45219	−0.69445	−0.85530	−1.03196	−1.08760	−1.01569	−0.82468	−0.53699	−0.18627	11.05775
0.45	−0.18926	−0.54559	−0.83788	−1.03196	−1.10502	−1.16460	−1.08760	−0.88306	−0.57501	−0.19946	11.84060
0.55	−0.19946	−0.57501	−0.88306	−1.08760	−1.16460	−1.10502	−1.03196	−0.83788	−0.54559	−0.18926	11.23481
0.65	−0.18627	−0.53699	−0.82468	−1.01569	−1.08760	−1.03196	−0.85530	−0.69445	−0.45219	−0.15686	9.31155
0.75	−0.15124	−0.43600	−0.66958	−0.82468	−0.88306	−0.83788	−0.69445	−0.46957	−0.30576	−0.10607	6.92633
0.85	−0.09848	−0.28390	−0.43600	−0.53699	−0.57501	−0.54559	−0.45219	−0.30576	−0.12348	−0.04283	2.54279
0.95	−0.03416	−0.09848	−0.15124	−0.18627	−0.19946	−0.18926	−0.15686	−0.10607	−0.04283	0.02542	−1.50897

NOTE: (*a*) If $r' \cong 1$, the column is critical.

(*b*) If (*a*) is not the case but $r \cong \pi$ (that is, $r \cong 1$) calculate $M(x)$ separately by steps 6–10 for $r = 0.9$ and $r = 1.2$ and interpolate between final results for $M(x)$.

6. Draw the moment diagram $m(x) = m(\eta l)$ for lateral loading as if the beam-column were a simply supported beam with no column load. Note that ordinates of $m(\eta l)$ at the ten finite values of η are to be used:

$$\eta = 0.05, 0.15, 0.25, \cdots, 0.95.$$

7. Calculate m_1 and m_2 by solving simultaneously Eqs. 7 with the aid of Table I (or similar appropriate table). Except as noted in 5(*b*), interpolation between table entries is permissible. This involves merely numerical integration of the right hand integrals of (7) by the use of the "$\gamma^2 \sin \gamma\eta$" column appended to the right of the table. Use of the "$\gamma^2 \sin \gamma\eta$" column inverted will furnish values of $\gamma^2 \sin \gamma(1 - \eta)$. Note that m_1, m_2 are *reactive* end moments; they are not *total* end moments unless $m(x) = 0$ at the bar ends.

8. Draw the "generalized" moment diagram

$$EIf(\eta l) = m(\eta l) + m_1 - (m_1 + m_2)\eta.$$

9. With the values of $EIf(\eta l)$ at the ten abscissae 0.05, 0.15, 0.25, $\cdots$, 0.95, enter Table I (or similar appropriate table) using interpolated values according to the value of r if necessary. Multiply each table entry in the row "0.05" by $EIf(0.05l)$; multiply each table entry in the row "0.15" by $EIf(0.15l)$, etc. Add the resulting columns. The resulting row of sums represents $-Py(\eta l)$.

10. Draw the final moment curve

$$M(\eta l) = - Py(\eta l) + EIf(\eta l).$$

NOTE: For a pin ended column, with no external applied moments, $m_1 = m_2 = 0$; therefore, steps 2, 3, 4, 7 and 8 are omitted while other steps proceed as before. In step 5, $r = r'$; in step 9, $m(\eta l)$ replaces $f(\eta l)$ in this case.

The author has worked several examples and found good agreement between analytical results (in simple cases, where these are possible) and the method presented here. This method is not made more difficult by complex side loadings as is the analytical method. One example, using the sample table, Table I, is presented below.

Example:

Concentrated load Q at center. Column ends partially restrained.

$c_1 = 4000$ in. lb. per radian, $EI = 2 \times 10^5$ lb. per sq. in.,
$c_2 = 3000$ in. lb. per radian, $l = 100$ in.
$P = 236$ lb.,

The calculation follows steps 1–10 above.

1. $(P_{CR})_{PIN} = 197$ lb.
2. $S_1 = 0.5$; $S_2 = 0.6667$; $\beta = 0.75$.
3. From Fig. 2, $c = 1.6$.
4. $P_{CR} = 1.6 \times 197 = 315$ lb.
5. $r = 1.2$; $r' = 0.75$; $\gamma = \pi \times \sqrt{1.2} = 3.45$.
   (*a*) Column is not critical.
   (*b*) No interpolation is needed; the table for $r = 1.2$ can be used directly.
6. $Q = 10$ lb.

η	0 05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95
$m(\eta l)$	27	75	125	175	225	225	175	125	75	25

7. Equations (31) are:

$$\begin{aligned} -1.199m_1 + 3.75m_2 &= 1130.9, \\ 3.75m_1 - 0.599m_2 &= -1130.9. \end{aligned}$$

The solutions are:

$$\begin{aligned} m_1 &= -267.04 \text{ in. lb.}, \\ m_2 &= 216.19 \text{ in. lb.} \end{aligned}$$

8.

η	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95
$EIf(\eta l)$	−239.5	−184.4	−129.3	−74.2	−19.2	−14.1	−59.0	−103.9	−148.8	−193.7

9.

	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95
0.05	−6.09	10.26	25.40	37.57	45.33	47.77	44.61	36.22	23.59	8.18
0.15	7.90	22.77	56.38	83.38	100.61	106.03	99.02	80.40	52.35	18.16
0.25	13.71	39.53	60.72	89.79	108.34	114.18	106.63	86.58	56.37	19.56
0.35	11.64	33.55	51.53	63.46	76.57	80.70	75.36	61.19	39.84	13.82
0.45	3.63	10.48	16.09	19.81	21.22	22.36	20.88	16.95	11.04	3.83
0.55	2.81	8.11	12.45	15.34	16.42	15.58	14.55	11.81	7.69	2.67
0.65	10.99	31.68	48.66	59.93	64.17	60.89	50.46	40.97	26.68	9.25
0.75	15.71	45.30	69.57	85.68	91.75	87.06	72.15	48.79	31.77	11.02
0.85	14.65	42.24	64.88	79.90	85.56	81.18	67.29	45.50	18.37	6.37
0.95	6.62	19.08	29.30	36.08	38.64	36.66	30.38	20.55	8.30	−4.92
	81.57	263.00	434.98	570.94	648.61	652.41	581.33	448.96	276.00	87.94

10.

0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95
−157.9	78.6	305.7	496.7	629.4	638.3	522.3	345.1	127.2	−105.8

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APPENDIX

List of Symbols

x, y	Abscissa and ordinate of point in loaded bar.
V	Vertical reaction at ends of bar (lb.).
δ_1, δ_2	Vertical deflection of linear springs at left and right hand ends of bar (in.).
k_1, k_2	Spring constants of linear springs at left and right hand ends of bar (lb. per in.).
m_1, m_2	Reactive moments at ends of bar under load.
c_1, c_2	Spring constants of torsional springs at left and right hand ends of bar (in. lb. per radian), that is, "end restraint coefficients" of bar.
θ_1, θ_2	Angular rotations of left and right hand ends of bar under load (radian).
P	Column load on bar (lb.).
l	Length of bar (in.).
$M(x)$	Bending moment on bar at any section x (in. lb.).
$m(x)$	Bending moment (in. lb.) on bar under transverse load when considered as a simple beam.
E	Elastic modulus of bar material (lb. per sq. in.).
I	Moment of inertia of any bar section (in.^{4}).
α	Defined by $\alpha^2 = P/EI$.
c	Column restraint coefficient appearing in Euler column formula for P_{CR}.
P_{CR}	Critical instability load on bar with general end restraints (lb.) defined by $\frac{c\pi^2 EI}{l^2}$ (Euler formula).
$(P_{CR})_{\text{PIN}}$	Critical instability load on bar with pin end restraints (lb.).
$S_1, S_2 = S$	Dimensionless "reciprocals" of end restraint coefficients c_1, c_2 defined by $$S_1 = \frac{EI}{lc_1}, \qquad S_2 = \frac{EI}{lc_2}.$$
S_L	Dimensionless parameter involving reciprocals of k_1, k_2 defined by $$S_L = \frac{EI}{l^3}\left(\frac{1}{k_1} + \frac{1}{k_2}\right).$$
γ	Dimensionless parameter defined by $\gamma = \alpha l$.
$D(S_L, S_1, S_2, \gamma)$ $D(S_1, S_2, \gamma)$	Transcendental stability expression involving dimensionless parameters. (See Eqs. 2 and 8.)
β	Ratio of S_1 to $S_2 = S$ (Fig. 2).
ξ	Auxiliary variable having dimensions of inches, same range as $0 \leqq x \leqq l$.

ζ	Auxiliary dimensionless variable having range $0 \leqq \zeta \leqq 1$.
η	Auxiliary dimensionless variable having same range as ζ.
Q	Concentrated bar transverse load (lb.) used in example.
r	$= \dfrac{P}{(P_{CR})_{\text{PIN}}}$.
r'	$= \dfrac{P}{P_{CR}}$.

Weston's Relative Humidity Indicator Now Has a Fan Attachment.—Accurate humidity readings can now be obtained quickly in factories, shops, rooms and even small, confined spaces near, or in operating machinery, thanks to a new motor-operated fan developed for attaching to any Weston Relative Humidity Indicator.

The small fan, powered by dry cells, will move air over the indicator's wick in sufficient volume to permit obtaining stable readings within 30 seconds. This makes possible complete analysis of humidity conditions in, for example, a textile mill, particularly in and around the machinery while it is operating, thus enabling the adjustment of the humidifiers and air-conditioning equipment for best results.

No calculations are required, as readings are immediately obtained on the instrument's simplified slide rule.

Weston Relative Humidity Indicators are now available with or without the auxiliary fan, or the auxiliary fan can be purchased separately. Complete information may be obtained from the Weston Electrical Instrument Corporation at 617 Frelinghuysen Avenue, Newark 5, N. J.

Fresh Orange Juice Keeps Flavor on Long Trips to Market.—Fresh, refrigerated orange juice, prepared and packaged by a new process, is now being shipped commercially from citrus-producing areas to distant markets without loss of flavor or other qualities, the U. S. Department of Agriculture announces.

Ordinarily, orange juice would lose much of its flavor after only a few days in transit and storage. But Bureau of Agricultural and Industrial Chemistry researchers at Pasadena, Calif., working in cooperation with citrus processors, have devised a large-scale, commercial method for producing and packing the juice so that it retains good taste quality and vitamin-C content for two weeks or longer. Even after 3 weeks in refrigerated storage, this product is superior to heat-processed canned orange juice.

Development of the new process, which provides an additional means of moving the country's increasing crop of healthful oranges into economical consumption, was undertaken in 1946 by Dr. E. A. Beavens, in charge of the Bureau's new Pasadena Research Laboratory, and one of his associates, Dr. J. C. Underwood. Most of their work was done at the laboratory maintained by the Bureau until recently at Los Angeles. Object of this research was to find a way to protect the flavor and nutritional quality of fresh, unpasteurized, single-strength orange juice for a period long enough to permit wide distribution.

The method now in commercial use involves careful reaming of the fruit, so that very little peel oil gets into the juice. Too much of this oil will hasten development of off-flavors. The prepared juice is quickly chilled to 30 degrees Fahrenheit and kept at this temperature until used.

Carload lots of this fresh orange juice, packed in 1-gal. cans and refrigerated with ice and salt, have been shipped successfully from Los Angeles to New York City. The present production in southern California, which amounted last year to almost 2½ million gallons, is currently distributed to nearer cities. Ten plants of varying size are now using the process on a year-round basis in California. A large part of their output is shipped by refrigerated truck to consumers within a radius of 500 miles.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS *

NEW USES FOR MAGNETIC FLUIDS

The magnetic fluid originally used in the electromagnetic fluid clutch developed at the National Bureau of Standards has several unique features that make possible other important applications of iron-oil mixtures. Further studies of the properties of these mixtures have revealed that magnetic fluids can be employed to good advantage in hydraulic systems, shock absorbers, and dash pots; to form casting molds; and as variable electrical resistors. The basic property on which all these applications depend is that the viscosity of a magnetic fluid is directly related to the strength of the applied magnetic field—the fluid may be changed from a liquid to a nearly solid state and back again at will.

In some applications extreme fluidity is required at all times, while in others a thixotropic characteristic is needed. Thixotropic fluids are characterized by isothermal reversible gel-sol-gel transformations under shearing and subsequent resting. Mayonnaise, a good example of a thixotropic fluid, resists flow when its container is turned over, but may be stirred easily with a spoon. Honey, a nonthioxtropic material, flows like all ordinary fluids but, because of its high viscosity, resists stirring.

When a magnetic fluid is used in a nonrotating device, the iron particles in the mixture must remain in suspension and not settle out. One way of achieving this is to use a thixotropic fluid. Another way is to maintain a small magnetic field in the mixture during idle periods. In rotating machinery, settling is less important as long as the iron powder can be readily remixed with the fluid. This can be achieved by adding a surface-active ingredient to the iron-oil mixture.

One engineering application proposed in the Bureau's preliminary investigation of new uses for magnetic fluids is the magnetic fluid dash pot. Basically, a dash pot consists of a piston moving in a fluid-filled cylinder with the viscosity of the fluid resisting the piston motion. In a conventional dash pot the rate of travel of the piston is determined by the external force, the fluid pressure on the piston, the dimensions of the fluid escape vent, and the fluid viscosity. Although the viscosity of the fluid depends to a large extent on its temperature, the speed of piston travel is not ordinarily regulated by controlling temperature. The only variable usually controlled is the pressure on the piston. The rate of piston travel can be adjusted mechanically by providing a variable orifice in the piston or in the cylinder wall—a method that presents a number of complications.

* Communicated by the Director.

On the other hand, if a magnetic fluid is used in the dash pot the rate of motion of the plunger can be readily controlled by magnetically varying the viscosity of the fluid. If a coil of wire is placed around the dash pot, the viscosity of the magnetic fluid will be a function of the current in the coil. Employing this idea in the design of shock absorbers for automobiles and trucks would provide an adjustable riding quality to meet various loading and roadway conditions. The rate or response of the magnetic fluid is high enough to provide virtually instantaneous changes in viscosity, thereby making possible a shock absorber with automatic compensation.

The electromagnetically controllable fluid offers interesting possibilities for use in hydraulic systems. If a hydraulic system is filled with a magnetic fluid, valving becomes extremely simple: By winding a coil of wire around a fluid-carrying pipe and controlling the amount of current through the coil, the flow of fluid past that point can be closely regulated from full flow to complete cut-off. An obvious advantage of this scheme is that various points in a hydraulic system can be remotely controlled from a central station.

Electromagnetic fluids are also being investigated for use in molding operations. A fluid is placed in a pot surrounded by a current-carrying coil, a model of the part to be cast is placed in the fluid, and the coil is then energized so that the fluid will solidify around the model. When the model is removed, a detailed impression remains outlined in the solidified magnetic fluid. Molding compound can then be poured into the mold and allowed to harden. After the coil current is turned off the molded replica can easily be removed from the liquid. In any application of this kind, the boiling point of the magnetic fluid must of course be higher than the temperature of the molten casting material.

An electrical resistor adapted to remote control can be made by immersing two electrodes in a magnetic fluid. When the fluid is in an unmagnetized condition, the resistance between the two electrodes will be extremely high because of the very loose contact among the conductive iron particles that are randomly distributed in the nonconductive oil. In the presence of a magnetic field, however, the iron particles apparently form chains along the lines of magnetic flux and draw into close physical contact. The flux density will determine the massiveness of the chain and, thus, the conductivity of the mixture. When the system is de-energized, the conductance does not drop back to its former very low level. This property of magnetic fluid resistors is attributed to the coherer effect that has been previously investigated by Branley, Marconi, and others.

The success or failure of any device utilizing magnetic fluid will depend to a considerable extent on the particular components in the iron-oil mixture, the choice of suspension fluid and iron powder in large measure being determined by the application for which the mixture is

intended. The iron powder is one component of the mixture not generally varied from one application to another, since certain properties are considered essential for satisfactory operation: In order to achieve maximum magnetic efficiency the iron powder must have high permeability; to minimize wear and abrasion on moving parts the particles should have smooth, continuous exteriors; and the iron powder must be chemically stable, resisting oxidation in the suspension fluid.

In mixtures where it is essential to prevent settling and sedimentation, the particles should be of uniform size and no larger than about 20 microns in diameter. A great many powders have been tried at the National Bureau of Standards including pure iron, alloys, oxides, and ferrites. The powder which has proven most universally successful is a carbonyl iron in the form of particles about 8 microns in diameter.

The choice of a suspension fluid is not so simple. Some of the factors which must be considered are chemical stability, flammability, vapor pressure, and viscosity. In any magnetic fluid device where there is considerable mechanical movement, the viscosity of the suspension fluid may be especially important, and if it is desirable to hold the viscous drag at a low value during non-energized periods, a low-viscosity fluid must be used. The fluids must also be chemically stable in the presence of the iron powder, and should not become sticky or gummy at high temperatures.

In earlier experiments at the Bureau it was found that many ordinary lubricating oils have enough sulfur in them to cause rapid oxidation and clumping of the iron particles. It was also found that many animal oils (including lard or sperm) tend to catalyze and become gummy and solid. One type of fluid that is remarkably stable in the presence of iron powder is a silicone liquid that has a viscosity of around 50 centistokes at 25 degrees Centigrade. It is excellent with respect to non-flammability and vapor pressure, and this versatile fluid will serve satisfactorily in nearly all but extremely high temperature applications. When it is necessary to operate a magnetic fluid device at elevated temperatures, special compounds such as fluorinated and chlorinated fluids can be used, but special precautions must be taken with the seals since the vapors from these fluids are quite toxic.

Government-sponsored Research. (Excerpt from Address by Karl T. Compton, Chairman, Research and Development Board, before the Executives' Club of Chicago).—Out of every dollar which you now pay in federal taxes, a little more than one cent is spent for research and development to provide more effective weapons, equipment, medicines and utilization of human resources for our national defense. Stated in another way, research and development takes a little less than four cents our of every dollar spent for our Military Establishment.

As citizens and taxpayers, you are rightly interested in the efficiency with which this money is used and the value of the results for which it pays. Are we spending enough or too much on this aspect of national security? What measures are being taken to evaluate these questions and to provide competent guidance and administration? These are questions which I welcome the opportunity of discussing with you today.

Let me first explain that the sum of approximately $550,000,000 about which I am speaking does not include the sums separately provided by Congress to the Atomic Energy Commission for its very important research and development program. These amount, for total contract authority in the current year, to about $150,000,000 and it is rather difficult to assign specific proportions of this to research and development versus production, or to military as opposed to peacetime purposes. It can be said that most of the research in this field, whether initially stimulated for military or non-military purposes, is applicable to both.

By a rough estimate, eighty per cent of all the expenditures under the atomic energy program could be justified by purely non-military objectives even though a considerable portion is now going into the production of atomic bombs. This is because the materials uranium and plutonium which are produced for atomic bombs are also the materials required for the production of atomic energy. If the happy time comes when we can "beat our swords into plowshares," then the stockpile of atomic bombs can be converted into the fuel for atomic power plants by methods which are now being developed within the program of the Atomic Energy Commission, and which should be operating in the early experimental pilot-plant stage within a couple of years.

THE FRANKLIN INSTITUTE

PROCEEDINGS OF THE 125TH ANNIVERSARY OF THE FOUNDING OF THE FRANKLIN INSTITUTE AND THE CELEBRATION OF THE 200TH ANNIVERSARY OF FRANKLIN'S ELECTRICAL EXPERIMENTS

April 20, 1949

Afternoon Session

At 2:00 P.M. a joint meeting of The Franklin Institute and the American Philosophical Society was held in the Lecture Hall. Mr. Richard T. Nalle, President of the Institute, graciously welcomed the members and their guests, among whom were many members of the American Philosophical Society, founded by Benjamin Franklin. The greetings of this Society were formally extended to the Institute by Dr. Edwin G. Conklin, President, in a short address whose text appears below. Dr. Robert A. Millikan then spoke about Benjamin Franklin, the printer, the businessman, the scientist and the statesman, emphasizing the importance of his electrical experiments. Dr. Millikan's talk is printed below. The session closed with an informative, illustrated lecture by Mr. C. Guy Suits, Vice-President of General Electric Company and Director of its Research Laboratories in Schenectady, on the subject "A Progress Report on American Research." Since this talk was based on a series of camera slides, it is not feasible to publish it in the JOURNAL. Following the meeting in the Lecture Hall, guided tours through the Museum and the Laboratories were arranged for those who were interested.

GREETINGS OF THE AMERICAN PHILOSOPHICAL SOCIETY TO THE FRANKLIN INSTITUTE ON THE 125TH ANNIVERSARY OF ITS FOUNDATION AND ON THE 200TH ANNIVERSARY OF BENJAMIN FRANKLIN'S ELECTRICAL EXPERIMENTS

EDWIN G. CONKLIN

President of the American Philosophical Society

I have been asked to bring the greetings of the American Philosophical Society to The Franklin Institute of Pennsylvania on this one hundred and twenty-fifth anniversary of the founding of the Institute. I am very happy to do this because the American Philosophical Society is proud to recognize The Franklin Institute as one of its daughters, or step-daughters, or younger sisters—at least a close relation—in the origin of which it played a part. Both the American Philosophical Society and The Franklin Institute have the same Patron Saint. Benjamin Franklin was the founder of the American Philosophical Society, its first Secretary, President for twenty-one years, chief contributor to its building, publications, library, and its European reputation. In the first Annual Report of The Franklin Institute six officers and twenty-four managers are listed. Both Vice-presidents and the Recording Secretary and Chairman of the Board, Editor of *The Franklin Journal* were members of the American Philosophical Society. Six of the twenty-five persons who are listed as chief founders of The Franklin Institute in Dr. Sidney L. Wright's "Story of The Franklin Institute" were members of the American Philosophical Society, *viz.* William H. Keating, Clement C. Biddle, Mathew Carey, George Fox, Thomas Gilpin, and Robert Harris, while ten others of these officers, members, and committee men, *viz.* Robert M. Patterson, William Strickland, Isaiah Lukens, Alexander Dallas Bache, James Rush, Robert E. Griffith, Matthias W. Baldwin, Thomas McEwen, Samuel Vaughan Merrick, and Frederick Fraley, soon after became members or officers of the American Philosophical Society. Merrick was the prime mover among these

founders in organizing the Institute; Robert M. Patterson was the eighth President of the American Philosophical Society; Bache, great grandson of Franklin, was the first President of Girard College and of the Central High School of Philadelphia, and twelfth President of the American Philosophical Society; while Frederick Fraley was the fifteenth President of the Society.

When Samuel Vaughan Merrick issued the first call for a meeting of those persons interested in what was to be virtually a technical high school, the place of meeting was specified as "Philosophical Hall." Unfortunately, no people attended this meeting and another meeting was called a few days later which was similarly neglected by the public, but on the ninth of December, 1823, Mr. Merrick, who was himself a young industrialist of Philadelphia, twenty-two years old, together with William H. Keating, Professor of Chemistry at the University, held a successful meeting of about a dozen citizens in the Hall of the American Philosophical Society and on this occasion they drew up a constitution and appointed a committee on membership. On February 5, 1824 a large general meeting was held in the County Court House, which is now known as Congress Hall, and the constitution of The Franklin Institute was adopted, and an invitation to join the Institute was followed by the acceptance of four or five hundred persons as members. This was the actual beginning of The Franklin Institute of Pennsylvania and it proceeded immediately in that year, 1824–25, to undertake the work for which it was organized, to secure a charter from the Legislature of the State, and to build the home of the Institute on Seventh Street between Market and Chestnut, which building it occupied for more than one hundred years, until it built and occupied the present magnificent structure on the Benjamin Franklin Parkway at Twentieth Street. This was intended to be, and indeed is, the greatest structural memorial to Benjamin Franklin in Philadelphia, though his name is associated with many of the scientific, educational, and humanitarian organizations of the city.

The earliest activities of the Institute consisted of lectures and classes on "Mechanic Arts" and the Institute was known as "Franklin High School" before there was any other high school in Philadelphia. Members were largely scientists, inventors, and business men, and their number was not limited, whereas the members of the American Philosophical Society were to be "Virtuosi or ingenious men" and the number of living members has never been more than five hundred. Indeed, the American Philosophical Society was modelled on the Royal Society of London and was intended by Franklin to be a kind of Royal or Premier Society of America. The Academy of Natural Sciences of Philadelphia, like The Franklin Institute, has no specific limit in numbers and its members have been largely naturalists and medical men interested in the scientific aspects of anatomy, biology, and physiology.

It may be useful to indicate the course of cultural differentiation which is presented by some of the organizations which have grown out of the American Philosophical Society, or which at least were stimulated by the Society in their early years. Franklin indicated in his call for the founding of the Society, "A Proposal for Promoting Useful Knowledge among the British Plantations in America," which was issued in 1743, that the membership should consist of "Virtuosi or ingenious men residing in the several Colonies" who were to "maintain a constant Correspondence." The subjects of the correspondence were to be: "New-discovered Plants . . . &c. their Virtues, Uses, &c. Methods of Propagating them . . . Improvements of vegetable Juices, as Cyders, Wines, &c.; New Methods of Curing or Preventing Diseases. All new-discovered Fossils in different Countries, as Mines, Minerals, Quarries; &c. New and useful Improvements in any Branch of Mathematicks; New Discoveries in Chemistry, such as Improvements in Distillation, Brewing, Assaying of Ores; &c. New Mechanical Inventions for saving Labour; as Mills, Carriages, &c. and for Raising and Conveying of Water, Draining of Meadows, &c.; All new Arts, Trades, Manufactures, &c. that may be proposed or thought of. Surveys, Maps and Charts of particular Parts of the Sea-coasts, or Inland Countries; Course and Junction of Rivers and great Roads, Situation of Lakes and Mountains, Nature of the Soil and Productions; &c. New Methods of Improving the Breed of useful Animals, Introducing other Sorts from foreign Countries. New Improvements in Planting, Gardening, Clearing Land &c.; And all philosophical Experiments

that let Light into the Nature of Things, tend to increase the Power of Man over Matter, and multiply the Conveniences or Pleasures of Life."

This was really a program of promoting all useful knowledge and its applications. Before the establishment of the Patent Office all sorts of inventions were reported to the Society, such as a machine for mowing with a horse, various improvements in ships and electrical rods or lightning conductors, description of a speedy elevator, methods of destroying pernicious insects, a new and simple nautical chart, improved hydrants and street lamps, and many other such practical applications of science to man's comfort and welfare which are now handled by many Government bureaus and private institutions.

The first Fund of the Society was established in 1786 by the gift of 200 guineas by John Hyacinth de Magellan, of London, for a gold medal to be awarded annually to the author of the best discovery or most useful invention in some of the fields which I have indicated.

But the American Philosophical Society was more than a stimulator of practical inventions for, like its founder, it also had a particular interest in what may be called the foundations of science and discovery or what in the eighteenth century was known by the name of "philosophy." It was particularly interested in the last of the "subjects of correspondence" listed above, namely "All philosophical Experiments that let Light into the Nature of Things, tend to increase the Power of Man over Matter, and Multiply the Conveniences or Pleasures of Life," and it is this part of Franklin's original program for the American Philosophical Society that today represents its principal function.

At present the fields represented by members of the American Philosophical Society are Mathematical and Physical Sciences, Geological and Biological Sciences, Social Sciences, and the Humanities, and the present functions of the Society are represented chiefly in the following standing committees, *viz.* the Committees on Finance, Meetings, Publications, Library, and Research. With the growth of the resources of the Society, chiefly in the present century, it has been possible to enlarge greatly the activities of the Society in all of these fields, and especially in the last named. The Society now allots grants for research in all fields of learning to a total sum of about $100,000 per year.

In 1812 naturalists and medical men of Philadelphia, and of the United States in general, organized the Academy of Natural Sciences of Philadelphia. Many leading members of the American Philosophical Society joined in this organization and over the course of the following century the Society turned over to the Academy its entire Cabinet of Natural History including many invaluable specimens collected and presented to the Society by Thomas Jefferson, Casper Wistar, and other distinguished members.

In 1824 the Historical Society of Pennsylvania took over in large part the functions of the Historical and Literary Committee of the American Philosophical Society, and in the same year The Franklin Institute took over much of the work of the Committee on Mechanics and Architecture and of another Committee on Trade and Commerce.

The early collections of the Society in archaeology and ethnology have been placed on permanent loan with the University Museum, and many volumes of the Library, in technical and special fields, no longer cultivated by the Society, have been distributed to other libraries in Philadelphia and elsewhere where they will be of greater service. Thus the functions and collections of this oldest learned society of America have gradually been distributed to these younger and more specialized organizations.

The American Philosophical Society felicitates The Franklin Institute on this auspicious birthday; it, in company with the world of science, congratulates the Institute on the notable contributions which it has made to science and civilization; it confidently anticipates in coming centuries and quarter-centuries still greater achievements in the service of man; and the Society hails the Institute with a hearty *God Speed!*

There is a fascination about Centennials that derives in part from the ease of marking off the constant flow of time in round numbers, and from the blessings of the decimal system for those who have difficulties with more complicated numerals. Such centennials also call attention in a striking way to the progress of civilization in successive periods of equal duration, and they furnish inspiration and call for new dedications for centuries to come. They are also

occasions of Campaigns for Centennial Funds, which sometimes, unfortunately, have *Ten-cential* results.

The first Centennial of which I have any recollection, although I was not present at the Celebration and followed the description of events only in the newspapers, was the great Centennial Celebration of American Independence in this City in 1876. Its influence on the progress of science and learning in America was simply incalculable. The effects of the Sesqui-Centennial Celebration of 1926 were not so great, although the progress of Civilization was probably greater in that half-century than in the previous century. But the rapid progress of science in the fraction of a century, indeed in a single lifetime, calls for looks backward and forward in every half- or quarter-century.

But in the endless flow of time how insignificant years, and lifetimes, and even centuries seem! And how devoutly we should echo Whittier's *Centennial Hymn:*

"Our father's God! from out whose hand
The centuries fall like grains of sand,
We meet today, united, free,
And loyal to our land and Thee,
To thank Thee for the era done,
And trust Thee for the opening one."

BENJAMIN FRANKLIN AND HIS ELECTRICAL EXPERIMENTS

DR. ROBERT A. MILLIKAN
California Institute of Technology

I have been asked to say something about Franklin on this double occasion for honoring him, but you will all agree that it is impossible to say something new about Franklin. There is no one who would challenge that statement. There is no American, living or dead, about whom so much has already been said and written as about Benjamin Franklin. I doubt whether any man has ever lived, other than the founder of a religion, whose name is so widely known as that of Benjamin Franklin. So what I shall attempt to do is merely to analyze why it is that Franklin has such publicity value.

There is one cause of his great notoriety for which he deserves no particular credit—call it the "Adam cause." In other words, the first man who enters any field and does things in it is always a very marked man. It can't be otherwise. Franklin was in fact the first man who ever did anything in the field of electricity that was analytical or that helped man in any way to understand this most destructive and amazing phenomenon of the natural world. Our age being the electrical age, it is inevitable and proper that Franklin should have a great part in celebrations having to do with the beginnings of "the age of electricity."

Benjamin Franklin is directly comparable with Sir Isaac Newton or should I say with Copernicus who is credited with the initiation of the great Copernican theory. How much he deserves credit for the whole of it is perhaps debatable. For Sir Isaac Newton certainly was the first in understanding the action of gravity. So both of them were "Firsts." Again in our own time, Roentgen has very wide notoriety primarily because of the fact that he was a First, that it was so thrilling a thing to all mankind to realize that there was such a thing as seeing your own skeleton, which is what Roentgen did. That type of publicity is a thing that is more or less accidental, but if you want, in order to understand the full scope of the popularity of Franklin, I'll have to give at least a "Who's Who" kind of a sketch of Franklin's life, which I shall divide into four periods.

The first period is from birth to the age of 17, and even then there were some things that were notable for which he had no responsibility—he was the 17th child, which is a thing that is quite out of the ordinary, and for which he certainly had no responsibility. He had no formal education, his only contact with the public schools being in the two years from age eight to age ten. At age 10, he became a printer's apprentice and, due to his own inner qualities, a master printer before he was 17. Not only did he become a master printer, but he began in those

seven years to cultivate the art of expression, studying English models that he might become a good writer, so that at 17 he was a master printer and a young publisher also.

That takes us to the second period of his life, which I call the period of the successful business man. That period lasted from the age of 17 up to the age of 42. It starts by his leaving Boston, taking a ship to New York and then getting over to Philadelphia with a few shillings in his pocket, mostly by walking. That shows some of the qualities in the man. Then when he got to Philadelphia, practically instantly, within a few months, he showed his geniality and his genius for making friends by having in the City of Philadelphia—I believe its population was the great number of 10,000 at that time—the friendship of the governor, Keith, who actually persuaded him to go to England in order to buy material for setting up a master printer's workshop in Philadelphia. Franklin went to England and stayed there for two years, but Keith, his patron, went back on him. This did not hurt him at all because he was already a master printer and could get work in printing, which he did, paying no attention to the fact that his patron had gone back on him. He returned to Philadelphia and started over again at the age of 20, becoming very rapidly the leader of the whole printing and publishing business in the colonies and dominating completely that field. Let me see if I can give you one or two illustrations of his diversified activities in that second period of his life.

When he was only 25 years old, he started "Poor Richard's Almanac" as a part of his venture and had an enormous circulation in the colonies with that Almanac. As a result of the profitableness of his publishing business, by the time he was forty-two he was ready and determined to retire and to do some things other than to make a living for himself and his family.

But even while he was making that living he was doing civic things. At the age of 25 he started the first circulating library in the new world. At the age of 30 he became a member of the General Assembly and he also organized the first Police and Fire Department in the Colonies. At about the same age, 30 or 31, he was active in founding a hospital and at almost the same time, probably a little later, he founded also the debating society which became the Philosophical Society, and also an academy which later became the University of Pennsylvania.

He was ready to retire at the age of 42 on the income from his publications. He had the sense to make a partnership, "Franklin and Hall," turning over the work to Hall but keeping a proper share of the income of the firm. He not only did that here in Philadelphia but also in New York. His partner in New York ran the business and Franklin's part of the proceeds was properly arranged so that when he retired he had an income of three thousand pounds a year, that is fifteen thousand dollars, which at the present value of American currency would be equal to fifty thousand dollars in buying power. That is then the brief history of Franklin as the successful business man.

The next period is the period of his scientific work. There were only seven years of it, for he started in 1747 and in 1753 he was taken away from his experiments and went into the last period, his political one, which lasted the rest of his life. It was in the scientific period that his great fame and publicity came, for he became at once a world figure, not merely a Colonial figure, because of the lightning rod incident.

The Lightning Rod incident began in 1750 with a letter to Peter Collinson, in which he described a way to prove that electricity, produced by rubbing a glass rod with silk, was the same thing as lightning. He wrote this letter suggesting that on some tall tower in Europe they put up a rod and collect the electricity from the clouds and store it in a Leyden jar. The paper was read in the Royal Society of London and completely shelved, if not laughed at. By some good fortune, the report about it got over to France and King Louis the Fifteenth had the sense to ask one of his subordinates to try the experiment. It was tried in Paris and it succeeded. At once Franklin became a world figure, for everywhere all over the world the experiment was repeated. In Russia in trying to repeat it one man was killed.

There came at once, naturally, the kind of publicity that comes from harnessing lightning—the most terrifying and the most awful phenomenon of nature. When it was learned that one man had tamed the lightning, and begun to understand it, the publicity value of that situation was infinite. Franklin was at once a man known the world over througout the rest of his life.

Because of the success of the Philadelphia experiment, performed in Paris, he was made a member of the Royal Society of London in 1753 and in the same year a member of the Paris Academy.

Scientific honors the world over were his. The kite experiment came two years after. It is the same thing as what had become known as the Philadelphia Experiment. It was simply another way of getting a sharp point up into the clouds. But at any rate, it got into the text books of every nation as Franklin's Kite Experiment, so that there is no wonder that Franklin's publicity value probably exceeds any other man living or dead, save that of those who have founded religions.

Franklin stopped his experimenting with electricity in 1753 and entered local politics for about four years, not too successfully. In 1758 now being a world figure, he was naturally chosen by the colonies to go to England, particularly because of the fact that he knew England so well from his earlier contact there. Save for two years, he was in England from 1758 to 1776 during which time he worked not as a master printer but rather worked with the British Government, trying to adjust things between the colonies and King George. By the way, one of the amusing incidents of that period is that inasmuch as it had been decided to place lightning rods on the top of Westminster and also on the Buckingham Palace—King George insisted that they be terminated by balls instead of points and there was a quarrel over that demand, an incident which shows how badly power is often misplaced in human affairs.

There is nothing further that I am going to add until 1776, when Franklin was sent as one of three commissioners to Paris to sell the case of the Colonies to what was then unquestionably the greatest monarchy on earth, namely that existing in France. And as everybody knows, he was able to adapt himself; whether it was in kissing the ladies of the court or whether it was in dealing with the qualities on the masculine courtiers, he showed himself a successful diplomat. He kept his press going in Passy too, so that to the end of his days he remained a master printer, and publisher. The last trip that he made to get back to this country was a very, very bothersome one because the days were long and he suffered a great deal from a stone in his bladder. I don't think they operated or that he had any relief from the pain of that stone, but he was here at home for five years, and in spite of his infirmities exerted an important influence in the constitutional convention. He died in 1790.

At his funeral the various societies which had been organized by him cast lots to see who should have the place of honor at his funeral. I think the Philosophical Society won first place and the University of Pennsylvania got second place. What they did, however, was to withdraw and let the Society of Printers go first, because they thought, properly, that it would please Franklin most that they should go first. He wanted to be known from the time he started to the time he died as Franklin, the printer and publisher.

Now, I don't want to close without presenting you with that which isn't only *about* Franklin but that which *is* Franklin. I suspect that a large number of this audience, if they have read the letters to Peter Collinson, will have forgotten them, and a large number of my audience will never have read them anyway. So, I want to read a few paragraphs from the first letter written when nobody knew anything whatsoever about electricity. Before Franklin only a few things had been discovered in that field. The rubbing of amber by Thales in 600 B.C. was the first, and not another thing appears until 1600 A.D., when Gilbert repeated the Thales experiment and showed that you could get the same effects with substances other than amber. A 100 years later, Stephen Gray discovered the facts of conductivity and then DuFay, a Frenchman, showed there were two kinds of electricity but he differentiated them only because he found attractions in one place when he found repulsions in another and therefore labeled them vitreous (or glass) electricity and resinous electricity after the two substances rubbed. He got what we now call electricities of a positive sign and of negative sign. Further, the so-called Leyden jar was accidentally discovered by Musschenbroeck in Holland in 1745, just at the time at which Franklin was arranging to retire. It was because of the fact that one could get larger shocks from a Leyden jar than one could get from a rubbed rod of glass or ebonite that electricity became quite a plaything of the courts, because one could shock another person more powerfully that way. Now comes the first letter of the Peter Collinson series by Franklin.

"To Peter Collinson—Philadelphia March 28, 1747
"Sir,

Your kind present of an electric tube, with directions for using it, has put several of us on making electrical experiments, in which we have observed some particular phenomena that we look upon to be new. I shall therefore communicate them to you in my next, though possibly they may not be new to you, as among the numbers daily employed in those experiments on your side [of]* the water, 'tis probable some one or other has hit on the same observations. For my own part, I never was before engaged in any study that so totally engrossed my attention and my time as this has lately done; for what with making experiments when I can be alone, and repeating them to my Friends and Acquaintance, who, from the novelty of the thing, come continually in crouds [sic] to see them, I have, during some months past, had little leisure for any thing else.

I am, &c.
B. FRANKLIN."

That was the first of the Collinson letters. Then on July 11th, of the same year, comes his next letter.

"Philadelphia, July 11, 1747
"Sir,

In my last I informed you that, in pursuing our electrical enquiries, we had observed some particular phenomena, which we looked upon to be new, and of which I promised to give you some account. . . ."

"The first is the wonderful effect of pointed bodies" [This had never been experimented on before. See how he does it.] "both in *drawing off* and *throwing off* the electrical fire. For example,

"Place an iron shot" [We'd call it a bullet—or rather a cannon ball. See how he describes this experiment. There isn't one teacher in a thousand that does this experiment today and does it anywhere near as well as he did it the first time it was ever done. Its quality lies in the way he handles little details. Note the way he does it to make it striking instead of merely showing a little spark, as one usually does.]

"Place an iron shot of three or four inches diameter on the mouth of a clean dry glass bottle. By a fine silken thread from the ceiling, right over the mouth of the bottle, suspend a small cork-ball, about the bigness of a marble; the thread of such a length, as that the cork-ball may rest against the side of the shot. Electrify the shot, and the ball will be repelled to the distance of four or five inches, more or less, according to the quantity of Electricity." [See, it is a beautiful lecture experiment when done as Franklin directs.] "When in this state, if you present to the shot the point of a long, slender, sharp bodkin, at six or eight inches distance, the repellency is instantly destroyed, and the cork flies to the shot. A blunt body must be brought within an inch, and draw a spark to produce the same effect. To prove that the electrical fire is *drawn off* by the point, if you take the blade of the bodkin out of the wooden handle, and fix it in a stick of sealing-wax, and then present it at the distance aforesaid, or if you bring it very near, no such effect follows"; [He is doing the experiment over again, only he is putting the bodkin on an insulator.] "but sliding one finger along the wax till you touch the blade, and the ball flies to the shot immediately. . . . If you present the point in the dark, you will see, sometimes at a foot distance and more, a light gather upon it, like that of a fire-fly, or glow-worm; the less sharp the point, the nearer you must bring it to observe the light; and at whatever distance you see the light, you may draw off the electrical fire, and destroy the repellency. . . ."

"To show that points will *throw off* as well as *draw off* the electrical fire; lay a long sharp needle upon the shot, and you cannot electrify the shot so as to make it repel the cork-ball. . . . Or fix a needle to the end of a suspended gun-barrel, or iron rod, so as to point beyond it like a little bayonet; and while it remains there, the gun-barrel, or rod, cannot by applying the tube to the other end be electrified so as to give a spark, the fire continually running out silently at the point. . . . The repellency between the cork-ball and the shot is likewise destroyed . . ." [and this is an extraordinary thing]. 3. By making a smoke about it from burning wood. 4. By candle-light, even though the candle is at a foot distance: these do it suddenly.

". . . The light of the sun thrown strongly on both cork and shot by a looking glass for a long time together, does not impair the repellency in the least. This difference between fire-light and sun-light is another thing that seems new and extraordinary to us." [He didn't understand it. Now he puts a footnote in the same letter—here is the footnote. He has been thinking it over.] "This different Effect probably did not arise from any difference in the light, but rather from the particles separated from the candle, . . ." [We call them ions now, produced by the

* Material in [ ] has been added by Dr. Millikan.

burning of the candle] "being first attracted and then repelled, carrying off the electric matter with them; . . ." [He hasn't gotten this quite right because what actually happens is, that you have ions formed at the point or near the point because of the intensity of the field rather than that the molecules themselves receive a charge and are then drawn to the shot. But it is an interesting thing that when I first studied electricity in Carhart's *Physics*, the description of the way the discharge of the points produced discharge was given exactly as Franklin here describes it. We didn't know anything about ionization in air at that time so Franklin's explanation lasted for about 200 years.]

"We had for some time been of the opinion, that the electrical fire was not created by friction, but collected, being really an element diffused among, and attracted by other matter, particularly by water and metals. We had even discovered and demonstrated its afflux to the electrical sphere, as well as its efflux, by means of little light windmill wheels made of stiff paper vanes, fixed obliquely and turning freely on fine wire axes. Also by little wheels of the same matter, but formed like water-wheels. . . . But now I need only mention some particulars not hinted in that piece, with our reasonings thereupon: though perhaps the latter might well enough be spared.

1. A person standing on wax, and rubbing the tube, and another person on wax drawing the fire," [that is, from the rubbed glass the second person touches it and gets a spark from it; both of them are on wax] "they will both of them, (provided they do not stand so as to touch one another) appear to be electrified, to a person standing on the floor; that is, he will perceive a spark on approaching each of them with his knuckle.

"2. But if the persons on wax touch one another during the exciting of the tube, neither of them will appear to be electrified.

"3. If they touch one another after exciting the tube, and drawing the fire as aforesaid, there will be a stronger spark between them than was between either of them and the person on the floor.

"4. After such strong spark, neither of them discover" [exhibit] "any electricity.

"These appearances we attempt to account for thus: We suppose, as aforesaid, that electrical fire is a common element, of which every one of the three persons abovementioned has his equal share, before any operation is begun with the tube. *A*, who stands on wax and rubs the tube, collects the electrical fire from himself into the glass; and his communication with the common stock being cut off by the wax, his body is not again immediately supply'd. *B*, (who stands on wax likewise) passing his knuckle along near the tube, receives the fire which was collected by the glass from *A*; and his communication with the common stock being likewise cut off, he retains the additional quantity received. . . . To *C*, standing on the floor, both appear to be electrified: for he having only the middle quantity of electrical fire, receives a spark upon approaching *B*, who has an over quantity; but gives one to *A*, who has an under quantity. If *A* and *B* approach to touch each other, the spark is stronger, because the difference between them is greater: After such touch there is no spark between either of them and *C*, because the electrical fire in all is reduced to the original equality. If they touch while electrifying, the equality is never destroy'd, the fire only circulating." [Here is the first description of an electrical current. Remember electrical currents weren't discovered until Volta's time, 50 years after this, but the whole of electrical current phenomena is described in that experiment, and with extraordinary care.] "Hence have arisen" [and look at the nomenclature that came into existence here at the first time that Franklin had done any electrical experimenting at all] "some new terms among us: we say *B*, (and bodies like circumstanced) is electrified *positively; A*, *negatively*. Or rather, *B* is electrified *plus; A*, *minus*. And we daily in our experiments electrify bodies *plus* or *minus*, as we think proper. . . . To electrify *plus* or *minus*," [Now this is also extraordinarily clever because he doesn't want to make any more assumptions than he needs to explain his phenomena] "no more needs to be known than this," [which is a great scientific principle which we physicists often don't use and the philosophers never use it, for they never stop when they've got enough to explain the thing they're working on. They try to get a universal out of it, and you never can get it, because we're finite beings.] "that the parts of the tube or sphere that are rubbed, do, in the instant of the friction, attract the electrical fire, and therefore take it from the thing rubbing" [the man]: "the same parts immediately, as the friction upon them ceases, are disposed to give the fire they have received, to any body that has less." [That is the description of that experiment.]

How unambiguously Franklin conceived his theory of electricity and described the most important properties of the electrical atom is shown by the following quotation of 1749, when he had had two years only of this type of experimenting, from a letter to Peter Collinson.

"1. The electrical matter consists of particles extremely subtle, since it can permeate common matter, even of densest metals, with such ease and freedom as not to receive any perceptible resistance.

"2. If anyone should doubt whether the electrical matter passes through the substance of bodies, or only over and along their surfaces, a shock from an electrified large glass jar, taken through his own body, will probably convince him." [Franklin built two Leyden jars—6 gallons each—and they were put together; the shock knocked him senseless to the floor; he had built his Leyden jars for killing turkeys, but he got a charge from them himself.]

"3. Electrical matter differs from common matter in this, that the parts of the latter mutually attract, those of the former" [those of the electricity] "mutually repel each other. Hence the appearing divergency in a stream of electrified effluvia.

"4. But though the particles of electrical matter do repel each other, they are strongly attracted by all other matter." [So, if you put this in terms of our present theories of of the atom in a neutral condition, made neutral because the positive charge on the nucleus is just balanced by the number of electrons around the outside (that's what the normal state of matter is), the matter in its normal state shows no electrical properties at all. Now, I'll go back to his original description.]

"But though the particles of electrical matter do repel each other, they are strongly attracted by all other matter." [That's why as stated, the atom when it has lost some electrons wants to get them back.]

"5. From these three things" [This is intensely penetrating] "the extreme subtilty [sic] of the electrical matter, the mutual repulsion of its parts and the strong attraction between them and other matter, arise this effect, that, when a quantity of electrical matter is applied to a mass of common matter, of any bigness or length, within our observation (which hath not already got its quantity) it is immediately and equally diffused throughout the whole.

"6. Thus common matter is a kind of spunge [sic] to the electrical fluid. And as a spunge would receive no water if the parts of water were not smaller than the pores of the spunge; and even then but slowly, if there were not a mutual attraction between those parts and the parts of the spunge; and would still imbibe it faster, if the mutual attraction among the parts of the water did not impede, some force being required to separate them; and fastest, if, instead of attraction, there were a mutual repulsion among those parts, which would act in conjunction with the attraction of the spunge. So is the case between the electrical and common matter.

"7: But in common matter there is (generally) as much of the electrical as it will contain within its substance. If more is added, it lies without upon the surface, and forms what we call an electrical atmosphere; and then the body is said to be electrified."

That's the end of the quotation. Let me quote again Franklin's phrase, "The electrical matter consists of *particles extremely* subtle." And so I say, I think the foregoing quotation clearly establishes the right of Benjamin Franklin to be considered the *discoverer* of the *atom* of *electricity*, that is, the *discoverer* of the *electron*.

The world has recently and properly celebrated the year 1947 as both the 200th anniversary of Franklin's discovery of the electron and the 50th anniversary of Sir J. J. Thompson's unambiguous establishment of the electron theory of matter. But remember that the atom of electricity itself *was discovered by Franklin, 200 years ago.* Thank you.

Evening Session

At 8:15 P.M. the final session of the day of celebration was opened in the Lecture Hall by Mr. Nalle, the President, who paid tribute to several other Philadelphia firms and institutions also observing their 125th anniversary this year. Mr. Nalle then thanked the representatives of The Electrical Association of Philadelphia, the United States Navy, and the Bell Telephone Company of Pennsylvania, for furnishing special exhibits for the Museum in honor of the occasion.

The remainder of the meeting proceeded as follows:

MR. NALLE: In celebrating our one hundred and twenty-fifth birthday we find that The Franklin Institute is to benefit by one of the great compensating devices for mitigating the yearly distress of increasing age—birthday presents. I think that I will give our Executive Vice-President and Secretary the pleasure of announcing our birthday gifts. Dr. Henry Butler Allen.

DR. ALLEN: Mr. President, Miss Florence Sibley, a good member of the Institute, and an officer of the French Legion of Honor, had the not too pleasant experience of being trapped

in the city of Verdun when it was occupied by the Germans in the last war. She won the respect and affection of the French by her good works in helping them during those sad days. So, when the people of Verdun were planning a contribution to come to America on the French "Thank you" Train, the ex-mayor of that city, President of the Association of War Prisoners of Verdun, wrote Miss Sibley for advice. He wanted her to indicate some institution in this country that would like the gift, a symbolical picture, when the gifts on the train were distributed.

Miss Sibley naturally thought of The Franklin Institute—Benjamin Franklin being a friend of France. Here it is. Mr. President, Miss Sibley is here, too, and perhaps if you ask her she will not mind briefly describing the picture to us.

MR. NALLE: Now, I think it would be most interesting to hear from Miss Sibley herself the story of this picture which we are so fortunate to have received. Just what, Miss Sibley, was your own part in bringing this picture to us?

MISS SIBLEY: Shortly before Christmas I received a letter from Monsieur Panau, President of the Association of the Prisoners of Verdun, saying that the Association was desirous of sending a gift to the "Merci Train" and that they had chosen a picture painted by one of their members, Monsieur Jeannet. Monsieur Panau at the same time asked me to suggest a society or a museum to whom to attribute their gift. I immediately thought of The Franklin Institute and got into touch with Dr. Allen who expressed much interest in the matter and said that he would be glad to accept the picture in the name of the Institute.

When the "Merci Train" arrived in Philadelphia, the car destined for Pennsylvania was detached and forwarded to Harrisburg, where the contents were exhibited during a number of weeks. After this the picture was sent to Dr. Allen, a few days before the celebration of the 125th Anniversary of the Institute.

The painting symbolizes tricolored France detached by an American tank led by the Star Spangled Banner from France covered with barbed wire. In the foreground stands one of the milestones marking the Way of Liberty leading from Normandy across France. Above the flag is a milestone of the Sacred Way running from Bar-le-Duc to Verdun and over which supplies passed during World War I. At the top of the picture rises the American monument at Montfaucon. In the center stands the Cathedral of Verdun and the Victory monument in the middle of the city. To the right is the Ossuary at Douaumont.

The Prisoners have expressed great satisfaction at having their gift go to The Franklin Institute and Monsieur Panau has begged me to convey to Dr. Allen their grateful appreciation of his cordial reception of it along with the hope that it may prove another tie in the friendly relations between our two countries.

MR. NALLE: Thank you, Miss Sibley.

DR. ALLEN: Not long ago, Mrs. Zimbalist sent us word that she had something in her house that she thought might interest the Institute—could we set a time when I could stop in to see it. I did, and found Mr. and Mrs. Zimbalist and Mrs. Geary. We went into the dining room and there on the table was the original 1757 Will of Franklin's, written in his own hand. Mrs. Zimbalist said that she had bought it for The Franklin Institute in memory of her father. I was spellbound. Mrs. Zimbalist let me thank her, informally, but sincerely. Those thanks were reaffirmed by our Board of Managers. I now make the formal announcement to the membership of the Institute of this unique and precious gift.

MR. NALLE: Mrs. Zimbalist, would you please rise so that we may all see the person to whom we are indebted for this valuable gift, which we will cherish among our most prized possessions.

MRS. ZIMBALIST: The Institute seems happy to acquire this will, but it does not begin to be as happy as I am to present it. People ask me how I procured it, and I just say from my wonderful friend, Miss Mabel Zahn, of Sessler's Bookshop, who had endeavored to interest the Historical Society and the American Philosophical Society, but neither had the means to purchase it. I knew of no one who might be interested. I could only think of my father and his tremendous interest in Benjamin Franklin, and of course in The Franklin Institute here, so I looked the will over and there was one highlight when I came to the final paragraph.

I could not let it go. To me its thought, its great religious strain, and the nobility of the expression he used—well, I could just not get away from it. To me the wording of that final paragraph where, after making his bequests, he thanked God, using the most marvelous phrases such as: "for producing me into being" and included the other things for which he was grateful to God, but with such deep feeling that I feel it ranks with the Gettysburg Address. I could not get away from it.

I would like to speak a moment about my father. There were, of course, some similarities between Franklin and Father. They were both boys that came from Boston to Philadelphia to make their fortune—both were printers. The *Saturday Evening Post* was picked up by my father and carried on, having been founded by Franklin. I remember hearing my father say many times "Strange, strange, no memorial to Franklin in this town," and it seems fitting that it was he who led the drive to make this building possible. It became my privilege to bring to the Institute this will of Benjamin Franklin, and I feel it a personal honor to have brought it here.

MR. NALLE: And now I call on Senator George Wharton Pepper, who needs no introduction, to express, as a member of The Franklin Institute, our real appreciation to Mrs. Zimbalist.

MR. PEPPER: At The Franklin Institute interesting things are constantly happening. Seldom however, is there an event to record which is as significant as the one upon which I now wish briefly to comment. I refer, of course, to the acquisition by the Institute of the holographic will written by Dr. Franklin in 1757 and presented to us by Mary Curtis Zimbalist n memory of her distinguished father, Cyrus H. K. Curtis.

It is my pleasant duty this evening to express to her the grateful appreciation of the Institute. In so doing, I shall begin in lighter vein and perhaps even with a trace of buffoonery.

I recently came into possession of an original cartoon by Carl Rose which depicts in delightful fashion Benjamin Franklin at the court of Louis XVI, in the very act of selling to the King a subscription to the *Saturday Evening Post.* This cartoon would have delighted Mr. Curtis; and it certainly should be in the hands of his daughter, Mary Curtis Zimbalist. I beg her to accept it from me but not to open it or pass it along while I am speaking, for otherwise Rose would steal the show.

Mrs. Zimbalist's gift of Franklin's Will of 1757 has an importance which cannot be overestimated.

At your place at the dinner table each of you found a little pamphlet, which I commend to your careful study, giving in Van Doren's excellent style an account of the circumstances under which the Will was written—how Franklin was appointed agent by the Assembly of Pennsylvania to go to London to interview the Ministry in an effort to break, if possible, the deadlock between the Assembly and the Governor; how he went to New York and cooled his heels for three weeks waiting for the packet to sail; how, calming his impatience, he reflected upon his existing will and decided to make a new one; how he wrote out the whole document in his own hand and signed it on the 28th of April, 1757, and then waited six long weeks before the packet sailed.

When we look back on the Revolutionary period we are apt to think of all the actors in the revolutionary drama as having been of equal age. Not so. When this Will was written, the Marquis de Lafayette and Alexander Hamilton were just being born. The next year Admiral Nelson and the younger Pitt were born, and it was not until ten years after the date of this document that the first Napoleon and the Duke of Wellington were born. It was into a distracted world that these men were ushered and Franklin was perhaps the only stabilizing element among them all.

This Will has two characteristics. One is its extraordinary clarity, evidencing as it does a layman's grasp of all the technicalities of will-making—so that, from the technical point of view, the Will is beyond criticism. The other is that final paragraph to which Mrs. Zimbalist has called attention. I venture to think that, through the publicity which will come to it now, the document will rank very high in the list of masterpieces of English prose.

I wish it were possible to expand some thoughts that I have on this subject, but I must compress my remarks into the ten minutes allotted to me. I am going to suggest that the circumstances under which this great gift has been made enable us without too much of a flight of fancy to picture a conversation between Mrs. Zimbalist, her father, Mr. Curtis, and Benjamin Franklin. There they are—a group of extraordinary interest. They have "struck fire" at their first meeting. Indeed, they fell for one another at a glance. You and I in imagination can see them sitting together as Mrs. Zimbalist turns to Dr. Franklin and says, "Dr. Franklin, I have acquired that will of yours written in 1757 and I am presenting it to The Franklin Institute. I hope you approve?"; and Dr. Franklin, rising and bowing from the waist replies, "My dear madam, I not only approve, but I greatly appreciate this honor that you have done me"; and then she, with that spark of coquetry which is indigenous in every woman, exclaims, "Why Dr. Franklin, I am giving this to The Franklin Institute in memory of my father and not of any other man." This does not disconcert Dr. Franklin at all. Says he, "My dear lady, I am entirely content that your father should have the honor and that the Institute should have the document provided that you will bestow upon me one of your radiant smiles." And oh! what a smile she gives him! From that moment Mr. Curtis begins to feel a little *de trop*, as if it were a case for the old maxim "Two's a company—and three's a crowd"; because when Dr. Franklin found himself in the presence of a charming woman emotion took possession of him—a form of electricity in which he was much interested. However, Dr. Franklin put him at once at his ease and said, "Mr. Curtis, your daughter has honored me very greatly while honoring you; and I should like to do something of the same sort. I look forward to the time when I shall have passed on and I am anxious that the *Saturday Evening Post* should ultimately find itself in your ownership because I trust you to make it a wholesome influence in American life." Then I can hear the little chuckle that Mr. Curtis gives, as he says to Dr. Franklin, "Dr. Franklin, you honor me greatly. All I can do is to recall the incident of the negro preacher who offered his services to the Lord and said 'Oh Lord, use me as Thou wilt, even if only in an advisory capacity.'" Then all three of them laughed heartily and parted from one another with deep regret.

Such is my conception of what would have happened had these three been contemporaries. It is easy for me, at my time of life, to forget differences in age. I think that I am perhaps one of the few who was not mentioned at the dinner as contemporary with the founding of the Institute. It is easy for me to obliterate the differences in age and to think of the concurrence of people who in fact chronologically lived long apart.

It is now my pleasant function to accept this great and significant gift on behalf of the Institute—one of the pleasantest and easiest duties that a man can be called upon to perform. We give you, Mrs. Zimbalist, our profound thanks and an expression of our highest appreciation. We shall cherish this document among the most precious treasures of the Institute, and I assure you that we shall do it in such a way that these three names, separated in time but alike in merit, shall be perpetually associated with one another in our annals—your father, whom you and we delight to honor, his daughter, who has made this priceless possession possible, and Dr. Franklin, for whom our prayer is that, undying, he may long continue to be a wholesome influence in the life of America.

MR. NALLE: Thank you, Senator Pepper. Now we come to a very appropriate part of this birthday celebration, the life story of our birthday child, and I have asked Mr. James H. Robins, as one of the immediate family, a member of our own Board of Managers and a great, great grandson of the first Chairman of the Institute's Board of Managers, to give us this story. Mr. Robins!

[Ed. Note: Mr. Robins' talk, "The Story of The Franklin Institute", appeared in the June issue of the JOURNAL OF THE FRANKLIN INSTITUTE.]

At the conclusion of Mr. Robins' talk, the meeting adjourned.

MUSEUM

From the time of its inception The Franklin Institute has concerned iteslf with the education of youth and has made use of different methods adapted to the circumstances and the times. During recent years the enlargement of display facilities offered by the opening of the Museum has permitted the Institute to extend its range and to bring a knowledge of scientific developments to a wider group of people than it had hitherto reached. The current summer has witnessed a new departure and further extension of these efforts to bring science to those who are not engaged in scientific pursuits. Reaching down to a lower age level than in any earlier trial the staff of the Museum's Educational Department organized an introduction to science for very young people called "Science Adventures for Small Fry."

During the months of June and July boys and girls between the ages of eight and twelve years enjoyed ten morning field trips to the Museum. Each session covered a specific topic, such as weather, marine transportation, aviation, stars, or simple mechanisms. The subjects were presented with the aid of motion pictures, demonstrations, and science games. Where possible the young students conducted their own experiments. At the conclusion of each Adventure the young people were given souvenirs which would enable them to continue with experiments at home. Typical of these souvenirs was that accompanying the Adventure in Electricity and Magnetism, which comprised a magnet, iron filings, a compass, dry cell, wires, and other materials for simple electrical demonstrations. After the Adventure in Weather the children were given cloud charts, thermometer, and a chemical humidity indicator to assist them in home weather forecasting.

The design and execution of such a program makes heavy demands upon the ingenuity of those responsible for its presentation but, in this case, the conductor possessed the requisite qualifications for the task. Miss Clara Teaf, who is with the Division of School Extension of the Philadelphia Board of Education, was assigned the duties of conducting the Adventures. Miss Teaf had formerly served as a Museum Teacher and was, therefore, informed upon the resources and work of the Museum.

The extraordinary success of the Science Fair held in the spring of this year had demonstrated the interest taken by local school students in scientific subjects, but with a constant need for basic science understanding and the absence of science classes in primary schools, the Museum staff hopes to continue offering further Adventures for Small Fry. The response to the first series justifies consideration of conducting future series not only during the vacation period but possibly during the school year as well.

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in the JOURNAL within the next few months:

MINORSKY, N.: Energy Fluctuations in a van der Poel Oscillator.

MARIN, JOSEPH: Stress-Strain Relations in the Plastic Range for Biaxial Stresses.

DEVLIN, JAMES A., WALLACE M. MCNABB AND FRED HAZEL: Preparation of Vanadium Pentoxide Sols by Ion Exchange.

COULSON, THOMAS: The Story of Aids to Navigation.

DUBILIER, WILLIAM: Development, Design and Construction of Electrical Condensers.

FANO, ROBERT M.: Theoretical Limitations on the Broadband Matching of Arbitrary Impedances.

FANO, ROBERT M.: A Note on the Solution of Certain Approximation Problems in Network Synthesis.

JACOBSEN, LYDIK, R. L. EVALDSON AND R. S. AYRE: Response of an Elastically Non-Linear System to Transient Disturbances.

MOON, PARRY AND DOMINA EBERLE SPENCER: A Modern Approach to Dimensions.

THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

***Abstract of* Combustion Studies With the Orthicon Spectrograph.**[1]—J. T. AGNEW, R. G. FRANKLIN, R. E. BENN, A. BAZARIAN.[2] Application of the image orthicon and electronic instrumentation to spectroscopy[3] has provided increased light sensitivity, greater spectral range of response, and time resolution of a higher order than can be obtained with a moving film or plate. The instrument is ideal for studying transient or rapidly varying light sources and has already proved useful in a preliminary combustion study.

The instrument consists essentially of a television pickup tube, the RCA image orthicon, whose photo-sensitive surface is placed in the focal plane of a large-aperture spectrograph. A curve of intensity *vs.* wavelength, at any chosen instant or succession of instants, appears on the screen of an oscilloscope whose scans are synchronized with those of the orthicon. Recurrent traces are photographed with a high-speed drum camera.

A recent study has been made of the explosion in a bomb of a mixture consisting of 3.5 per cent n-heptane, 62.8 per cent oxygen, and 33.7 per cent nitrogen, by volume. This can be used as an illustration of the advantages of the orthicon assembly. Single sweep scans were

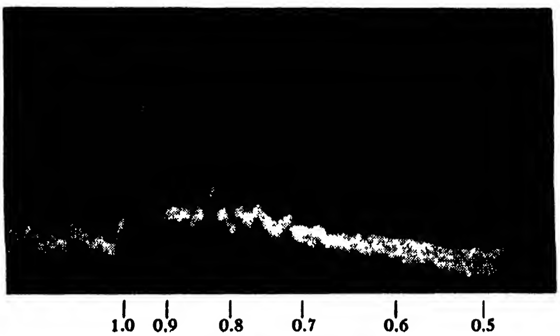

FIG. 1. Radiant energy emission during one millisecond interval from 10th to 11th millisecond.

made at the rate of one every millisecond. They were photographed on a rotating-drum camera whose speed was such that successive traces were conveniently separated. Each scan represents the integrated intensity of the preceding millisecond.

A few representative traces from the series have been reproduced in the accompanying figures. From these and the original traces, a few general observations are apparent.

The growth with time of various emission lines and bands is clearly visible. Characteristic radiation first appeared in the vicinity of 0.93 to 0.96 micron (see Fig. 1). Red and yellow bands have appeared in 15 milliseconds (Fig. 2) and strong green bands in 25 milliseconds (Fig. 3).

The appearance of the bands and lines in the order red to green and their decay in the order green to red strongly suggests the important role of temperature in these phenomena. For the mixture studied, it appeared that the temperature increased rapidly during the first part of the combustion, reached a maximum which was maintained without great change for a time, and then gradually decreased. Paralleling the temperature changes was the appearance of the red bands and lines and then the addition of the other spectral regions, the green appear-

[1] "Letter to the editor," published in May, 1949 issue of *Journal of the Optical Society of America*. Reprinted with permission of the editor.

[2] The Franklin Institute Laboratories, Philadelphia, Pa.

[3] R. E. BENN, W. S. FOOTE, AND C. T. CHASE, "The Image Orthicon in Spectroscopy," to be published.

ing only at maximum temperature and decaying after the temperature had again fallen below that necessary to excite the region.

It might be pointed out that with a single explosion, data comprising the whole spectral region from 0.45 to 1.0 micron are obtained, and some bands are visible which might possibly escape detection if a standard photographic method were used. Also, the feature of obtaining complete spectra of the explosion at a rate of one every millisecond is unique due to the high sensitivity and storage feature of the orthicon tube.

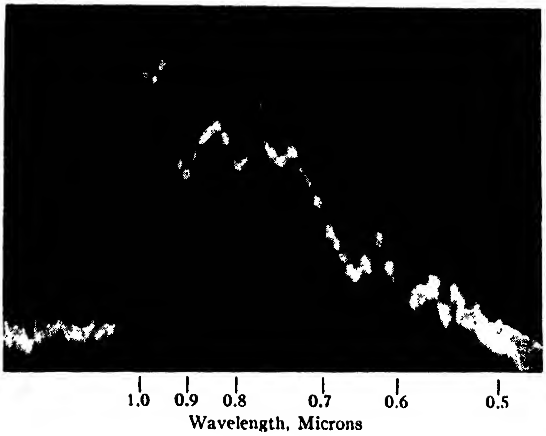

FIG. 2. Radiant energy emission during one millisecond interval from 14th to 15th millisecond.

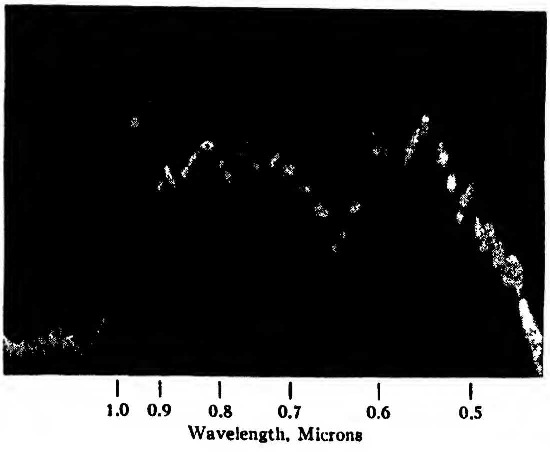

FIG. 3. Radiant energy emission during one millisecond interval from 24th to 25th millisecond.

It is also important to note that the rate of one trace per millisecond was arbitrarily chosen for this particular explosion. Faster rates could be used if the source were of shorter duration, particularly so if it were more intense. The maximum rate, using commercial components, would be approximately 15,750 traces per second, in which case the storage time would be equal to the tracing time of approximately 63 microseconds. The actual time consumed in producing the traces shown in Figs. 1–3 was 63 microseconds while the storage time was 1 millisecond.

Engravings loaned through the courtesy of the Journal of the Optical Society of America.

BARTOL RESEARCH FOUNDATION

Abstract of **Resistance of Oxide Cathode Coatings for High Values of Pulsed Emission.***— W. E. DANFORTH[1] AND D. L. GOLDWATER.[1] Measurements are described of the potential variation of fine ribbon probes embedded in standard barium and strontium oxide coatings. Observations were made with 19 tubes at values of pulsed emission as high as 50 amperes/cm^2. Results were as follows: (a) values of resistance were found ranging from 2 to 230 ohms-cm^2.; (b) potential gradients were adequate to admit dielectric breakdown as a cause of sparking; (c) the curve of resistance *vs.* emission passes through a maximum and, at different temperatures, this maximum occurs at the same probe potential; (d) with BaO or mixed cathodes of higher values of resistance the resistance is largely at the interface, for low resistance coatings no observations on this matter were obtained; (e) the two best tubes as regards high emission without sparking had the lowest values of coating resistance; (f) the resistance of SrO cathodes is an order of magnitude higher than that of BaO or mixed cathodes; (g) super-position of d.c. upon pulsed emission gives a pronounced decrease of resistance.

* Paper appears in *Journal of Applied Physics*, Vol. 20, No. 2, 163–173, February, 1949.
[1] Bartol Research Foundation of The Franklin Institute, Swarthmore, Pa.

Abstract of **Neutron Spectrum from N^{15} (d, n).***[,1]— E. L. HUDSPETH[2] AND C. P. SWANN.[2] Gas targets of normal and of N^{15}-enriched N_2O have been bombarded with 1.2 Mev. deuterons, and the neutron spectra have been observed on photographic plates placed at 0° to the bombarding beam. Two groups previously reported[3] from N^{14} (d, n) were observed, with evidence for the production at our bombarding voltage of a third group of lower energy. Neutron groups from N^{15} (d, n) were observed at 11 Mev. and at approximately 4.5 Mev. The latter group was broader and several times as intense as the group at 11 Mev. There is evidence for a third group at 2.5 Mev., but it may belong entirely to the N^{14} reaction.

Excited states of O^{16} in the vicinity of 6.5 Mev. are well-known from studies by other experimenters of the reaction F^{19} (p, α) and from the β-decay[4] of N^{16}. Our results reveal these levels (unresolved), and no lower excited state is observed.

* Assisted by the Joint Program of ONR and AEC.
[1] Paper as yet unpublished.
[2] Bartol Research Foundation of The Franklin Institute, Swarthmore, Pa.
[3] W. E. STEPHENS, K. DJANAB, AND T. W. BONNER, *Phys. Rev.*, Vol. 52, p. 1079 (1937).
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Abstract of **The Disintegration Scheme of Cerium** (141).*[,1]—C. E. MANDEVILLE[2] AND E. SHAPIRO.[2] The beta rays of Ce^{141} have a maximum energy of 0.44 Mev. An absorption curve in lead of the gamma radiation yields a quantum energy of 0.13 Mev. This gamma ray is coincident in time with the beta ray spectrum. Beta-beta coincidences are present in the disintegration of Ce^{141}, showing that the gamma ray is to some extent converted. The beta rays of Pr^{143} have a maximum energy of 0.835 Mev.

* Assisted by the Joint Program of the Office of Naval Research and the Atomic Energy Commission.
[1] Paper as yet unpublished.
[2] Bartol Research Foundation of The Franklin Institute, Swarthmore, Pa.

Abstract of **Cosmic-Ray Variations at High Altitudes Resulting from Meteorological Variations.***[,1]—W. F. G. SWANN[2] AND S. H. FORBES.[2] K. M. Kupferberg has called attention to latitude and seasonal variation of cosmic-ray intensity resulting from variations of the distance between the layer of mesotron production and the place of observation, which distance operates through mean-life on the cosmic-ray intensity. We have developed an expression for the percentage change of intensity at any altitude resulting from unit change in the aforesaid distance, this expression taking into account the energy distribution function and the lower limit of energy as determined by the energy necessary to penetrate the apparatus. Data supplied by the Weather Bureau show that observations extending over a single day may

experience a change of 100 meters or even more in the aforesaid distance. Our calculations show that with a change of 100 meters at an altitude of 30,000 ft. with a lower energy limit determined by 9 cm. of lead, the effect on the cosmic-ray intensity amounts to 2.3 per cent. With a lower limit determined by 18 cm. of lead, the effect is 2.1 per cent.

* Assisted by the Joint Program of ONR and AEC.
[1] Paper presented at the Cambridge, Mass., meeting of the American Physical Society, June 18, 1949.
[2] Bartol Research Foundation of The Franklin Institute, Swarthmore, Pa.

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NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

Molecular Weights of Desoxyribonucleic Acid Polymers.—LAURA E. KREJCI, LUCILE SWEENY AND JOHN HAMBLETON. During an investigation of the enzymatic liberation of phosphorus from nucleic acid (1), estimates of molecular weight were required for a graded series of desoxyribonucleic acid polymers. The sedimentation-viscosity method (2) was utilized for this purpose because of the small amount of each polymer available.

MATERIALS AND METHODS

Desoxyribonucleic acid was isolated from calf thymus by the method of Mirsky and Pollister (3). Aliquot portions of a 0.3 per cent solution were depolymerized by the following agents:

- X-rays (100,000 r)
- Hydrogen peroxide (1 per cent at 40° C. for 60 min.)
- Heat (60° C. for 60 min.)
- Treatment in a sonic oscillator for 1, 3, 10 and 30 min., respectively
- Hydrolysis in 1 *N* hydrochloric acid for 1, 1½ and 2 hr., respectively
- Hydrolysis with desoxyribonucleic acid depolymerase.

The details have been described elsewhere (1).

Sedimentation velocity measurements were made in a Beams type analytical ultracentrifuge (4, 5, 6) equipped with a Philpot-Svensson optical system (7). Each polymer was examined at a concentration of about 0.25 per cent in 0.2 *N* sodium chloride solution; the samples showing least extensive depolymerization were also examined at lower concentrations. The viscosities of all solutions were measured at 30° C. in an Ostwald viscometer. The densities were measured by the method of Barbour and Hamilton (8) at the same temperature.

EXPERIMENTAL RESULTS

Sedimentation Constants. The sedimentation constants, measured at temperatures varying from 24.1 to 26.8° C., were corrected to the basis of sedimentation in water at 20° C., or to s_{20}, by use of the viscosity and density of the solutions (9). The sedimentation constants so corrected (Table I) were independent of the weight concentration of the polymer for the solutions showing most extensive depolymerization (those subjected to the action of depolymerase, acid hydrolysis, and

sonic oscillation for 10 or 30 min.). For the remaining solutions (those depolymerized by X-rays, heat, hydrogen peroxide, and sonic oscillation for 1 or 3 min.) the corrected constant increased with weight concentration; for these the lower value was used in estimating the molecular weight. The sedimentation boundaries for the acid- and enzyme-treated preparations did not completely separate from the meniscus. The heat-treated preparation showed extensive boundary spreading. The sedimentation boundaries for all other preparations were well-defined throughout the entire period of centrifuging, and showed no pronounced departures from symmetry.

TABLE I. *Molecular Weights of Desoxyribonucleic Acid Polymers.*

Depolymerizing Agent	Concn. %	Relative Viscosity	Intrinsic Viscosity	Axial Ratio	Frictional Ratio	s_{20}	M
None	0.242	3.3144	856	124	4.51	18.38	
	0.046	1.2553	852	123	4.49	10.42	691,000
Sonic 1	0.242	2.2946	593	100	4.07	12.76	
	0.048	1.2107	681	108	4.22	9.25	526,000
X-ray	0.242	2.4790	648	105	4.16	13.14	
	0.048	1.2059	666	107	4.20	9.09	509,000
H_2O_2	0.242	1.9490	477	88	3.83	10.67	
	0.048	1.1515	502	90	3.87	9.44	476,000
Heat	0.242	2.2742	587	99	4.05	16.64	
	0.121	1.3466	425	82	3.70	9.35	439,000
Sonic 3	0.242	1.7621	405	81	3.68	9.56	
	0.048	1.1373	458	86	3.78	7.72	341,000
Sonic 10	0.242	1.3672	223	57	3.13	7.15	
	0.048	1.0705	242	58	3.15	7.01	224,000
Sonic 30	0.242	1.2207	142	44	2.78	5.88	142,000
	0.048	1.0473	164	48	2.89	5.86	
Acid 1	0.242	1.0225	15.9	11.4	1.61	1.76	10,300
Acid 1½	0.242	1.0162	11.5	8.8	1.48	1.62	8,000
Acid 2	0.242	1.0162	11.5	8.8	1.48	1.10	4,500
Depolymerase	0.227	1.0183	12.9	9.7	1.53	0.82	3,000

Frictional Ratio. The frictional ratio for each solute was determined from the intrinsic viscosity by the use of the Simha equation (10, 11) relating intrinsic viscosity and axial ratio, and the equation of Perrin (12) and of Herzog, Illig and Kudar (13) relating axial ratio and frictional ratio.

Partial Specific Volume. For the partial specific volume the value $V = 0.58$, the average of the published values (14, 15), was used throughout.

Molecular Weights. The molecular weights were computed from the above values by means of the equation (2):

$$M = [6(f/f_0)\pi\eta s_{20}N(3V/4\pi N)^{1/3}/(1 - V\rho)]^{3/2}$$

in which f/f_0 represents the frictional ratio of the solute molecule, s_{20} the sedimentation constant, η the viscosity of water at 20° C., N the Avagadro number, V the partial specific volume of the solute, and ρ the density of water at 20° C.

All values are listed in Table I.

DISCUSSION

The purpose for which this graded series of nucleic acid polymers was to be used (1) necessitated a knowledge of the number average molecular weights. Ideally the sedimentation-viscosity method probably gives a weight-weight average molecular weight; for an inhomogeneous sample this is greater than the number average because it stresses the larger molecules. The procedure and approximations which were used, however, as shown below, tended to weight the data in favor of the smaller molecules, in the direction of the number average molecular weight.

For elongated molecules the random molecular orientation required by the Simha equation is realized only at very low velocity gradients. Such gradients give a high limiting value for the viscosity which is a function of the weight average of the squared axial ratios (2), and which thus stresses the largest molecules. The relatively high gradients obtained with the Ostwald viscometer minimize the influence of the larger molecules and might therefore be expected to give a value which more nearly approximates the number average when both large and small polymers are present. When the size range is narrow and the average axial ratio high, on the other hand, these gradients give too low a value for the frictional ratio.

For the solutions showing the least extensive depolymerization, the sedimentation constants corrected for the viscosity and density of the solutions increased with the weight concentration of the polymer. These are the solutions for which the viscosities measured in the Ostwald viscometer probably gave too low an estimate of the axial and frictional ratios. The use of the sedimentation constant found with the more dilute solution (0.048 per cent), instead of the still lower value which would be obtained by extrapolation to infinite dilution, should have compensated somewhat the under-evaluation of the frictional ratio, since both errors, opposite in their effect on the molecular weight, should increase with chain length.

The molecular weights listed in Table I may be subject to considerable absolute error, because hydration has been ignored, and the value

for the partial specific volume is uncertain and may vary with chain length. Relative to one another, however, they probably approximate the number average molecular weights within the limits of accuracy required by the experiments in which they were utilized. They show a striking difference in the degree of depolymerization effected by hydrolytic agents on the one hand, and by physical and oxidizing agents on the other. Under the conditions used, hydrochloric acid and desoxyribonucleic acid depolymerase produced fragments which were smaller by an entire order of magnitude than those produced by treatment with X-rays, heat, sonic vibration, or hydrogen peroxide.

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BOOK REVIEWS

PULSES AND TRANSIENTS IN COMMUNICATION CIRCUITS, by Colin Cherry. 317 pp., drawings and tables, 14 × 22 cm. London, Chapman and Hall, Ltd., 1949. Price, 32/.

This is an up-to-date book which teaches fundamentals of a particular phase of communications engineering, a many-sided profession. Although the book presents no original material, it places within its 300 pages the basic concepts of network analysis in the briefest possible form. In addition to providing a rapid summary of first principles, the author has applied the theories of transient response to subjects of recent importance such as multistage amplifiers and vestigial sideband transmission.

The book is essentially an elementary abridgement of E. A. Guillemin's "Communications Networks" to which constant reference is made. It is more enjoyable to study, and progress is more easily made than in Guillemin's books because of the omission of all but the more elementary mathematics. Excellent lists of twenty-five or thirty references at the end of each chapter not only indicate the author's familiarity with the important literature but make this a handy bibliography for the professional engineer.

The American reader will find throughout the text a number of interesting electrical terms peculiarly British. The author is a lecturer at the University of London with a background in the Research Laboratories of the General Electric Co., Ltd. He has written articles for American and British periodicals on a variety of communication subjects.

The value of the book results from the author's sorting out and applying uniform notation to the basic portions of important articles on transient analysis published in the past by key men of the communications field. Specific solutions to circuit problems are not given in sufficient detail, and the author has not been able in the space available to go deeply enough into his subject to relieve the reader of having to refer to original material. This, however, is true of most textbooks in the relation they bear to the practical solution of an actual engineering problem. The undergraduate engineer specializing in communications will benefit from the book's contents alone. In a different sense the practicing engineer will find, within the limited scope of the subjects covered, that the volume is useful as a tabular summary and reference catalogue.

C. W. HARGENS

RADIOACTIVE MEASUREMENTS WITH NUCLEAR EMULSIONS, by Herman Yagoda. 356 pages, illustrations, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1949. Price, $5.00.

A review of this is at once simplified by the fact that it need not be compared with similar books, since it is essentially the first of its kind. It is a careful recording of the research (published and otherwise) to date, dealing with the detection and study of radioactive emanations as brought into visual evidence by the photographic film or emulsion. The Geiger tube and the dramatically popular click-click of its counter have practically obscured the achievements of emulsions in popular nuclear studies and unjustifiably overshadowed their success even in the scientific mind.

All the research described in the book is based on the experimental fact that a charged radioactive emission or ionizing radiation of reasonable energy will sensitize the particles of silver halide granules to render them reducible on developing.

After describing in detail laboratory methods of preparation of radioactive samples and the use and treatment of nuclear emulsions, a large fraction of the book is devoted to the study of alpha-particle tracks. They are considered carefully with reference to their fundamental properties (for example, range), occurrence in minerals, and biology.

The beta-particle receives much less attention due to its inferior applicability to emulsion technique. Thus betas usually result in spot blackening while alphas produce discernible

tracks, as is admirably illustrated in the fascinating and numerous illustrations of this volume. Later chapters deal with gamma radiation, nuclear physics studies and cosmic rays, the latter being an especially interesting and well-illustrated section.

This book, since it contains so much experimental detail, is not conducive to continuous reading. This, perhaps, lends even more brilliance to the note on p. 231 whereon radioactive tomato slices are being discussed: "The cutting of extra-thin slices of tomatoes and other foodstuffs is a highly perfected art as practiced by restauranteurs. Details of these guild techniques have not been published."

With the many references appended alphabetically, this publication should be an indispensable handbook for workers in this field, and can provide interesting reading for the scientific curiosity seeker if he omits the laboratory details. The keynote of the book, and an admirable thought for the scientist in general, is found, perhaps, in a statement by P. Morrison quoted by Mr. Yagoda. In complimenting the cosmic ray work done at the Univeristy of Bristol, Morrison says, ". . . they use no complicated series of coincidence circuits, no microsecond timing circuits, no magnets. They employ a photographic plate, a microscope and plenty of insight, patience and skill, always the best of apparatus."

ALBERT L. MYERSON

ELEMENTS OF AERODYNAMICS OF SUPERSONIC FLOWS, by Antonio Ferri. 434 pages, illustrations, 16 × 24 cm. New York, Macmillan Co., 1949. Price, $10.00.

This book is the first of its kind in which the basic mathematical conception and development of the theory of supersonic flow are included with their aerodynamic application. Many practical aspects of the flow phenomena are presented. In general, the work covers the theoretical fundamentals of two and three dimensional flow, the application of theory to aeronautical engineering as to instruments, effusors, diffusors and wing design, and the correlation between theory and the physical phenomena.

Some highlights are chapters dealing with interaction and reflection of shock waves and expansion effusors, characteristic system for three dimensional phenomena, pressure drag of supersonic wings, and lift and induced drag of supersonic wings. One error was noted on p. 381 in which an expression is given for the total drag coefficient; it should have read as the "induced" drag coefficient.

There is an appendix listing bibliography, symbols and five tables for assistance in numerical computation.

The book is of special interest since under one cover a good background of supersonic flow theory is presented along with the current conception of application of the theory to the physical phenomena. The book seems to be well edited.

MAX W. BERG

COLLOID CHEMISTRY, by Harry Boyer Weiser. Second edition, 444 pages, illustrations, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1949. Price, $5.50.

Colloid chemistry had its beginning with the discovery of dialysis and its subsequent applications to experiment by Thomas Graham in 1860. Dr. Weiser, in this second edition, attempts to bring up to date much of the work presented in his first book. The general purpose of his book is to outline systematically the entire field of colloid chemistry, from its infancy up to the present time. The author offers the important classical work of the early colloid chemists, and presents many of the most recent theories by experts in the field. A very gratifying feature of this treatise is that Weiser presents and correlates these theories, discussing their weaknesses as well as their strong points without falling into the fault of stating as fact, that which is still only in the conjectural stage. He illustrates many of the principles and theories by showing their scientific and technical applications such as the use of phenomenon of adsorption in the function of the modern gas mask, and the purification by electrophoresis of special clays for use in the manufacture of porcelain.

The book is divided into six parts: namely, (1) Adsorption, (2) Sols, (3) Gels, (4) Emulsions and Foams, (5) Aerosols and Solid Sols, and (6) Application of Colloidal Chemical Principles

to Contact Catalysis and Dyeing. The arrangement of this edition, as the author states in his preface, follows Freundlich's "Kappilarchemie," and this approach to the subject matter seems to be most satisfactory. In an attempt to cover the entire field in one volume there must be a great deal of generalization, but the unusually complete list of references more than amply compensates for this, while at the same time provides a source of more detailed information for those who are interested in some particular phase of the subject.

The completeness of the book is evidenced by noting many of the subjects give a thorough discussion and analysis, which other authors omit or merely mention despite their obvious importance. For example: Verwey's theory of the stability of lyophobic colloids, the application of the ultracentrifuge, the work of Svedberg and Pederson, and a critical evaluation of the theory of the electrical double layer. The one criticism that could be made is that the treatment of macro-molecules is much too brief, but realizing the scope of the subject under discussion and the fact that at times only a perfunctory dissertation is possible, the author left a certain amount of deductive work to the student.

The text is profuse with equations and derivations pertinent to the phenomena under discussion. The author presupposes that the reader has a well established background in advanced mathematics, chemistry, and physics to understand thoroughly the material covered. It is quite obvious that the book is not meant to be an introductory approach to the field, but rather an exhaustive survey of classical and modern research in colloid chemistry, suitable for graduate students or for research workers. It should serve equally well as a reference book and a text book, but in either case any person interested in this branch of chemistry will find in this book a wonderful source of established work as well as the latest advances in colloid science.

DONALD H. RUSSELL

PROBABILITY THEORY FOR STATISTICAL METHODS, by F. N. David. 230 pages, 14 × 22 cm. Cambridge, University Press, 1949. Price, $3.50.

This treatment of probability theory is designed to furnish a theoretical background such as would be useful in the study of statistical methods. While it is quite plain that the justification of procedures used in statistical analysis must be based upon some conception of probability, very often this background is difficult to find or to determine. This book makes a valuable attempt to set down in a small space the ideas necessary to such a justification.

There are three important ways in which this task is undertaken. Firstly, there is to be found, in the opening chapters and from place to place throughout the book, a number of informal but extremely lucid discussions of the fundamentals of probability. Paradoxes and differences of opinion are explained and numerous references are noted for further study.

Secondly, the important theorems of probability, ranging from the binomial theorem to the study of characteristic functions, are set down, explained, and proofs given.

Thirdly, there are a large number of problems worked out in considerable detail. There is, for example, a valuable chapter on genetical applications. However, it should be noted that physical applications are not mentioned. In the main, the author has confined himself to the traditional subject matter of the theory, without venturing into mathematical physics.

The book is noteworthy for the clarity and simplicity of its style. It might serve as an illuminating introduction to the study of the theory of probability.

F. R. INNES

THEORY OF OSCILLATIONS, by A. A. Andronow and C. E. Chaikin. 358 pages, illustrations, 16 × 24 cm. Princeton, Princeton University Press, 1949. Price, $6.00.

In 1937 the authors published, in Russia, the first extensive treatment of non-linear oscillations. This volume, edited by Professor Lefschetz of Princeton University, is a somewhat condensed English language edition of that work. For many years the original publication remained little referred to, primarily because of the language barrier, but between 1944 and 1946 Dr. N. Minorsky, then of the David Taylor Model Basin, introduced the work of a whole

school of Russian mathematicians in his collected reports entitled "Introduction to Non-Linear Mechanics." This work opened new interest in this field.

Engineers often discover the limitations in the applications of the theory of linear harmonic oscillations because most practical problems are inherently those pertaining to non-linear oscillation which can be mathematically described by a set of non-linear differential equations. While the practice of linearizing equations is an expedient not to be overlooked, the existence of large deviations more often than not causes the approximate solutions to be too far removed from actual occurrence. Hence the great interest in the non-linear cases.

The original book, which developed much general theory of non-linear oscillations, has been scaled down in this volume to emphasize the numerous practical examples which appeared in the Russian version.

The basic topics considered are non-linear systems of one degree of freedom and related oscillations; in particular, the important questions of steady state and stability are repeatedly discussed.

The material is introduced through a consideration of linear systems, affording a stepping stone for the newcomer in the field. Phase space and the phase plane as applied to harmonic oscillations are introduced, for several electrical and mechanical physical systems, giving rise to the various general classes of phase space diagrams. The following three chapters are devoted to non-linear conservative and non-conservative systems and to dynamical systems described by a single differential equation of the first order. These four chapters require the least mathematical preparation, but this does not detract from the importance of the material presented. The authors have chosen to explain the methods of non-linear equations through examples met with in common engineering work; witness the following problems considered in the early part of the book: electrical and mechanical systems with negative damping; oscillating circuit with iron core; vacuum tubes with discontinuous characteristics; various kinds of relaxation oscillators; and the theory of the clock.

The advanced theories of non-linear oscillations developed primarily by Poincaré and Liapunoff are presented in the fifth and sixth chapters, which discuss dynamical systems described by two first order differential equations. Again emphasis is placed upon an investigation of the phase portraits for these systems, based upon a study of the characteristic roots for the differential equations. These results are applied in various ways in the remaining three chapters to study: discontinuous oscillations and the effects of parasitic parameters; systems with cylindrical phase surfaces, such as a pendulum with friction and an applied torque and the parallel operation of electric generators; and a quantitative investigation of non-linear systems with emphasis on the methods described by Van der Pol.

Three appendices and a bibliography of the more accessible books and papers treating the subject matter close the book.

The translators and editor have produced an excellent readable book; their efforts should not go unnoticed by the mathematicians, research engineers and scientists working with oscillatory phenomena, who should find this book extremely profitable and pleasant reading.

S. Charp

Indentation Hardness Testing, by Vincent E. Lysaght. 288 pp., drawings, tables, graphs, and photographic illustrations, 16 × 23.5 cm. New York, Reinhold Publishing Corp., 1949. Price, $5.50.

Interest in hardness testing can be traced back to early times when man first became aware of the different properties of the natural materials in his environment. This interest has increased greatly during the past fifty years with our growing knowledge and development of new products and new machines, and the necessity of controlling the properties of the materials utilized within certain, well-defined limits.

This excellent book will enable anyone interested in hardness testing to became familiar with the various types of equipment used and to obtain a clearer understanding of the different methods employed, since the author describes the development of numerous hardness testing machines, showing their advantages, disadvantages, limitations, and uses.

The book includes chapters on hardness concepts; on various types of hardness testers, such as the Scleroscope, Rockwell Vickers, etc.; on tests under varying conditions of temperature and of sample shape and size; on tests for both metallic and non-metallic materials; and on other related topics.

The chapter on hot hardness testing is of particular value at this time to those working with alloys used for high temperature applications, such as turbines, jet propulsion, and similar purposes.

The chapter on microhardness testing (of metals) fills a real need for more information in a field that is gaining increasing interest. Exploratory tests can be made on thin and extremely small areas, including microconstituents at high magnifications. It is possible to obtain hardness values on materials with a wide range of hardness, using appropriate loads for each material.

STUART KINGSBURY

ELECTRIC AND MAGNETIC FIELDS, by Stephen S. Attwood. Third edition, 475 pages, illustrations, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1949. Price, $5.50.

This text was written to facilitate the coordination of underclass work in mathematics, mechanics and physics with the professional work in the last two years of the undergraduate course. The author, a professor of Electrical Engineering, has prepared this text from material presented in a course in electricity and magnetism at the University of Michigan.

The text has four general subdivisions in which are discussed: (1) the electric field; (2) the magnetic field under conditions of constant permeability; (3) the ferromagnetic field; and (4) combined electric and magnetic fields.

The author elects to discuss the electric field first since he believes that studies of this field often receive too little emphasis and, moreover, that a study of this field more readily enables the student to develop the field concepts of potential, field intensity, flux density and others.

The second part of the text is so written as to emphasize current, rather than magnetic poles, as the fundamental source of the magnetic effect. In other respects the treatment of this subject is conventional.

The discussion of ferromagnetic fields is founded on a very practical basis. For example, magnetization curves and a discussion of the properties of a wide variety of ferromagnetic materials are included. Some treatment is also given to subjects involving diamagnetic and paramagnetic materials. Nearly all of the suggested problems are consistent with questions which arise in practice.

The treatment of combined electric and magnetic fields is so written as to enable the student to establish a firm foundation for later advanced studies related to the theory of propagation of electromagnetic waves

Considerable attention has been given to methods of "field mapping," and in the preparation of the illustrations, great care has been taken to insure that the shape of line of flux and equipotential surfaces are correct.

In view of the trend toward the use of the mks. system of units, this system, in rationalized form, is used throughout the text.

D. W. JENSEN

PRINCIPLES OF ELECTRICITY AND ELECTROMAGNETISM, by Gaylord P. Harnwell. Second edition, 670 pages, illustrations, 16 × 23 cm. New York, Murray-Hill Book Co., 1949. Price, $6.00.

In this second edition of Dr. Harnwell's book on basic and fundamental phenomena in electricity and magnetism, the author has primarily added and revised those sections of the original work which bear upon the tremendous advances in electronics, atomic and nuclear physics which occurred in the past decade. Continuing emphasis is placed upon fundamentals and basic physical theories and laws, especially from the point of view of the experimental

physicist rather than that of the electrical engineer, although in many respects such distinction is rather artificial for a book of this kind.

The text is written for the advanced undergraduate student in either physics or electrical engineering who has had previous courses in introductory physics including heat and mechanics, differential and integral calculus, elementary differential equations, and the vector notation. The subject matter, drawn from classical physics and advanced electrical engineering, requires such background preparation in order for the reader to understand those physical phenomena which can best be expressed in terms of mathematical relationships.

The classical order of presentation of the subject matter is utilized for the most part, albeit the classical viewpoint has in many places given way to modern theoretical thought, especially where it was found necessary to integrate the new and recent developments and discoveries with the older concepts and practices.

The first subject considered is electrostatics; the study of charge configurations, their resultant mechanical forces and energy; the elementary vector concepts; and the physical characteristics of dielectrics and conductors.

The electric effects of charges in motion, non-magnetic in nature, are introduced in a chapter on direct current circuits, this being followed by chapters devoted to: non-ohmic circuit elements and alternating currents; chemical, thermal and photoelastic effects; thermionic vacuum tubes; and the conduction of electricity in gases. These phenomena, usually not so fully covered in a general textbook, are treated in an extensive manner, the descriptive illustrations being drawn from current applications of modern circuit theory and developments.

Magnetic phenomena are introduced by considering the electromagnetic effects of moving charges and the electromagnetic effects of charging electric currents. This is followed by a discussion of the magnetic properties of matter, a subject which usually precedes the former material. The remainder of the book is devoted to subjects of prime interest to the electrical engineer and of considerable interest to anyone engaged in physical research and communications. These include: electromagnetic machinery; linear constant parameter filters, and transmission lines; vacuum tube circuits; and electromagnetic radiation. These latter chapters place some emphasis on ultra-high frequency techniques and phenomena, a natural extension from the earlier edition. The material on radiation is based, of course, upon appropriate solutions of Maxwell's equations for dielectric and conducting media, illustrated by their application to wave guides and antennae.

Throughout the copiously illustrated book appear numerous references to the current literature. The problems found at the end of each chapter serve to test the student's understanding of the basic principles of the subject matter; in many cases they have been chosen to illustrate applications of a very practical industrial or research nature. In addition to its classroom application, the book is recommended as a basic reference text for practicing electrical engineers and physicists who desire a thorough review of this subject matter from the physicist's point of view.

S. CHARP

BOOK NOTES

STEEL AND ITS HEAT TREATMENT. VOLUME III. ENGINEERING AND SPECIAL-PURPOSE STEELS, by D. K. Bullens and The Metallurgical Staff of the Battelle Memorial Institute. Fifth edition, 606 pages, illustrations, diagrams and tables, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1949. Price, $7.50.

This present third volume of Bullens covers the subject of Engineering and Special-Purpose Steels, which was treated in Volume 2 of the fourth edition. The considerable increase of knowledge of the subject in the last decade has necessitated a quite thorough rewriting and enlargement of the text. Particular attention has been paid to the importance of hardenability data and the realization that composition in an alloy constructional steel is of importance only as it affects structure. The possibilities of using alternate steels are indicated.

The main sections are: Engineering alloy steels; constructional alloy steels for heat treating; special steels. As a correlated presentation of the known facts about the important alloy and special steels, this volume continues to serve a useful function.

ELECTRON TUBES. VOLUMES I AND II, 1935 TO 1948 (The Ninth and Tenth Volumes of the *RCA Technical Book Series*), edited by A. N. Goldsmith and others. 475, 454 pages, illustrations, 15 × 22 cm. Princeton, *RCA Review*, 1949. Price, $2.50 per volume.

The present two volumes are the ninth and tenth in the RCA Technical Book Series which reprint significant contributions by RCA authors in various fields of electronics. These are the first which have been devoted exclusively to tubes, Volume I covering the period 1935–1941, while Volume II brings the record through 1948.

Following the pattern set in earlier volumes, some papers are presented in full, while others are given only in a summary. Each volume is divided into four sections: general; transmitting; receiving; and special. In addition to the articles reprinted, a full bibliography of all papers on tubes by RCA authors has been included, which, for the sake of completeness, notes those articles on the applications of tubes which are considered in other volumes of the series.

The volumes will serve as a useful compilation of the many contributions made to electron tube research by RCA engineers.

ANNALES DES TRAVAUX PUBLICS DE BELGIQUE. Numero jubilaire 1843–1948. 213 pages, illustrations, 24 × 31 cm. Brussels, G.I.G., 1949. No price.

In recognition of its hundredth year of publication, the *Annales des Travaux Publics de Belgique*, established in 1843, has just issued a special jubilee number. Because of its function as the organ of the Belgian Ministry of Public Works, the *Annales* offers in this number several historical articles on the various types of public works executed in Belgium since 1830, as well as other subjects in the field of civil engineering. After an introductory section outlining the origin and purpose of the publication, the first section is devoted to roads, with three articles. Of these, one is devoted to planting along the highways, discussing briefly the various problems encountered and the need for planning. Canals are treated at some length and the accompanying maps depict graphically the great development since 1914. Buildings and bridges are also treated in separate chapters.

Louis Baes has written a summary article on the resistance of materials, the theory of elasticity and of the plasticity of solid bodies. Although this has been treated in limited scope, it makes ample reference to the literature and should prove useful. Other articles consider such varied topics as research laboratories, developments in metal construction, railroad bridges, local rapid transit, metallurgy and water works.

With many illustrations this is a worthy issue to note the hundredth anniversary of the *Annales*, and it will prove valuable as a reference source on the history of engineering in Belgium.

TABLES OF BESSEL FUNCTIONS OF FRACTIONAL ORDER, prepared by the Computation Laboratory of the National Applied Mathematics Laboratories, National Bureau of Standards. Volume II. 365 pages, tables, 20 × 27 cm. New York, Columbia University Press, 1949. Price, $10.00.

The JOURNAL for December 1948 carried a note on the appearance of Volume I of these tables. The present volume comprises tables for Bessel functions of fractional order: $I_\gamma(x)$ for $\nu = \pm \frac{1}{4}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{3}{4}$. The value and usefulness of tables such as these needs no further comment.

BIBLIOGRAPHY ON SPRAYS, by Kalman J. DeJuhasz. 98 pages, 21 × 28 cm. New York, Texas Company, Refining Department, Technical and Research Division, 1948. No price.

The recent development in gas turbines and jet engines has encouraged the author to bring up-to-date an earlier bibliography on liquid jets, sprays and nozzles, published in 1932.

In the present listing, in addition to the oil engine applications of the earlier list, other industrial applications of sprays have been noted, as well as such borderline subjects as dusts, powders.

The bibliography itself was prepared in connection with two research projects, one for the U. S. Navy Department, Bureau of Ordnance, and the other for The Texas Company.

The entries are arranged alphabetically by author, and generally full bibliographical data are given, with the titles of foreign entries both in the original language and translated into English. Abstracts are included. About ten per cent of the entries would seem to be cited without having been seen. It is to be hoped that in undertaking a revision, more of these will be examined, since a considerable part of those cited would seem to be relatively available. A subject index is included.

The method of reproduction unfortunately leaves something to be desired, since some entries run off the bottom of the page. Despite these deficiencies, the bibliography presents evidence of a quite extensive coverage of the literature which will prove useful and the author should be encouraged in his plans to keep it up-to-date.

TIN, ITS MINING, PRODUCTION, TECHNOLOGY AND APPLICATIONS, by C. L. Mantell. Second edition, 573 pages, illustrations, tables, maps, 15 × 23 cm. New York, Reinhold Publishing Corp., 1949. Price, $10.00.

In revising this work first published twenty years ago, the author has amplified the text by nearly a half. The same general arrangement has been followed, with the addition of a chapter on electrolytic tin plate and the division of the chapter on alloys into three chapters. In these the greater knowledge available on the different tin alloy systems has been reviewed.

German practice has been more extensively covered on the basis of a German translation of the first edition and the information gathered by the technical teams who studied German metallurgy after World War II. In its expanded form, "Tin" remains an outstanding work in its field.

INSTALLATION AND SERVICING OF LOW POWER PUBLIC ADDRESS SYSTEMS, by John F. Rider. 204 pages, illustrations, 14 × 19 cm. New York, John F. Rider Publisher, Inc., 1949. Price, $1.89.

A practical book on low power public address systems, limited primarily to those up to and including 50 watts. Microphones and phonograph pickups, amplifiers, and loudspeakers are treated at some length in separate chapters. Suggestions are given for the installation of public address systems under varying conditions and a concluding chapter discusses the problem of servicing.

CURRENT TOPICS

The Spectra Direct Reading Color Temperature Meter.—The SPECTRA Direct Reading Color Temperature Meter made by Photo Research Corporation, San Fernando, California, is the first instrument offered to the general public for measuring the exact color of a source of illumination. It operates photo-electrically and is therefore independent of individual eyesight and personal judgment.

Everyone is familiar with the fact that the appearance of objects depends greatly upon the illumination which falls on them. Even an unobservant person cannot fail to note the difference between objects seen under sunlight and candlelight, and every woman knows that colors which match under tungsten light do not match in daylight.

These, however, are large differences of which almost everyone is aware. In recent years, the growing importance of color photography, and the increasing need of precise color matching in many fields, has made it necessary to deal with much smaller differences—differences which the human eye is totally unable to perceive because of its ability to adapt itself to them, yet which can seriously impair the correctness of color matching or of a color photograph.

Practically no light source in common use furnishes light of constant, known color. Sunlight varies in color with the time of day, the season, the weather, the altitude, and many other factors. Tungsten light is somewhat more constant, but serious variations occur which depend on the wattage of the lamp, its age, the voltage at the lamp (which varies widely in most communities) and other influences.

For a decade or more there has been a need for an instrument which would measure the color of a source of illumination in a precise way, so that suitable corrective measures might be applied. That need has finally been met in the SPECTRA. The SPECTRA is simple in principle, and no special knowledge or skill is required for its operation.

In order to measure the color of illumination, we must have some sort of units in which to measure it. Fortunately, such a scale of measurement already exists in the laboratory. It is known as Color Temperature. It is not necessary to understand Color Temperature in order to use the SPECTRA, but for those who are interested in understanding how it operates, here is a brief explanation:

The Color Temperature of a light source is the Kelvin temperature to which a black body would have to be heated in order to give off light of the same color. The Kelvin temperature is measured in degrees Centigrade above Absolute Zero.

When we know the Color Temperature of a light source, we can say exactly how much red, yellow, green, blue and violet is contained in the illumination. Similarly, if we know how much of these colors is in the light, we can compute the Color Temperature.

Fortunately, it is not necessary to follow such a complicated procedure. It is sufficient to measure the amount of any two colors in the light source, and the ratio between those two colors will correspond to one Color Temperature, and one only. This is what is done in the SPECTRA.

First, the SPECTRA is pointed at the light source which we wish to measure. It is now measuring the red content of the light. We adjust a ring so as to bring the needle on the scale to a red mark. This adjusts our red reading to a standard value. Then we pull the trigger on the SPECTRA. This moves the red filter out of the way and swings a blue filter in front of the light sensitive cell. With the trigger held back, the needle now tells us on the scale the exact Color Temperature of the light source. That is all there is to it.

If we are taking a color photograph on Daylight Type film, the light should be 5900 K. For Type A or Photoflood film, it should be 3400 K., and for Type B or Tungsten, 3200 K. If the SPECTRA shows a departure from the correct figure, corrective measures must be taken, such as the use of filters, as explained in the directions furnished with the meter.

An important feature of the SPECTRA is the Spectrasphere, a diffusing hemisphere which is placed over the instrument when light of mixed character is to be measured. Out of doors, for example, direct sunlight and light coming from the sky are not of the same color. Indoors, several lamps may be in use which are not of exactly the same color. The Spectrasphere mixes and averages up the various light sources, and provides a figure which is the best guide to the most favorable results.

Farm Alcohol Makes Low-Octane Motor Fuel Equal to Regular Gasoline.—On two recent trips of nearly 1000 miles each, a U. S. Department of Agriculture test truck ran successfully on relatively inexpensive low-octane gasoline fortified by periodic injections of alcohol and water. Results of these long-distance road trials, according to the Department's Bureau of Agricultural and Industrial Chemistry, show how using alcohol in motor vehicles can help conserve petroleum and provide a potentially vast new market for surplus grains. The tests were made as part of the motor-fuels work financed under the Research and Marketing Act.

Bureau researchers report that in the first trial the truck operated as well on 58-octane gasoline plus alcohol-water injection as it would on regular gasoline, which has an octane rating of 74 or better. The alcohol-water mixture (85 per cent alcohol, 15 per cent water) was injected automatically into the truck's carburetor when the engine was under heavy load, as in climbing hills, accelerating in traffic, or passing other vehicles. This enabled the engine to give knock-free performance even with the low-octane gasoline.

For the second trial run the engine of the truck was equipped with a high compression head (8.25-to-1 compression ratio). This made it necessary to supply the engine with fuel having an effective octane rating of at least 85. On this trip 74-octane gasoline plus alcohol-water injection was used satisfactorily.

Because of the tremendous consumption of motor fuels—more than 80 million gallons a day—general adoption of alcohol-water injection could open up a wide new outlet for surplus grains and other agricultural commodities. Alcohol can be made from a variety of these products. The usual method is

by fermentation of wheat, corn, or other starchy crops, and scientists at the Peoria laboratory are working to lower the cost of alcohol and to increase the value of feed byproducts produced by this process. They are also developing methods for converting corncobs and other farm residues to alcohol.

The Laboratory's researchers also point out the potential importance of alcohol-water injection in extending the country's supplies of petroleum. Alcohol is equivalent to 100-octane fuel, and it can be used to increase the effective octane rating—or anti-knock quality—of gasoline. It thus makes possible wider use of low-octane fuels, which are cheaper and can be produced from petroleum in greater quantity than high-octane gasoline.

New ACF-Talgo Train (*Diesel Power*, Vol. XXVII, No. 6).—A unique train, first to be built in America based on the Spanish "Patentes Talgo," has been completed by the American Car and Foundry Company for testing and demonstration. Designed by ACF and Talgo engineers and constructed at the Berwick, Pa., and Wilmington, Del., shops, this revolutionary streamliner is one of three such trains, the other two being destined for revenue service in Spain.

The "ACF-Talgo" train represents entirely different concepts in railroad passenger car construction. A comparison with streamline equipment on the railroads today shows the ACF-Talgo to have floors 2 ft. 9 in. lower, a weight reduction of nearly 75 per cent due to both its design and all-aluminum construction and an overall height lower by almost four feet. Completely new throughout, the train presents a very dramatic appearance due to its extremely low center of gravity. Moreover, it is equipped and decorated in keeping with its modern design. The name "Talgo" is derived from Tren (train) Articulado (jointed) Ligero (light) Goicoechea (the inventor) and Oriol (the man backing the train).

Each Talgo for Spain will consist of a diesel-electric locomotive, a baggage unit, and three coaches, the last of which features an observation lounge. The over-all length of the train is approximately 370 ft. and is for 5 ft., 6 in. gauge track. The design is unique in that a coach consists of four articulated passenger units and one equipment unit, each of which has only one pair of wheels in the rear. The front is supported by a special coupling arrangement on the unit ahead of it, while the first unit is supported by the locomotive.

Entrance to the coaches is gained through the equipment unit which is in the center of each coach. These equipment units contain various functional items such as air-conditioning equipment, control lockers, washrooms, and kitchenettes for serving light meals.

Each coach is 100 ft. long and seats 64 passengers, 16 in each passenger unit. The coach containing the observation lounge is 7 ft. longer. The entire train seats 176 passengers with 16 additional seats in the observation lounge.

An experimental train of this type has undergone successful tests in Spain. ACF, realizing that the ideas embodied in the Spanish train presented a challenge to American industry, entered into a partnership agreement with "Patentes Talgo" which included exclusive manufacturing rights and sales representation in the United States and Canada.

The train for experimental purposes is of identical design to those destined for Spain with the exception of length and gauge, the latter being generally 4 ft.

8½ in. in the Western hemisphere. It consists at this time of only a diesel-electric locomotive, a baggage unit and a coach of five units, one of them being for equipment and another serving as an observation lounge. The train is 168 ft. long or only approximately one-third of its eventual length.

This unit has been planned to show just one of the many possibilities of interior design. Wide-vision windows, a 42-in. seat spacing, wardrobes and luggage storage facilities in each coach, individual ash trays at each seat, kitchenettes to provide meal service at the seats and an observation end from which passengers have extra-wide visibility are some of the ideas which have been incorporated. It can be adapted to both long and short hauls, coach or sleeper service, depending on the requirements of individual railroads and its passenger carrying capacity increased or decreased to suit the density of traffic.

Obviously, the Talgo is a complete break with tradition in the construction of railway equipment. It will be interesting to watch its progress. Many advantages are offered to the railroad industry that heretofore have never been available.

Engineered for rapid and economical production and incorporating such features as the use of standard automotive type diesel engines for power, elimination of approximately 75 per cent of the dead weight and facility of maintenance and servicing, these units should make an attractive, economical package.

Rocket-Jet Airplane (*Mechanical Engineering*, Vol. 71, No. 7).—What is believed to be one of the first flights ever attempted under dual rocket and jet power was made recently by the D-558-2 Skyrocket, the Navy Department's Bureau of Aeronautics revealed. The research craft took off from the desert runways of Muroc, Calif., test base.

After exhausting its rocket fuel, the Navy test ship continued upon a routine research flight and landed under turbojet power.

For the past year the Douglas-built explorer has made numerous routine research flights powered by its turbojet while awaiting final development of the jet-rocket engine recently installed.

Designed and developed for the Navy by Douglas Aircraft Company, Inc., in cooperation with the National Advisory Committee for Aeronautics, the Skyrocket is obtaining valuable knowledge on advanced power plants and speeds near the sonic range of flight.

The D-558-2 Skyrocket is the advanced sequel to the Navy's D-558 Skystreak, straight-winged red jet which in the course of its research investigations broke the world speed record twice within five days in August, 1947.

Though of advanced aerodynamic design and incorporating the latest power plants, the Skyrocket performs as a conventional aircraft taking off and landing under its own power.

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DEVELOPMENT, DESIGN AND CONSTRUCTION OF ELECTRICAL CONDENSERS *

BY

WILLIAM DUBILIER [1]

Any electrical apparatus, however complex, is composed, essentially, of one or more devices employing *inductance*, *resistance*, or *capacity*. They are the building blocks of electrical engineering.

The use of inductances permitted the development of transformers, electromagnets, electric motors, generators and similar appliances.

The use of resistors made possible the development of electric lights, electric heaters, controls, and the like.

But modern electrical engineering would have been impossible without the use of condensers—millions of condensers, of many types and sizes—some of them not much larger than a match head, others as big as a room.

THE COMING OF THE MODERN CONDENSER

The condenser is an old invention but a recent development.

The first electrical condenser, the Leyden jar, was invented in 1746 by Deen Van Kleist. It was referred to by Benjamin Franklin as an "accumulator" of electrical energy. Later, it was used extensively in medical and wireless telegraph equipment. The Leyden jar remained practically unchanged for more than 200 years, retaining essentially the same shape, design and construction.

Forty years ago, in 1910, we already had a sizable electrical industry—all of it based on electrical devices employing primarily only inductance and resistance. The only practical power condenser then available was the glass Leyden jar, or its equivalent, the glass plates.

* Based on a paper presented before the Seminar of Electrical Engineering, Princeton University, Princeton, N. J., February 28, 1949.

[1] Founder and Technical Director, Cornell-Dubilier Electric Co., New York, N. Y.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

The telephone and telegraph industry, operating with only minute energies, had a primitive low-voltage, low-power paper condenser. For higher voltages and higher powers we used glass plates, or the equally inefficient, cumbersome, fragile Leyden jars.

The standard, fragile glass jar was then used in the many thousands of high-tension electro-medical machines, in thousands of wireless telegraph installations. Radio communications with ships at sea, and between the ships of all the navies of the world, all depended on the Leyden jars.

Here is an interesting footnote to history. Up to the end of 1911, practically all of the Leyden jars were made in Germany. All commercial and military radio installations used German-made Leyden jars. The German government subsidized the industry and discouraged foreign development.

The British Navy realized that its entire communications system was dependent on a foreign power. When I was visiting England in 1911, the government invited me to assist them in finding a substitute for the Leyden jar. Thus the modern power capacitor was born.

THE ROCKY ROAD OF PROGRESS

We began with crude tools. We were lone workers, shaping the individual bricks of knowledge. Today, the edifice of knowledge is a towering vastness: new bricks are being added daily, raising it ever higher—and I doubt that a single human mind could encompass all the knowledge it contains. We did not know nearly as much about electrons and atomic structure as we do today; wave mechanics were nearly two decades in the future.

In the beginning, our progress was slow. We gained new knowledge experimentally, by trial and error. This gave us experience and developed our imagination. Experience and imagination make an ideal marriage.

The rocky road of progress is hard. There are many obstacles to be overcome, and the inertia of public opinion is not the least of these. If radio broadcasting had been developed before the telephone, and then someone had invented a method of making broadcasting secret, by simply guiding the waves along a thin metal wire, he would have received public acclaim.

A little more than a century ago, when trains were first developed, a speed of twelve miles an hour was considered the limit of human endurance. The medical profession, newspapers and others, were quick to point out their terrible menace and danger to humanity. They predicted that a speed of fifteen miles an hour would cause nosebleeds, deafness and even death.

I recall that in my own boyhood, automobiles were prohibited in many sections: they frightened horses, and a speed of twenty-five

miles an hour—"without tracks," as an old ordinance put it—was "dangerous." There were local laws to keep automobiles off the roads; and there was broken glass to cut our tires, and stretched wire to cut our necks. The man who first drove an automobile through Central Park in New York was arrested for disorderly conduct.

Long ago, when I was a lad, a bearded German professor named Roentgen discovered some mysterious X-rays which made it possible to photograph bones in the living body. The now-defunct *Pall Mall Gazette*, of London, was horrified. It protested against what it called

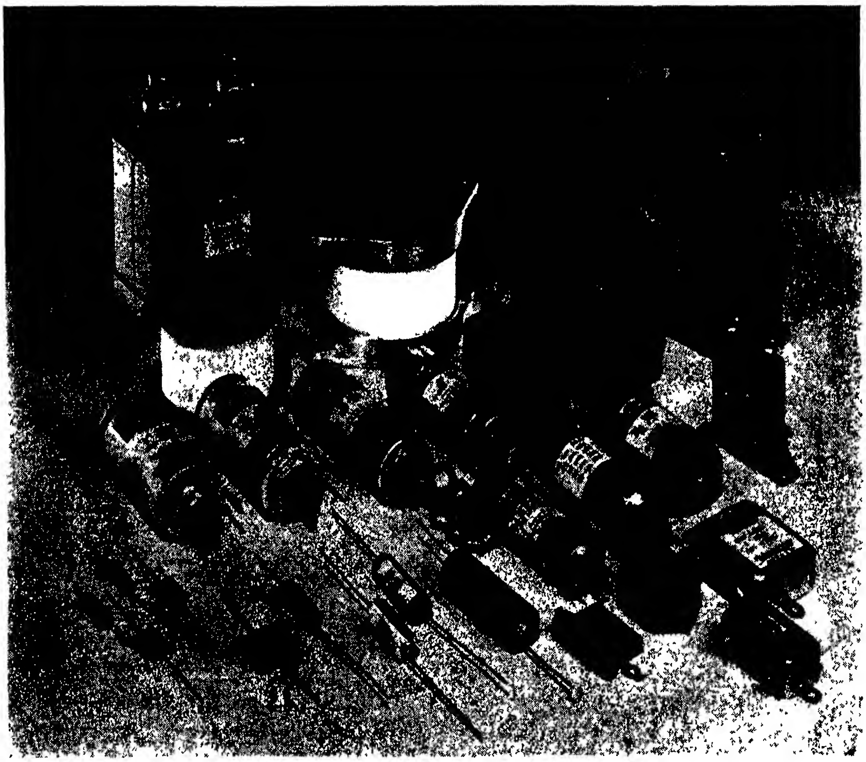

FIG. 1. Representative group of small electrical condensers showing variety of shapes and sizes.

the "revolting indecency of making pictures of our insides," and hoped something would be done to "thwart the shameless experimenters who were beginning to supply the discovery." This newspaper stated that it would be possible to see through the clothes of people as they walked in the streets.

We, too, had similar obstacles to overcome.

The condenser appears to be the simplest of all electrical devices—a pair of conducting sheets separated by insulating layers. Actually, the condenser is probably the most complex of the electrical elements.

It has more hidden problems than any other electrical device, and is more difficult to produce.

Today, we take our condensers for granted, and expect them to do their job with precision. We can do so only because much ingenuity and many inventions resulting from patient research went into their design and construction.

The eventual adoption of the new compact and efficient power capacitors permitted the development of many branches of the electrical industry, with its high-frequency equipment and vacuum tube applications. Powerful broadcasting and communications stations, and other high-frequency equipment, would have been impractical because of the prohibitive size, cost and inefficiency of the glass jar.

THE MICA CAPACITOR

The first development that broke the German Leyden-jar monopoly was our *mica condenser.*

Up to 1910, all technical publications and text-books were in agreement that mica was, theoretically, a more efficient and suitable material than glass. Yet there was no practical mica condenser, one that could withstand for long periods of time high voltages or high power without deterioration and breakdown. The phenomena of corona, hysteresis, eddy currents, ionization, and above all mechanical losses in capacitors, were either misinterpreted or unknown.

In those days whenever higher voltages were involved, it was the customary practice to make the insulating material thicker and larger.

I found that corona, or brush discharge, was particularly destructive, and that it invariably started at potentials of around 1000 volts. To overcome this, I did not "make it thicker and larger." We made and tested hundreds of condensers, employing materials of various thicknesses, applying to them different potentials. These condensers were then carefully dissected and examined. Tests showed that, at five times the thickness of the insulating material, the corona began at *less than two times* the previous corona potential. I therefore concluded that the voltage across any two adjacent electrodes must always be less than 1000, irrespective of the total voltage applied to a condenser. This was the origin of the corona-free condenser.

After the corona was eliminated, some condensers still became hot and broke down. We found that in some cases certain spots in the armatures became discolored because of concentrated heat.

I recalled my school days and the teacher's demonstration of the "talking book" telephone. The "talking book" was a loosely arranged condenser made by placing light metal foils between the pages of a book. When connected to a microphone and a battery, the book "talked." I connected the book across a 60-cycle supply, and it gave

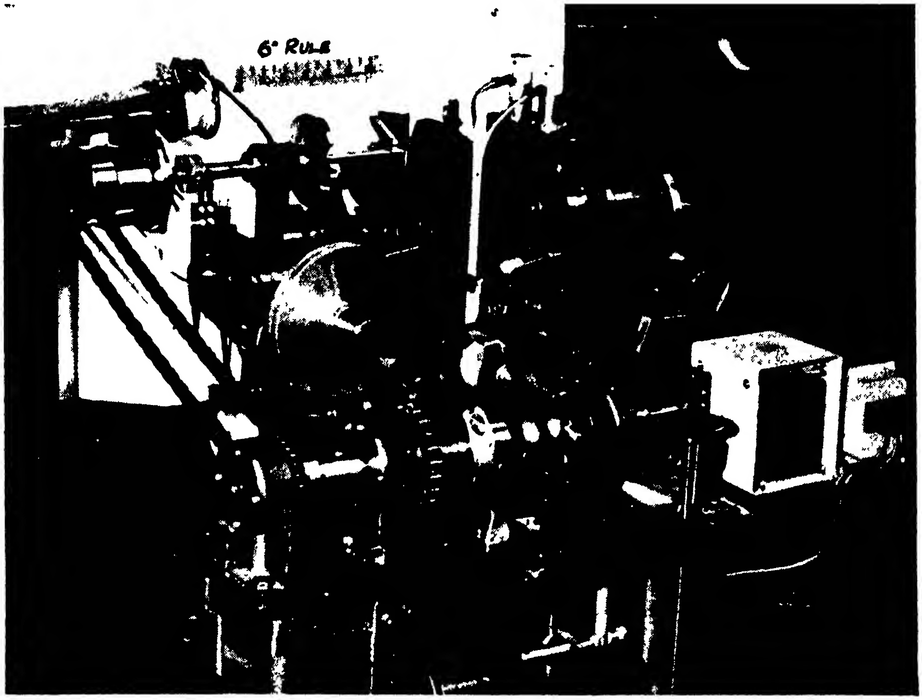

FIG. 2. An automatic machine for making mica condensers.

FIG. 3. Finished condensers being ejected from the machine shown in Fig. 2.

a tremendous hum. I recalled that I tried to stop it by pressing the book with my hand, and soon found that the book became hot.

It was this school lesson that helped me to eliminate the hot spots. I reasoned that, whatever the frequency, be it a hundred thousand or half a million cycles instead of sixty or a few hundred, a minute move-

FIG. 4. The author, with a large mica condenser rated at 2250 KVA. for operation at 15,000 volts, 150 amperes.

ment of the electrodes resulted in heavy power losses. Hence, it was necessary to obtain an intimate contact between the foil electrodes and the mica sheets.

I found that the surface of mica sheets, although it appears perfectly smooth to the eye, is in reality full of microscopic hills and dales. Therefore, a special type of soft metal foil was developed, one that would flow and imbed itself into the fine crevices of the mica sheets, ensuring an intimate contact throughout the condenser. This was the second major improvement which made the mica condenser practical.

With heavy currents, such as are encountered in high-frequency equipment, X^2R loss became a serious factor. To reduce this loss to a minimum, changes were made in the shapes, sizes, and assembly. The units were evacuated and impregnated with certain compounds at various temperatures. We were able to reduce the size of the mica condenser to less than 10 per cent of the equivalent Leyden jar, and the losses by more than 90 per cent.

Ours being a new development, we were subjected to most rigid specifications. Although the size (and therefore the radiating surface) was considerably reduced, and consequently heat generation due to losses was also reduced, instead of allowing us a temperature rise equivalent to that allowed in Leyden jars, we were limited to 10° C., in places where the equivalent Leyden jar became so hot that many times the glass melted, and cooling fans were therefore necessary.

Not only did we reduce the overall dimensions of the capacitor, but we also eliminated a great deal of such associate equipment. The importance of this can be realized when we recall that in a radio installation on board a battleship, Leyden jars usually occupied more than 50 per cent of the equipment space.

For a while, the government distrusted our new and revolutionary condenser, as lives and ships depended on its continued safe operation. To meet the exacting government specifications, we produced units where the heat radiation was equal to the heat generation, and thermal stability was reached within less than 10° C.

Today, it is a source of some satisfaction to us that every radio broadcasting station, every radio transmitting station on land, at sea or in the air, and other high-frequency electrical equipment use mica condensers as originated by us forty years ago.

PAPER CONDENSERS

Paper condensers employ impregnated paper as the dielectric. Paper condensers look quite simple, but their appearance is deceiving. They are extremely difficult to engineer and produce.

Consider the paper that serves as the dielectric. Many of our capacitors contain enough paper to cover the walls of a room. A

single microscopic defect, a mere pinpoint, will cause the entire condenser to fail. In a modern condenser factory, the materials controls which apply to the paper begin with the manufacturer of the paper itself. He must carefully control the chemical content of the water used for washing the pulp from which the paper will be made. He must control, within close limits, the ash content, the acidity, the alkalinity, the porosity, and the residual moisture in the finished product. Minute metal particles from paper-making machines find their way into the paper, impairing its dielectric efficiency. Paper thickness is exceedingly important, as the cost and the bulk of condensers increase as the square of the thickness.

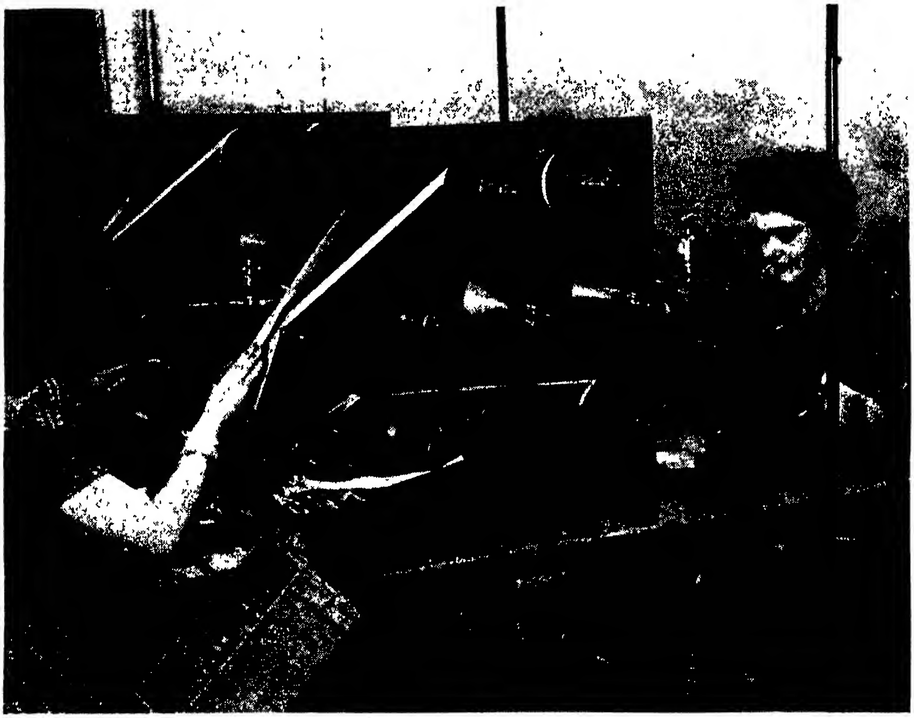

FIG. 5. Winding paper condensers.

The design of metal electrodes, and the composition of the metal foil, are equally important. Under certain operating conditions, metal particles detach themselves from the foil and penetrate the paper, resulting in disintegration and breakdown of the condenser. Aluminum foil has been found to minimize such ionic effects.

Paper condensers are impregnated with various insulating materials, such as waxes, oils, chlorinated diphenyl preparations, and others. These must be constantly tested before using, and must be free from contamination. Blending some of the impregnants results in improve-

ments; other blends produce poor results. Condensers are impregnated under vacuum, and the production cycle sometimes takes more than 150 hr. The vacuum is as low as 100 microns, and the temperatures are about 250° F.

The manufacture of paper condensers is a controlled precision operation from start to finish. Materials must be kept free from contamination. We must even guard ourselves against the effect of perspiration of the assembly workers.

The fibrous paper and the impregnants alike have a great affinity for moisture. Unless thoroughly sealed, they can absorb a great deal of moisture from the atmosphere. On an ordinary humid day, in a few hours the units may become unfit for use. To guard against moisture absorption, paper condensers are assembled and impregnated under carefully controlled conditions, and then hermetically sealed. The seal itself must withstand the internal expansion and contraction over a wide temperature range.

There are many other, unseen dangers. A slightly defective condenser may operate satisfactorily for a length of time, but under power strains, internal heat is generated due to the various losses. Unless proper precautions are taken, fatty acids may be formed inside the unit. In many instances, inhibitors must be added to the impregnants.

The completed unit is tested at approximately twice its normal operating voltage, and its terminals at five times. It is then passed through long heated ovens for detection of minute flaws. Finally, the unit is given an accelerated heat test under power overload.

These are only some of the problems in paper condenser manufacture. Many other, unforeseen problems arise, which must be solved.

Here is an interesting puzzle that gave us many a sleepless night. We made a large number of paper condensers, rated at 600 volts, for a new high-voltage anode supply source operating over long continuous periods. Before shipment, our condensers were all tested at 6000 volts. The breakdown tests showed more than 10,000 volts. The condensers were impregnated with a refined paraffin wax, as we had practiced successfully for many years, and under normal conditions would have lasted almost indefinitely.

A few months later, many of these condensers failed in service and were returned to us. We were mystified. Investigation showed that dark streaks and spots had developed on the paper inside the condensers. It took much time and work to analyze and solve this riddle.

We discovered that when the condenser units were removed from the impregnating tanks, the outside of the units cooled rapidly and formed a hard solid shell. But the center retained its heat for a much longer period, keeping the wax soft and fluid. As the center finally cooled and contracted, minute spaces were formed *internally*, sealed by the solid outer crust. Gases filled these spaces, and the gases glowed

at potentials of 200 volts or less. Our condensers, in effect, contained miniature glow lamps.

When subjected to long continued periods of operation, enough heat was generated to destroy the insulation, causing breakdowns. In addition, the uneven contraction of the wax also imposed severe mechanical strains and distortion along the insulating layers. Thus, condensers made to operate reliably at thousands of volts may break down at a potential of a few hundred volts, unless precautions are taken to prevent ionic discharges within the unit.

This was a serious problem, and one that we had to overcome at once. Finally, one of my assistants suggested cooling the condensers under oil. This allowed the entire impregnated mass to cool without the hard shell.

This cooling process increased the life of the condenser tenfold, and its rated safe operating voltage from three to five times. This simple manufacturing improvement proved to be one of the most important and revolutionary developments in wax-impregnated condensers, and now is universally used in the manufacture of high-voltage capacitors.

ELECTROLYTIC CONDENSERS

Up to 1925, one of the basic precautions taught to every condenser worker was to be always on guard against any risk of contamination of condenser materials by outside chemicals—alkalis or acids. We knew, from our own sad experiences, that a single drop of water or acid in a gallon of impregnating oil or wax would make the entire batch of material unfit for use.

We were so thoroughly trained in strict precautions against chemical contamination of our condenser materials, that the very idea of making a *chemical* condenser seemed at first frightening. Although we knew that metallic oxides were good insulators, we also knew that strong acids are needed for the formation of such oxides on aluminum foil—and strong acids could never be tolerated in or near a condenser plant.

These were the mental roadblocks that the new electrolytic condenser had to overcome. Yes, we were slow and cautious in accepting the electrolytic condenser. But today, it has found its rightful place in industry, especially for starting small a-c. motors, and in filtering circuits.

Although the electrolytic condenser seems to the eye to be much more complex, the problems of its manufacture are nowhere near as great as those of mica and paper condensers.

CERAMIC CONDENSERS

During the First World War, the Germans purchased, through Holland, a small number of our transmitting condensers, and copied them, Japanese fashion. Those were shipped by us in 1916, and I knew that they were destined for Germany. After the War, in 1920,

I visited Berlin. The Telefunken Company, which was then the largest radio organization in the world, showed me thousands of units made exactly like ours. Most of them failed after a few weeks' service. I knew they would fail—another proof that experience is necessary for success.

Ceramic condensers originated in England, about a quarter of a century ago. However, they received their highest development in Germany, after the First World War—again with the aid of financial subsidies from the German government.

At the end of the First World War, the new mica condensers were used extensively, in transmitters and other electrical equipment. Germany had no mica supply of her own. The best mica came mostly from England and India. Therefore, Germany heavily subsidized and encouraged the development of ceramics to replace mica.

The resultant ceramics were much stronger and had lower losses than the old wet-process porcelains. Units were made of many materials, such as magnesium, and later the titanates. Their production required higher firing temperatures and closer manufacturing tolerances and controls.

The development of low-loss ceramic insulators, particularly those with high-capacitance possibilities, soon affected the electric power industry.

Ceramic condensers were gradually improved until the overall efficiency of the best German-made condensers during the last War was about equal to mica condensers of similar dimensions.

Since then, American manufacturers concentrated on the development of titanates, mostly for use in low-voltage applications. Titanate compounds have the advantage of possessing a large capacity, and occupy but a small space.

We have made some ceramic condensers with a K as high as 500,000. The losses, however, were more than 30 per cent. (The K of mica is about 7; that of paper, around 4; of paraffin, around 2.)

Improvement in ceramic condensers has been rapid. At present, large quantities of ceramic condensers are being made, with a K of between 3000 and 5000, with losses as low as 2 per cent. Recent developments in low-loss ceramics show most encouraging results.

WHAT OF THE FUTURE?

The future progress in electricity and radio is inseparable from progress in condenser design.

Inductances and resistances have never presented serious design or manufacturing problems: they are not likely to present them in the future. But the third of the building blocks of electrical engineering—the condenser—remains a limiting factor in applied electrical and radio engineering.

Forty years ago, our horizons limited by the old Leyden jar, we could not foresee the variety, the complexity, the utility of modern electrical apparatus. New and better condensers were a major factor in making the modern electrical and radio industry possible. Today, forty years later, our eyes can see farther, and our vision is clearer. With our mind's eye we can see the magnificent vistas which the development of newer and better condensers may bring about comparatively few years from now.

There always is something newer and better waiting to be developed. Forty years ago, the new compact mica capacitor replaced the Leyden jar. Today, its supremacy is already threatened by the newer types, such as for instance the ceramic capacitor, and the yet-newer vacuum-tube type. And yet, such is the excellence of the modern mica condenser that it will never be eliminated or supplanted by others; rather, the newer capacitors will open new fields, will make possible new apparatus as yet in the dream stage.

It is a thankless task to prophesy; but, given capacitors smaller and more efficient than the best we have today, such obvious new devices as vest pocket telephone transmitters, capable of reaching a relay station a few miles away, would be a practical possibility. Those same condensers would also permit us not merely to bounce a few radar pulses against the surface of the moon, as was done some two years ago, but to maintain communications with the space ships which will inevitably venture forth in a few years, ranging farther afield than the moon, into the solar system, perhaps beyond.

ENERGY FLUCTUATIONS IN A VAN DER POL OSCILLATOR

BY

N. MINORSKY, Ph.D.[1]

1. INTRODUCTION

By analogy with the differential equation of the harmonic oscillator

$$\ddot{x} + x = 0 \tag{1.1}$$

one can consider the equation

$$\ddot{x} - \epsilon(1 - x^2)\dot{x} + x = 0 \tag{1.2}$$

as representing a special, *van der Pol oscillator* (abbreviation VDP). The important feature of the harmonic oscillator is the fact that (1.1) has the first integral

$$\tfrac{1}{2}\dot{x}^2 + \tfrac{1}{2}x^2 = h, \tag{1.3}$$

which expresses the law of conservation of energy (we assume here for the sake of simplicity $m = c = \omega_0 = 1$ in the usual form of the equation $m\ddot{x} + cx = 0$ of the harmonic oscillator).

Although the VDP oscillator does not possess this feature, the introduction of *energy* as a dependent variable is of a certain interest in applications and this will form the subject of this note.

If one uses the phase-plane representation ($\dot{x} = y$), (1.3) is

$$x^2 + y^2 = r^2 = \rho = 2h. \tag{1.4}$$

That is, $r^2 = \rho$ represents twice the total energy of the harmonic oscillator and, moreover, this energy is constant throughout the cycle. In order to simplify the writing we will assume that $r^2 = \rho = 2h$ is the total energy of the oscillator thus disregarding the factor 2.

Equation 1.1 written as a system of two equations is

$$\dot{x} = y; \qquad \dot{y} = -x.$$

Transforming these equations into ρ, θ coordinates one gets:

$$\frac{d\rho}{dt} = 0; \qquad \frac{d\theta}{dt} = -1. \tag{1.5}$$

Eliminating time between these equations one obtains the integral curve (or "trajectory")

$$\frac{d\rho}{d\theta} = 0. \tag{1.6}$$

[1] Department of Electrical Engineering, Stanford University, Stanford, Calif.

The first equation (1.5) expresses the law of conservation of energy; (1.6) represents a circle in the ρ, θ plane and the second equation (1.5) gives $\theta = -t$, as the constant of integration can obviously be assumed to be zero by a proper choice of the origin of either θ or t. The sign minus appearing in this equation is of no special significance and arises merely from the fact that the angles are counted as positive in the trigonometric sense (counterclockwise) while positive direction on the integral curve is clockwise.

If one applies this procedure to (1.2) one obtains similar equations

$$\frac{d\rho}{dt} = 2\epsilon(1 - \rho \operatorname{Cos}^2 \theta)\rho \operatorname{Sin}^2 \theta, \tag{1.7}$$

$$\frac{d\theta}{dt} = -1 + \epsilon(1 - \rho \operatorname{Cos}^2 \theta) \operatorname{Sin} \theta \operatorname{Cos} \theta, \tag{1.8}$$

$$\frac{d\rho}{d\theta} = \frac{2\epsilon(1 - \rho \operatorname{Cos}^2 \theta)\, \rho \operatorname{Sin}^2 \theta}{\epsilon(1 - \rho \operatorname{Cos}^2 \theta) \operatorname{Sin} \theta \operatorname{Cos} \theta - 1}. \tag{1.9}$$

These equations reduce to Eqs. 1.5 and 1.6, respectively, when $\epsilon = 0$. Since it is known that the VDP oscillator is periodic, its integral curve is closed, that is

$$\int_0^{2\pi} \frac{d\rho}{d\theta} d\theta = \int_0^{2\pi} d\rho = 0. \tag{1.10}$$

If, moreover, $\epsilon \ll 1$ the VDP oscillator differs but little from a harmonic oscillator so that $\rho(\theta) \cong \rho_0 = \text{Const.}$ In this case neglecting the small term with ϵ in the denominator of (1.9) one can write

$$\frac{d\rho}{d\theta} \cong -2\epsilon(1 - \rho_0 \operatorname{Cos}^2 \theta)\rho_0 \operatorname{Sin}^2 \theta,$$

and the condition (1.10) of periodicity gives

$$\int_0^{2\pi} d\rho = -2\epsilon\left[\rho_0 \int_0^{2\pi} \operatorname{Sin}^2 \theta d\theta - \rho_0^2 \int_0^{2\pi} \operatorname{Cos}^2 \theta \operatorname{Sin}^2 \theta d\theta\right] = 0, \tag{1.11}$$

whence $\rho_0 = r_0^2 = 4$, which is well known (**1**).[2] If, however, $\epsilon = 0$, condition (1.10) is satisfied identically in (1.11) and one has no right to equate the square bracket in (1.11) to zero. In this case ρ_0 cannot be determined from (1.11) which is also obvious since in this case the VDP oscillator degenerates into a harmonic oscillator whose energy content is arbitrary depending on the initial conditions: For a VDP oscillator, on the contrary, the initial conditions ultimately do not play any role and the oscillator "selects," so to speak, a definite energy content $\rho_0 = 4$ as long as $\epsilon \ll 1$ but not zero.

So far we were able merely to retrace some well-known facts,

[2] The boldface numbers in parentheses refer to the references appended to this paper.

starting with a new dependent variable ρ, the energy. In what follows we shall endeavor to elaborate this line of argument.

2. PHASE TRAJECTORY

We will consider first the differential equation (1.9) of the phase trajectory assuming $\epsilon \ll 1$. Expanding the right hand of (1.9) into a series one gets

$$\frac{d\rho}{d\theta} = -2\rho[\epsilon(1 - p \operatorname{Cos}^2 \theta) \operatorname{Sin}^2 \theta + \epsilon^2(1 - \rho \operatorname{Cos}^2 \theta)^2 \times \\ \times \operatorname{Sin}^3 \theta \operatorname{Cos} \theta + \epsilon^3(1 - \rho \operatorname{Cos}^2 \theta)^3 \operatorname{Sin}^4 \theta \operatorname{Cos}^2 \theta + \cdots], \quad (2.1)$$

where $\rho = \rho(\theta)$ is an unknown periodic function of θ which we will endeavor to determine by the perturbation method. If one sets $\rho = \rho_0 + \epsilon\rho_1 + \epsilon^2\rho_2 + \cdots$ in the preceding equation, one obtains a series of successive approximations arranged according to the order of ϵ.

(*a*) *Approximation of order zero*

$$\frac{d\rho_0}{d\theta} = 0; \qquad \rho_0 = K_0 = \text{Const.}$$

(*b*) *First approximation*

From (2.1) one has

$$\frac{d\rho_1}{d\theta} = -2(1 - \rho_0 \operatorname{Cos}^2 \theta)\rho_0 \operatorname{Sin}^2 \theta,$$

whence

$$\rho_1 = K_1 - \rho_0(1 - \tfrac{1}{4}\rho_0)\theta + \tfrac{1}{2}\rho_0 \operatorname{Sin} 2\theta - \tfrac{1}{16}\rho_0^2 \operatorname{Sin} 4\theta, \quad (2.2)$$

K_1 being an integration constant. The second term on the right of (2.2) is clearly a *secular term* and cannot exist in a stationary state. Determining the constant ρ_0 so as to eliminate this term one obtains again $\rho_0 = 4$ so that (2.2) becomes

$$\rho_1 = K_1 + 2 \operatorname{Sin} 2\theta - \operatorname{Sin} 4\theta. \quad (2.3)$$

The constant K_1 is determined in the next approximation.

(*c*) *Second approximation*

Equating the terms with ϵ^2 in (2.1) one obtains

$$\frac{d\rho_2}{d\theta} = -2\rho_1 \operatorname{Sin}^2 \theta + 4\rho_1\rho_0 \operatorname{Sin}^2 \theta \operatorname{Cos}^2 \theta - 2\rho_0 \operatorname{Sin}^3 \theta \operatorname{Cos} \theta + \\ + 4\rho_0^2 \operatorname{Sin}^3 \theta \operatorname{Cos}^3 \theta - 2\rho_0^2 \operatorname{Sin}^3 \theta \operatorname{Cos}^5 \theta.$$

Replacing ρ_1 and ρ_0 by their values, passing to the multiple arguments, integrating, and eliminating the secular term (which yields $K_1 = 0$)

one gets

$$\rho_2 = K_2 - \frac{3}{4}\operatorname{Cos} 2\theta + \frac{1}{4}\operatorname{Cos} 4\theta + \frac{5}{12}\operatorname{Cos} 6\theta - \frac{1}{4}\operatorname{Cos} 8\theta, \quad (2.4)$$

where K_2 is a constant of integration.

(*d*) *Third approximation*

One obtains $K_2 = \frac{3}{8}$ and

$$\rho_3 = -\frac{1}{12}\operatorname{Sin} 2\theta - \frac{5}{192}\operatorname{Sin} 4\theta + \frac{23}{72}\operatorname{Sin} 6\theta - \frac{37}{192}\operatorname{Sin} 8\theta - \frac{5}{48}\operatorname{Sin} 10\theta + \frac{5}{64}\operatorname{Sin} 12\theta + K_3. \quad (2.5)$$

If one limits the approximation up to the terms of the order ϵ^4, the periodic function $\rho(\theta)$ is then given by the following expression arranged by the orders of ϵ:

$$\begin{aligned}\rho(\theta) = 4 &+ \\ &+ \epsilon(2\operatorname{Sin} 2\theta - \operatorname{Sin} 4\theta) + \\ &+ \epsilon^2\left(\frac{3}{8} - \frac{3}{4}\operatorname{Cos} 2\theta + \frac{1}{4}\operatorname{Cos} 4\theta + \frac{5}{12}\operatorname{Cos} 6\theta - \frac{1}{4}\operatorname{Cos} 8\theta\right) + \\ &+ \epsilon^3\left(K_3 - \frac{1}{12}\operatorname{Sin} 2\theta - \frac{5}{192}\operatorname{Sin} 4\theta + \frac{23}{72}\operatorname{Sin} 6\theta - \frac{37}{192} \times \right. \\ &\qquad \left. \times \operatorname{Sin} 8\theta - \frac{5}{48}\operatorname{Sin} 10\theta + \frac{5}{64}\operatorname{Sin} 12\theta\right) + \\ &+ \cdots. \end{aligned} \quad (2.6)$$

If the terms of this array are arranged according to the order of harmonics one gets

$$\begin{aligned}\rho(\theta) = (4 + \tfrac{3}{8}\epsilon^2 + \cdots) &+ \\ &+ \left[\left(2\epsilon - \frac{1}{12}\epsilon^3 + \cdots\right)\operatorname{Sin} 2\theta + \left(-\frac{3}{4}\epsilon^2 + \cdots\right)\operatorname{Cos} 2\theta\right] + \\ &+ \left[\left(-\epsilon - \frac{5}{192}\epsilon^3 + \cdots\right)\operatorname{Sin} 4\theta + \left(\frac{1}{4}\epsilon^2 + \cdots\right)\operatorname{Cos} 4\theta\right] + \\ &+ \left[\left(\frac{23}{72}\epsilon^3 + \cdots\right)\operatorname{Sin} 6\theta + \left(\frac{5}{12}\epsilon^2 + \cdots\right)\operatorname{Cos} 6\theta\right] + \\ &+ \left[\left(-\frac{37}{192}\epsilon^3 + \cdots\right)\operatorname{Sin} 8\theta + \left(-\frac{1}{4}\epsilon^2 + \cdots\right)\operatorname{Cos} 8\theta\right] + \\ &+ \cdots. \end{aligned} \quad (2.7)$$

The non-written terms in brackets are ascending powers of ϵ. Unfortunately, the calculations beyond the third approximation are so

long that it becomes impracticable to continue them unless, perhaps, with the aid of some mechanical computers. It is possible, however, to form certain general conclusions regarding the series (2.7).

1. The coefficients $a_0, a_1, b_1, a_2, b_2 \cdots$ of the trigonometric terms in (2.7) are power series in ϵ approaching zero when $\epsilon \to 0$, except a_0 which approaches the value 4. The amplitudes $\sqrt{a_1^2 + b_1^2} \cdots$ of the various harmonics also approach zero when $\epsilon \to 0$. As to their phases, they approach at the limit $\epsilon \to 0$ the phase of the term whose coefficient contains a lower power of ϵ.

2. It seems likely (although the approximations in our case did not progress far enough to be able to assert this) that for a sufficiently small ϵ, the coefficients a_n, b_n of the trigonometric series (2.7) approach zero when $n \to \infty$. If such is the case the trigonometric series (2.7) is actually a Fourier series.

3. The constant term $(4 + \frac{3}{8}\epsilon^2 + \cdots)$ may be regarded as a "component" of a harmonic oscillator while the trigonometric terms represent merely the *energy fluctuations* between the VDP oscillator and the external source. It should be noted that the preceding discussion relates only to the integral curve $\rho(\theta)$ of the VDP oscillator inasmuch as *time* does not enter here. In the following section we shall investigate the dependence on time, that is, the actual fluctuations of energy. It may be useful to say a few words regarding the convergence of the expansion (2.1). Inasmuch as this expansion was obtained from the function appearing on the right side of (1.9) it is clear that the only case when it is divergent is when $d\theta/dt = 0$. This case is obviously ruled out for $\epsilon \ll 1$ in which we are interested here. It can be shown, however, (section 5 below) that, even in the case when ϵ is large, $d\theta/dt \neq 0$ although it may become of the order of $1/\epsilon^2$ for some special intervals of θ. In these intervals the convergence becomes so slow that the perturbation method becomes impracticable in view of the impossibility of carrying the subsequent approximations indefinitely on account of a rapidly increasing complexity of calculations.

For this reason the preceding procedure is applicable only for $\epsilon < 1$ and, preferably, for $\epsilon \ll 1$. In many applications, however, one is interested precisely in the range of small ϵ so that the perturbation procedure can be used then. As regards the so-called "relaxation range" ($\epsilon \gg 1$), although the perturbation method ceases to be applicable, the energy fluctuations in a VDP oscillator can be still explored in connection with the data obtained by the isocline method as will be shown in section 5.

3. ENERGY FLUCTUATIONS

The preceding study concerns the function $\rho(\theta)$ in which the time does not appear. In order to investigate the energy fluctuations $\rho(t)$ in

a VDP oscillator it is necessary to deal directly with Eqs. 1.7 and 1.8 containing time. In other words, it is necessary to determine the *motion* $\rho(t)$ on the trajectory $\rho(\theta)$.

The perturbation procedure remains the same as before with the exception that it is to be carried out simultaneously in both variables ρ and θ considered as functions of time.

Equations 1.7 and 1.8 can be written

$$\frac{d\rho}{dt} = \epsilon[\rho(1 - \operatorname{Cos} 2\theta) - \tfrac{1}{4}\rho^2(1 - \operatorname{Cos} 4\theta)], \tag{3.1}$$

$$\frac{d\theta}{dt} = \epsilon[(\tfrac{1}{2} - \tfrac{1}{4}\rho) \operatorname{Sin} 2\theta - \tfrac{1}{8}\rho \operatorname{Sin} 4\theta] - 1. \tag{3.2}$$

Since the time does not appear explicitly in these equations, there is no difference between t and $t + t_0$, t_0 being arbitrary. This means that to a given trajectory (or integral curve) $\rho(\theta)$ corresponds an infinity of motions $\rho(t)$, $\theta(t)$ differing by an arbitrary phase t_0 depending on the initial conditions.

The perturbation procedure is carried out by setting

$$\rho = \rho_0 + \epsilon\rho_1 + \epsilon^2\rho_2 + \cdots,$$
$$\theta = \theta_0 + \epsilon\theta_1 + \epsilon^2\theta_2 + \cdots.$$

Since θ appears only under the sign of the trigonometric functions we have to expand Sin 2θ, Cos 2θ, $\cdots$ into a Taylor's series around $\theta = \theta_0$, which gives

$$\operatorname{Cos} 2\theta = \operatorname{Cos} 2\theta_0 - 2(\epsilon\theta_1(t) + \epsilon^2\theta_2(t) + \cdots) \operatorname{Sin} 2\theta_0 + \cdots,$$
$$\operatorname{Sin} 2\theta = \operatorname{Sin} 2\theta_0 + 2(\epsilon\theta_1(t) + \epsilon^2\theta_2(t) + \cdots) \operatorname{Cos} 2\theta_0 + \cdots.$$

Rearranging the results according to the powers of ϵ one finds

$$\begin{aligned}\frac{d\rho}{dt} = \epsilon[\rho_0(1 - \operatorname{Cos} 2\theta_0) - \tfrac{1}{4}\rho_0^2(1 - \operatorname{Cos} 4\theta_0)] + \\ + \epsilon^2[2\rho_0\theta_1 \operatorname{Sin} 2\theta_0 + \rho_1(1 - \operatorname{Cos} 2\theta_0) - \\ - \rho_0^2\theta_1 \operatorname{Sin} 4\theta_0 - \tfrac{1}{2}\rho_0\rho_1[1 - \operatorname{Cos} 4\theta_0)] + \epsilon^3[\cdots] + \cdots\end{aligned} \tag{3.3}$$

$$\begin{aligned}\frac{d\theta}{dt} = -1 + \epsilon[(\tfrac{1}{2} - \tfrac{1}{4}\rho_0) \operatorname{Sin} 2\theta_0 - \tfrac{1}{8}\rho_0 \operatorname{Sin} 4\theta_0] + \\ + \epsilon^2[(1 - \tfrac{1}{2}\rho_0)\theta_1 \operatorname{Cos} 2\theta_0 - \tfrac{1}{4}\rho_1 \operatorname{Sin} 2\theta_0 - \\ - \tfrac{1}{2}\rho_0\theta_1 \operatorname{Cos} 4\theta_0 - \tfrac{1}{8}\rho_1 \operatorname{Sin} 4\theta_0] + \epsilon^3[\cdots] + \cdots.\end{aligned} \tag{3.4}$$

The approximation of the zero order yields again

$$\frac{d\rho_0}{dt} = 0; \qquad \frac{d\theta_0}{dt} = -1, \qquad \text{that is} \qquad \theta_0 = -t,$$

which coincides with Eqs. 1.5 for the harmonic oscillators. The elimination of the secular term in the first approximation gives again $\rho_0 = 4$. There appears, however, another constant of integration in the second equation which must be determined by the *initial conditions*, the independent variable being now t. The simplest way of introducing the initial condition is to assume that for $t = 0$, $\theta(0) = \theta_1(0) = \theta_2(0) = \cdots = 0$.

The first approximation becomes then

$$\rho_1(t) = K_1 - 2\,\mathrm{Sin}\,2t + \mathrm{Sin}\,4t;$$

$$\theta_1(t) = \tfrac{3}{8} - \tfrac{1}{4}\,\mathrm{Cos}\,2t - \tfrac{1}{8}\,\mathrm{Cos}\,4t, \qquad (3.5)$$

where K_1 is an integration constant determined in the second approximation. The procedure from now on becomes obvious, *viz:* for the second approximation we collect the terms with ϵ^2 in (3.3) and (3.4) and it is observed that $d\rho_2/dt$ and $d\theta_2/dt$ are expressible now in terms of ρ_0, θ_0, ρ_1 and θ_1. Integrating these equations, eliminating the secular term in ρ_2 (which leads to the determination of K_1) and determining the constant of integration in θ_2 by the assumed initial conditions, one obtains the second approximation ρ_2 and θ_2; one finds

$$\rho_2 = K_2 + \mathrm{Cos}\,2t - \frac{7}{4}\,\mathrm{Cos}\,4t + \frac{2}{3}\,\mathrm{Cos}\,6t,$$

where $K_1 = 0$.

It is to be noted that the expression for $\rho_2(t)$ is not the same as that for $\rho_2(\theta)$ and likewise for other approximations so that, in general, the trigonometric series for $\rho(t)$ is not the same as that for $\rho(\theta)$. The reason for that is due to the non-uniformity of rotation of the radius vector as is seen from the expression for $\theta_1(t)$ which gives in this approximation

$$\theta = -t + \epsilon(\tfrac{3}{8} - \tfrac{1}{4}\,\mathrm{Cos}\,2t - \tfrac{1}{8}\,\mathrm{Cos}\,4t).$$

This shows that there exists a kind of a "phase modulation" affecting all trigonometric terms.

The fluctuation of energy in time up to the terms of the order ϵ^3 is then

$$\rho(t) = 4 + \epsilon(-2\,\mathrm{Sin}\,2t + \mathrm{Sin}\,4t) + \\ + \epsilon^2\left(\mathrm{Cos}\,2t - \frac{7}{4}\,\mathrm{Cos}\,4t + \frac{2}{3}\,\mathrm{Cos}\,6t\right) + K_2,$$

where the constant of integration K_2 is determined in the next approximation and so on.

4. EFFECT OF THE FORM OF THE VAN DER POL EQUATION ON THE FREQUENCY SPECTRUM

So far we have been concerned with Eq. 1.2 originally formulated by van der Pol (1). Later on, this equation was generalized by A.

Liénard (2), E. & H. Cartan (3), N. Levinson and O. K. Smith (4) and others. The general form of the van der Pol equation is

$$\ddot{x} - \epsilon f(x, \dot{x})\dot{x} + g(x) = 0, \tag{4.1}$$

where the functions f and g must satisfy certain conditions (4). We shall give a few examples relative to the electron tube circuits assuming that they do not involve any non-linear capacities in which case $g(x) = x$, and consider $f(x, \dot{x}) = f(x)$ in the Liénard sense, *viz:* $f(x)$ is an even function of x so that $F(x) = \int f(x)dx$ is odd and positive for small values of x and becomes negative beginning with $x = x_0$, decreasing monotonically for $x > x_0$. The polynomials of the form $F(x) = ax - bx^3$; $F(x) = ax + bx^3 - cx^5$ etc., clearly, satisfy this condition and are frequently used in applications. Thus, for instance, when $F(x)$ is a polynomial of the fifth degree, the van der Pol equation has the form

$$\ddot{x} - \epsilon(1 + \alpha x^2 - \beta x^4)\dot{x} + x = 0. \tag{4.2}$$

Using the preceding procedure one gets

$$\frac{d\rho}{d\theta} = -2\rho[\epsilon(\ )\operatorname{Sin}^2\theta + \epsilon^2(\ )^2\operatorname{Sin}^3\theta\operatorname{Cos}\theta + \epsilon^3(\ )^3\operatorname{Sin}^4\theta\operatorname{Cos}^2\theta + \cdots],$$

where $(\ ) = (1 + \alpha\rho\operatorname{Cos}^2\theta - \beta\rho^2\operatorname{Cos}^4\theta)$. The perturbation method limited to the zero, and the first approximation gives

$$\rho_0 = K_0 = \text{Const};$$

$$\rho_1 = -2[\rho_0\int\operatorname{Sin}^2\theta d\theta + \alpha\rho_0^2\int\operatorname{Cos}^2\theta\operatorname{Sin}^2\theta d\theta - \\ - \beta\rho_0^3\int\operatorname{Cos}^4\theta\operatorname{Sin}^2\theta d\theta] + K_1$$

and the elimination of the secular term gives

$$\rho_0 = \alpha/\beta + \sqrt{(\alpha/\beta)^2 + (8/\beta)},$$

so that, up to the terms of the order ϵ^2 one gets

$$\rho = \rho_0 + \epsilon\rho_1 = \rho_0 - \epsilon\left[\frac{1}{2}\left(\rho_0 + \frac{1}{64}\beta\rho_0^3\right)\operatorname{Sin}2\theta + \right. \\ \left. + \frac{1}{16}\rho_0^2\left(\alpha - \frac{1}{2}\beta\right)\operatorname{Sin}4\theta - \frac{1}{96}\beta\rho_0^2\operatorname{Sin}6\theta\right] + K_2.$$

There exists obviously no ρ_0 for $\beta = 0$ since the damping term is then negative for all values of x. For $\alpha = 0$, $\beta = 1$, (4.2) becomes

$$\ddot{x} - \epsilon(1 - x^4)\dot{x} + x = 0, \tag{4.5}$$

in which case

$$\rho_0 = 2\sqrt{2}.$$

Interesting cases arise when the form of the van der Pol equation for an electron tube oscillator is modified by the presence of certain

variable conductors in the oscillating circuit. We shall consider one such case in connection with the circuit shown in Fig. 1. If all parameters (L, C and R) are constant, it is well known (5) that the van der Pol equation in this case is of the form

$$\frac{d^2\nu}{d\tau^2} - (\beta + 2\gamma\nu - 3\delta\nu^2)\frac{d\nu}{d\tau} + \nu = 0, \tag{4.6}$$

where $\nu = \nu_g/\nu_s$ is a dimensionless variable (ν_g is the grid voltage; ν_s the

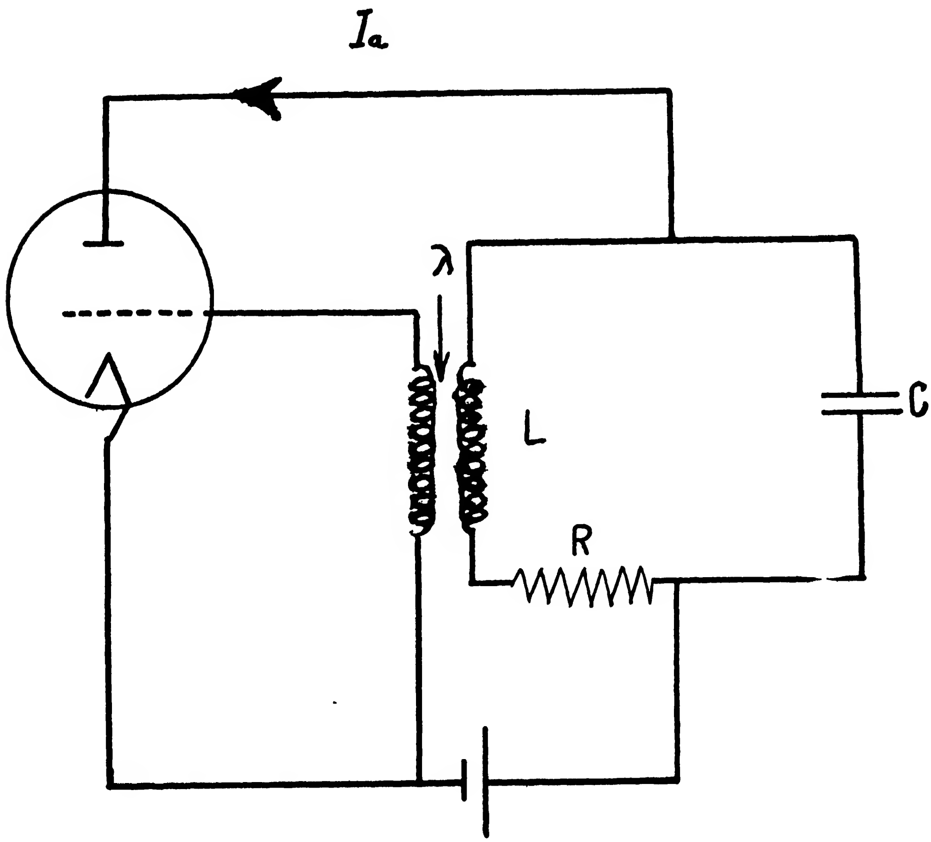

FIG. 1.

constant "saturation voltage"), $\tau = \omega_0 t$; $\omega_0 = \sqrt{1/LC}$; $\beta = (\lambda\beta_1 - RC)\omega_0$

$$\gamma = \lambda\gamma_1\omega_0; \qquad \delta = \lambda\delta_1\omega_0; \qquad \beta_1 = S_1; \qquad \gamma_1 = S_2\nu_s; \qquad \delta_1 = -S_3\nu_s^2,$$

and the non-linear function, the anode current, is assumed to be represented by the polynomial

$$I_a = \phi(\nu_g) = I_0 + S_1\nu_g + S_2\nu_g^2 + S_3\nu_g^3.$$

Let us modify the problem by replacing the constant resistor R by a variable one $R(|i|)$, depending on the absolute value of the current and

assume, moreover, that the non-linearity of $R(|i|)$ appears *before* the non-linearity of the electron tube is felt. In other words, we wish to consider the case when the electron tube operates still on the rectilinear part of its characteristic (that is, $\gamma = \delta \cong 0$). Equation 4.6 in such a case becomes

$$\frac{d^2\nu}{d\tau^2} - [\lambda\beta_1 - R(|i|)C]\frac{d\nu}{d\tau} + \nu = 0. \tag{4.7}$$

It is necessary to make a certain assumption regarding the function $R(|i|)$. Assume, for example, that the resistance of the non-linear resistor is of the form

$$R(|i|) = m + n|i|,$$

where m and n are positive constants, that is, the resistance increases as a linear function of the absolute value of the current which is generally the case for conductors having a positive temperature coefficient. On the other hand

$$\nu_g = \frac{1}{C}\int idt,$$

whence

$$i = C\nu_s\dot{\nu} = C\nu_s\omega_0\frac{d\nu}{d\tau}$$

so that

$$|i| = C\nu_s\omega_0\left|\frac{d\nu}{d\tau}\right|$$

and (4.7) becomes

$$\frac{d^2\nu}{d\tau^2} - \left(\epsilon - b\left|\frac{d\nu}{d\tau}\right|\right)\frac{d\nu}{d\tau} + \nu = 0,$$

where

$$\epsilon = (\lambda\beta_1 - mC); \qquad b = nC^2\nu_s\omega_0.$$

The preceding equation becomes

$$\frac{d^2\nu}{d\tau^2} - \epsilon\left(1 - K\left|\frac{d\nu}{d\tau}\right|\right)\frac{d\nu}{d\tau} + \nu = 0, \qquad K = \frac{b}{\epsilon}.$$

The equivalent system is

$$\frac{d\nu}{d\tau} = y; \qquad \frac{dy}{d\tau} = \epsilon(1 - K|y|)y - \nu.$$

Assuming that ϵ and r are small, Eq. 1.9 in this case is

$$\frac{d\rho}{d\theta} \doteq \frac{2\epsilon(\ )\rho \operatorname{Sin}^2\theta}{\epsilon(\ )\operatorname{Sin}\theta\operatorname{Cos}\theta - 1}$$

$$= -2\rho[\epsilon(\ )\operatorname{Sin}^2\theta + \epsilon^2(\ )^2\operatorname{Sin}^3\theta\operatorname{Cos}\theta + \epsilon^3(\ )^3\operatorname{Sin}^4\theta\operatorname{Cos}^2\theta + \cdots],$$

where

$$(\ ) \doteq (1 - Kr|\operatorname{Sin}\theta|) = (1 - K\rho^{\frac{1}{2}}|\operatorname{Sin}\theta|).$$

The perturbation method gives

$$\frac{d\rho_0}{d\theta} = 0; \qquad \rho_0 = K_0 = \text{Const.}$$

$$\frac{d\rho_1}{d\theta} = -2(1 - K\rho_0^{\frac{1}{2}}|\text{Sin }\theta|)\rho_0 \text{Sin}^2 \theta =$$

$$= -\rho_0 + \rho_0 \text{Cos } 2\theta + K\rho_0^{\frac{1}{2}}(|\text{Sin }\theta| - \text{Cos } 2\theta|\text{Sin }\theta|).$$

Replacing

$$|\text{Sin }\theta| = \frac{2}{\pi} - \frac{4}{\pi}\left(\frac{1}{1.3}\text{Cos } 2\theta + \frac{1}{3.5}\text{Cos } 4\theta + \frac{1}{5.7}\text{Cos } 6\theta + \cdots\right)$$

$$\text{Cos } 2\theta|\text{Sin }\theta| = -\frac{2}{3\pi} + \frac{28}{15\pi}\text{Cos } 2\theta - \frac{76}{105\pi}\text{Cos } 4\theta - \cdots$$

and eliminating the secular term one obtains

$$\rho_0 = r_0^2 = \left(\frac{3\pi}{8K}\right)^2,$$

which starts the perturbation procedure which we will not continue here. It is worth mentioning merely that in this case we encounter a situation in which already in the first approximation there appears the whole trigonometric series instead of trigonometric polynomials containing but a few terms as this was the case for Eqs. 1.2 and 4.5. The frequency spectrum of energy fluctuations in an oscillator of this kind is, therefore, far richer in harmonics than in the cases just mentioned.

These examples show that, following this procedure, it is always possible to form an idea as to the frequency spectrum of energy fluctuations in a VDP oscillator for a given non-linear function $F(x)$ provided it satisfies the Liénard condition and provided ϵ is sufficiently small.

5. CASE WHEN ϵ IS LARGE

The perturbation method ceases to be applicable when ϵ is not small. Certain additional conclusions can be still obtained from (1.7), (1.8) and (1.9) also, in this case, with the aid of the data yielded by the method of isoclines.

It is to be noted that the maxima and minima of both polar curves $\rho(\theta)$ and $r(\theta)$ are situated on the same radii-vectors since the $\rho(\theta)$ curve is obtained merely by squaring the radii vectors of the $r(\theta)$ curve for each θ and, conversely if $\rho(\theta)$ curve is given, the $r(\theta)$ curve can be obtained by an inverse operation. We shall speak preferably of the energy curve $\rho(\theta)$ inasmuch as in this manner it will be possible to derive certain physical conclusions regarding the VDP oscillator but occasionally we shall refer also to the integral curve $r(\theta)$ in view of the above relation between these two closed curves.

From (1.7) it follows that the maxima and minima of energy occur when $\rho(\theta) = 1/\text{Cos}^2 \theta$ which in cartesian coordinates is obviously $x = \pm 1$.

For these particular values of θ the term with $\dot{x}$ in (1.2) vanishes and the VDP oscillator coincides instantaneously with the harmonic oscillator of this particular (ρ_{max} or ρ_{min}) energy content. One can also verify this circumstance by replacing $\rho = 1/\text{Cos}^2\,\theta$ in (1.8) which gives $\dot{\theta} = -1$ which in conjunction with the condition $\dot{\rho} = 0$ gives conditions (1.5) for a harmonic oscillator. In order to distinguish maxima from minima it is sufficient to calculate $\ddot{\rho}$ which gives

$$\ddot{\rho} = 2\epsilon[\dot{\rho}(\text{Sin}^2\,\theta - 2\rho\,\text{Sin}^2\,\theta\,\text{Cos}^2\,\theta) + 2\rho\dot{\theta}(\text{Sin}\,\theta\,\text{Cos}\,\theta + \rho\,\text{Sin}^3\,\theta\,\text{Cos}\,\theta - \rho\,\text{Sin}\,\theta\,\text{Cos}^3\,\theta)].$$

Replacing in this, expressions $\dot{\rho} = 0$, $\dot{\theta} = -1$, $\rho = 1/\text{Cos}^2\,\theta$, one finds

$$\ddot{\rho} = -4\epsilon\,\text{Tan}^3\,\theta,$$

which shows that the maxima of energy are located in the first and the third quadrants of the polar curve $\rho(\theta)$ and minima in the second and in the fourth quadrants which also agrees with the data yielded by the method of isoclines. Similarly, when ϵ increases the isocline method shows that the maxima increase and the minima decrease but since the condition $\rho\,\text{Cos}^2\,\theta = 1$ is independent of ϵ, it is clear that the values of θ corresponding to ρ_{max} approach the y axis and those for ρ_{min} recede to the x axis while still being in their respective quadrants. This also agrees with the data furnished by the isocline method. There exists thus a definite relation between the magnitude of the maxima and minima of energy and their phase in the polar curve.

For a sufficiently large ϵ and in the neighborhood of maxima of the energy content, the rate of change of energy $\dot{\rho}$ varies very rapidly even for a small variation $\Delta\theta$ around the value $\theta = \theta_{max}$. Thus, for instance, from the isocline curve for $\epsilon = 30$, $\rho_{max} \cong 1300$ and $\theta_{max} \cong 88.4°$. For $\Delta\theta = \pm\,0.1°$, $\dot{\rho}$ changes from about -13000 (for $\Delta\theta = -\,0.1°$) to about $+11000$ (for $\Delta\theta = +\,0.1°$) in the dimensionless units used here, whereas in the neighborhood of minima the same variation $\Delta\theta$ accounts for a small part of one per cent of the above variation. If one wishes to give a physical analogue of the behavior of the VDP oscillator in the neighborhood of its maxima energy points, at least as far as energy fluctuations are concerned, perhaps a pneumatic hammer or a similar device may give an adequate analogy. In fact in a device of this kind a rapid acceleration of the hammer with the incident increase in its energy content (which is the kinetic energy here) is followed by a not less rapid deceleration when the hammer strikes an energy absorbing medium and thus loses its energy.

In addition to one maximum and one minimum of energy per one half cycle (the other half cycle being identical) the VDP oscillator has also one *stationary* point for $\theta = 0$ (for the other half-cycle this stationary point occurs for $\theta = \pi$) at which the energy passes through a stationary value without being either maximum or minimum. One sees

this from (1.7) which shows that $\dot{\rho}$ passes through zero at these points without changing its sign. One notes also that $\ddot{\rho} = 0$ at this point. When ϵ is large these stationary points are close to the minima points of energy. The investigation of the behavior of an integral curve in the neighborhood of these stationary points is far from being simple, if ϵ

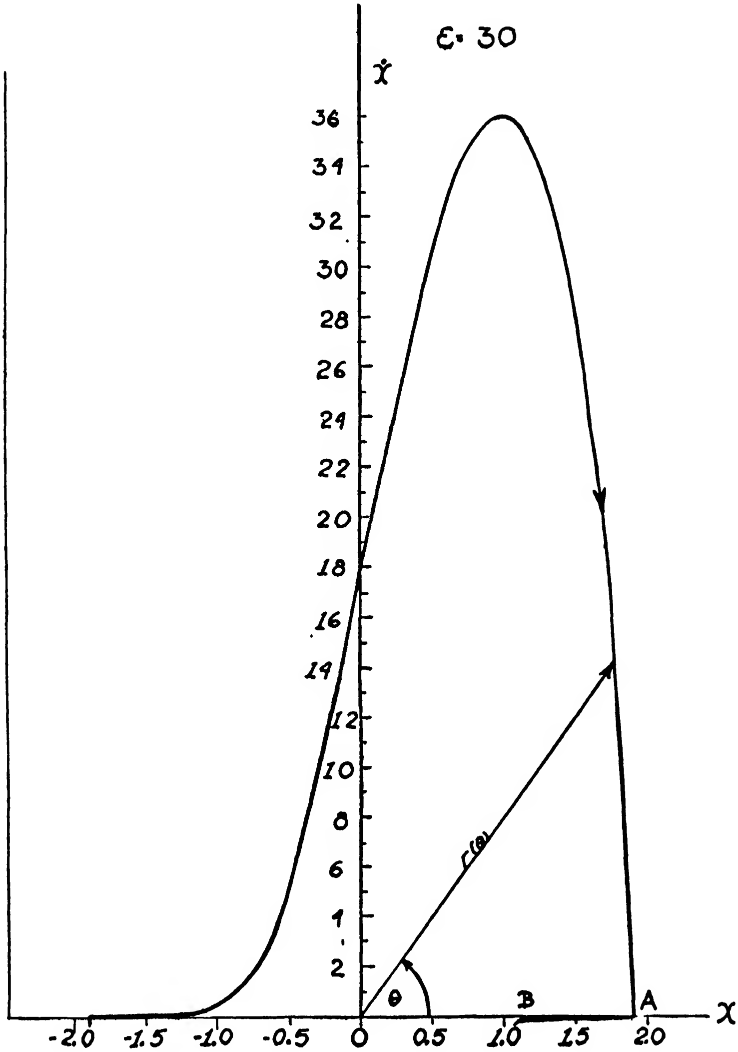

FIG. 2.

is large enough and this is probably one of the principal difficulties of the van der Pol equation. The method of isoclines shows, in fact, that there is a sharp bend of integral curves in this "relaxation region" as can be seen from Fig. 2, representing the integral curve $r(\theta)$ for $\epsilon = 30$ constructed by the method of isoclines.[3] If one follows this integral curve

[3] The author is indebted to Miss E. Yost for this construction.

in the positive direction (that is, clockwise) one finds that it cuts the x-axis at right angles (point A) but immediately after that the integral curve has an almost 90° bend and follows practically the x-axis toward the origin. The accuracy of the graphical construction is obviously insufficient to be able to specify the details of what happens in this critical region and it is necessary to make a closer analysis.

We shall indicate first an approximation based on the results of the isocline data and shall analyze the situation later independently of the method of isoclines.

Let us assume, as an approximation, that in this "relaxation region" (say between $x \cong 1.9$ and $x \cong 1.2$ in Fig. 2) the integral curve is parallel to the x-axis, that is, coincides with the zero isocline whose equation is

$$y = x/\epsilon(1 - x^2).$$

If one replaces this value of y into the expression for $\dot{\theta}$ in the x, y coordinates, *viz:*

$$\dot{\theta} = -1 + \epsilon(1 - x^2)xy/\rho,$$

after a few transformations, one finds the following approximate expression for $\dot{\theta}$ in this region

$$\dot{\theta} \doteq -\frac{1}{\epsilon^2(1 - x^2)^2},$$

which shows that $\dot{\theta}$ is small of the order $1/\epsilon^2$ and negative as everywhere else on the integral curve. In view of this, from (1.8) we get

$$\epsilon(1 - \rho \operatorname{Cos}^2 \theta) \cong 1/\operatorname{Sin} \theta \operatorname{Cos} \theta,$$

so that (1.7) becomes

$$\frac{d\rho}{d\theta} \cong 2\rho \operatorname{Tan} \theta \cong 2\rho\theta. \tag{5.1}$$

Since $\theta < 0$ in this region is small and changes very slowly, if one takes its average value $\bar{\theta}$ in this interval, (5.1) yields an approximate formula

$$\rho = \rho_0 e^{2\bar{\theta} t} \tag{5.2}$$

where ρ_0 corresponds to the beginning of this "relaxation" region. This approximate relation shows that the energy is slowly drained away from the system in a manner in which the electrostatic energy of a charged condenser disappears gradually owing to an imperfect dielectric.

This somewhat crude analysis is merely a physical interpretation of the results yielded by the method of isoclines.

For that reason it is preferable to investigate the behavior of the functions $\dot{\rho}(\theta)$ and $\dot{\theta}(\theta)$ in a small interval $(\theta_1, -\theta_1)$; $\theta_1 > 0$ around $\theta = 0$ (the argument applies also to $\theta = \pi$). This will permit obtaining certain conclusions directly without relying on the method of isoclines.

Differentiating (1.8) with respect to θ and setting $\theta = 0$ after the differentiation one gets

$$\left(\frac{d\dot{\theta}}{d\theta}\right)_0 = -\epsilon(\rho_0 - 1); \qquad \left(\frac{d^2\dot{\theta}}{d\theta^2}\right)_0 = 0;$$

$$\left(\frac{d^3\dot{\theta}}{d\theta^3}\right)_0 = \epsilon(10\rho_0 - 4) \text{ etc.,} \qquad \rho_0 = \rho(0),$$

so that

$$\dot{\theta}(\theta) = \dot{\theta}_0 + \theta\left(\frac{d\dot{\theta}}{d\theta}\right)_0 + \frac{\theta^2}{2}\left(\frac{d^2\dot{\theta}}{d\theta^2}\right)_0 + \cdots$$

$$= -1 - \epsilon(\rho_0 - 1)\theta + \epsilon(5\rho_0 - 2)\frac{\theta^3}{3} + \cdots. \qquad (5.3)$$

Since θ is small and ρ_0 is of the order 4 one can limit this expansion to the linear term in θ. Under this assumption, $\dot{\theta}$ becomes very small for $\theta<0$ such that $\epsilon(\rho_0-1)|\theta|$ is of the order of unity, that is $|\theta|\cong 0(1/3\epsilon)$. In such a case the term with $\epsilon\theta^3$ can be neglected as we did. If, therefore, the interval $\theta_1, -\theta_1$ in which we propose to investigate the behavior of $\dot{\rho}(\theta)$ and $\dot{\theta}(\theta)$ is fixed on this basis, $\dot{\theta} < 0$ is a monotonically increasing function. In fact $\dot{\theta} = -1 - K\theta$ has the values $|\dot{\theta}|_{\theta_1} = 1 + K\theta_1$; $|\dot{\theta}|_0 = 1$ and $|\dot{\theta}|_{-\theta_1} = 1 - K\theta_1$.

In a similar manner differentiating (1.7) with respect to θ and setting $\theta = 0$ after the differentiation, one gets

$$\left(\frac{d\dot{\rho}}{d\theta}\right)_0 = 0; \qquad \left(\frac{d^2\dot{\rho}}{d\theta^2}\right)_0 = -2\epsilon\rho_0(\rho_0 - 1).$$

As, moreover, $\dot{\rho}_0 = \dot{\rho}(0) = 0$, one gets

$$\dot{\rho}(\theta) = \dot{\rho}(0) + \theta\left(\frac{d\dot{\rho}}{d\theta}\right)_0 + \frac{\theta^2}{2}\left(\frac{d^2\dot{\rho}}{d\theta^2}\right)_0 + \cdots \cong -2\epsilon\rho_0(\rho_0 - 1)\theta^2. \qquad (5.4)$$

Since $\epsilon|\theta|$ is assumed to be of the order one, the preceding expression shows that $\dot{\rho}(\theta)$ undergoes a small variation of the order θ in the interval $(\theta_1, -\theta_1)$. As first approximation we can assume, therefore that $\rho \cong \rho_0$ in this interval. We shall indicate later how a second approximation can be obtained.

It is useful now to specify the interval $(\theta_1, -\theta_1)$, $\theta_1 > 0$ in which the preceding assumptions apply. In the construction of the curve of Fig. 2, $\epsilon = 30$ was assumed which is a purely arbitrary value selected with a view to go a little beyond the value ($\epsilon = 10$) which van der Pol used in his construction. This permits obtaining a more pronounced "relaxation region" between the points A and B. On the other hand, van der Pol, in his analysis of a relaxation oscillation circuit with constants commonly encountered in applications, finds $\epsilon \cong 3.10^5$, that is $K \cong 10^6$. This gives the value of $\theta < 0$, for which $\dot{\theta}$ becomes very

small, of the order of 1 sec. of arc. This means that the motion of the representative point which is practically tangential (with respect to the radius vector) for $\theta = 0$, becomes radial for $\theta \cong -1$ sec., which gives an idea as to the order of magnitude of the interval $(\theta_1, -\theta_1)$ in question for the various values of ϵ.

As regards the function $\dot{\rho}(\theta)$, (5.4) shows that, under the assumed approximation, it is an even function of θ; moreover $\dot{\rho}(\theta)$ is negative in this interval, that is, the energy decreases between θ_1, and $-\theta_1$.

We propose now to calculate the loss of energy $-\Delta\rho_1$, and $-\Delta\rho_2$ in the half-intervals $(\theta_1, 0)$ and $(0, -\theta_1)$, respectively, when the radius vector traverses the small arc $(\theta_1, -\theta_1)$. One has

$$-\Delta\rho_1 = \int_{\theta_1}^{0} \dot{\rho} dt; \qquad -\Delta\rho_2 = \int_{0}^{-\theta_1} \dot{\rho} dt.$$

From (5.3) and (5.4) we have

$$\dot{\theta} = -(1 + K\theta); \qquad \dot{\rho} = -2\rho_0 K\theta^2,$$

where $K = \epsilon(\rho_0 - 1)$. Thus

$$\dot{\rho} dt = 2\rho_0 K \frac{\theta^2}{1 + K\theta} d\theta$$

and, therefore,

$$-\Delta\rho_1 = 2\rho_0 K \int_{\theta_1}^{0} \frac{\theta^2}{1 + K\theta} d\theta; \qquad -\Delta\rho_2 = 2\rho_0 K \int_{0}^{-\theta_1} \frac{\theta^2}{1 + K\theta} d\theta.$$

Under this assumption

$$-\Delta\rho_1 = -\frac{2\rho_0}{K^2}\left[\tfrac{1}{2}(1 + K\theta_1)^2 - 2(1 + K\theta_1) + \ln(1 + K\theta_1) + \tfrac{3}{2}\right], \quad (5.5)$$

$$-\Delta\rho_2 = \frac{2\rho_0}{K^2}\left[\tfrac{1}{2}(1 - K\theta_1)^2 - 2(1 - K\theta_1) + \ln(1 - K\theta_1) + \tfrac{3}{2}\right]. \quad (5.6)$$

Since in the half-interval $(0, -\theta_1)$ the quantity $1 - K\theta_1 > 0$ becomes very small, $\ln(1 - K\theta_1)$ is a very large negative number so that $\Delta\rho_2 \gg \Delta\rho_1$. On the other hand, since $-\Delta\rho_1$ and $-\Delta\rho_2$ represent the decrease of the radius vector in $(\theta_1, 0)$ and $(0, -\theta_1)$, respectively, one obtains a situation depicted in Fig. 3 which explains the origin of a sharp bend in the curve of energy $\rho(\theta)$ (and also the integral curve $r(\theta)$) in the negative neighborhood $(\theta < 0)$ of the stationary point A. Once the dissymmetry of $\dot{\theta}(\theta)$ has been ascertained starting from the assumption that $\rho \cong \rho_0$ in this angular interval $(\theta_1, -\theta_1)$, it is possible to carry out the second approximation using, for example, the values $\rho_0 + \Delta\rho_1/2$, $\rho_0 - \Delta\rho_2/2$ instead of ρ_0 (and the corresponding values K_1 and K_2) in both half-intervals which gives a more correct estimate for $\Delta\rho_1$ and $\Delta\rho_2$ and thus calculate the shape of the $\rho(\theta)$ curves in this neighborhood.

The sharp bend of the curves $\rho(\theta)$ and $r(\theta)$ is thus due to a very marked assymmetry of the function $\dot{\theta}(\theta)$ in this region while the function $\dot{\rho}(\theta)$ remains symmetrical about these stationary points. From a physical standpoint this circumstance means that, on account of a smaller velocity $\dot{\theta}$ in the $(0, -\theta_1)$ half interval the same drain of energy (for the same θ) persists for a longer time in the negative half interval than in the positive one which is traversed rapidly, so that the actual decrease of energy is greater in $(0, -\theta_1)$ than in $(\theta_1, 0)$. It must be noted that for very large values of ϵ, like those mentioned by van der Pol, the bend of integral curves becomes so sharp that the analyticity of the curve in this region is bad. For that reason the Taylor expansion in this region becomes rather complicated as compared to the remaining portions of the curve. It is likely that these *local* complications in the

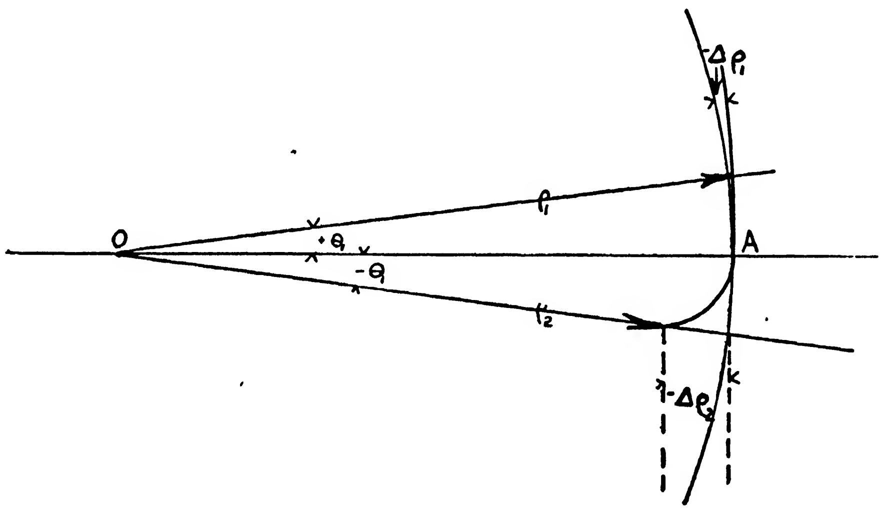

FIG. 3.

analytical structure of the integral curve at the beginning of this "relaxation" region, is the main reason why no explicit solution of the van der Pol equation (for example, in the form of a series) could be found as yet, in spite of nearly twenty years of effort, although considerable progress has recently been accomplished in connection with a qualitative analysis of this difficult problem (6).

CONCLUDING REMARKS

The most interesting feature of the VDP oscillator is the fact that its behavior is widely different for different values of the parameter ϵ.

At one end of the range ($\epsilon \ll 1$) the VDP oscillator differs very little from the harmonic oscillator and thus offers practically a perfect physical image of the latter. Probably it is not an exaggeration to say that, since the time of Galileo who introduced the concept of the

harmonic oscillator, we were able to actually observe a practically perfect physical image of the latter only when the VDP oscillator became available relatively a short time ago. In fact, very likely, there exists no better physical image of a simple harmonic motion than that which is produced by modern high quality electron tube oscillators, particularly those whose frequency is stabilized by quartz units. But this almost perfect physical image contains a germ of an immense complexity due to the presence of an infinite spectrum of frequencies with which the energy fluctuates between the oscillator and the source of energy. These fluctuations escape our observation, however, if ϵ is very small and, as the result of this, what is actually observed is only the constant term in the infinite array of terms (2.7). However, it is precisely this hidden complexity of vanishingly small energy fluctuations which permits obtaining a simple harmonic oscillation in its apparently pure form.

On the other end of the range ($\epsilon \gg 1$) the behavior of the VDP oscillator is radically different from that of a harmonic oscillator. If one has to look for an analogy, as far as energy fluctuations are concerned, in this range the VDP oscillator resembles a pneumatic hammer or a similar device in which the energy fluctuations (from absorption to dissipation and vice versa) occur in a quasi-discontinuous manner.

On the other hand, a closer investigation of the behavior of the oscillator reveals that in spite of this apparent difference of its behavior for different values of ϵ, there exists a feature common to all cases, namely the energy resumes *exactly* the same value after one period although during the period it fluctuates between the oscillator and the source with infinitely many frequencies of even harmonics. At times the energy is absorbed, at times it is dissipated but the time integrals of all these fluctuations with different frequencies over the period are zero and there remains only the average constant term around which these fluctuations occur. If these fluctuations are vanishingly small in comparison with the constant energy content of the VDP oscillator, it behaves approximately as a conservative system. If $\epsilon \gg 1$, on the contrary, the fluctuations of energy dominate everything else to such an extent that it becomes even impossible to answer the question: around which average constant value of energy do these fluctuations occur?

In addition to these fluctuations of energy, there exist also fluctuations in the angular velocity of the radius vector of the representative point moving on the integral curve. If ϵ is small enough these fluctuations are also small so that, from this standpoint, the motion on integral curves does not differ much from the corresponding motion for the harmonic oscillator in which case these fluctuations are rigorously zero. For an increasing ϵ these fluctuations in angular velocity become more and more pronounced and, for very large values of ϵ, the angular velocity becomes of the order $1/\epsilon^2$, that is very small, in the neighborhood of the

points at which the energy passes through a stationary value. In this neighborhood one observes the interesting phenomenon of a "relaxational" drain of energy from the system and results in a sharp bend in the integral curve.

In spite of a number of common features between a harmonic, and a VDP oscillator for $\epsilon \ll 1$, there exists a fundamental difference between the two oscillators, namely: a harmonic oscillator can oscillate with *any* energy content prescribed by the initial conditions, whereas the energy content of a VDP oscillator does not depend at all on these conditions. In fact, the oscillator "selects" so to speak its own energy content consistent with the form of the non-linear function $F(x)$ as well as with the parameters of the system. This remarkable peculiarity of the VDP oscillator, as we saw, is due to the fact that a stationary periodic motion in this case is possible only in the case when *all* secular terms are reduced to zero, if we wish to think in terms of the perturbation theory. The requirement for the elimination of secular terms in the infinite frequency spectrum of energy fluctuations thus imposes a requisite number of additional conditions owing to which only *one* periodic motion is possible, instead of infinity of such motions as in the case of a harmonic oscillator.

Acknowledgments

The author is indebted to Prof. M. Schiffer for valuable discussions of this matter.

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New Etching Process.—A revolutionary new etching process to prepare metal surfaces for microscopic examination and photography has been developed by the Ford Motor Company.

The unique metal-etching method—never before used in industry—utilizes ionized atoms. The process is known as cathodic vacuum etching.

Etching is widely used in industry to examine metal structures for determining quality and aiding in development of new materials or manufacturing processes.

Cathodic vacuum etching has brought an important improvement. It enables the taking of an unusually clear photograph of flow lines—a curious strata-like pattern which indicates the direction the metal flowed when forged under high pressure.

The new etching technique works this way: The metal sample is placed in a partial vacuum with argon, a rare gas found in the air. Twelve thousand volts of electricity are charged into the vacuum, creating argon ions which bombard the metal surface and knock off minute particles.

Under the newly developed method the physical features of metal can be examined under a microscope and measured accurately as to their effect on strength and performance in whatever part of the Ford cars the metal ultimately will become a fixture.

Flow lines in steel samples previously undetected now have been revealed by cathodic bombardment.

Comparison of the structure pattern produced by the best chemical etching method revealed that the new cathodic treatment brings out the true microstructure of the metal with greater detail and clarity.

A special pressure-rolled Ford front wheel spindle was examined experimentally by the usual microscopic technique. No superficial layer of flowed metal could be detected by the chemical etching process. However, cathodic bombardment etching revealed the entire superficially rolled structure, showing individual flow lines which indicated that deformation was only .005-inch deep.

Results also were obtained which indicate that grain structure of metals also can be more clearly shown by cathodic bombardment. Ford engineers have begun an extensive program to explore this phase of cathodic etching.

Photographs illustrating cathodic etched structures were awarded first prize in an exhibition sponsored by the American Society for Metals.

MINIMUM AND MAXIMUM SENSITIVITIES OF PHOTOGRAPHIC EMULSIONS *

BY

A. P. H. TRIVELLI [1]

INTRODUCTION

In 1940, Bruscaglioni (1)[2] investigated the minimum sensitivity of photographic plates by measuring the quantic absorption of the incident radiation. The present author has investigated the same problem by means of Silberstein and the author's quantum theory of exposure (2), which is an extension of Silberstein's original theory (3). The original theory was restricted to the study of 1-quantum grains, which are grains made developable by the effective absorption of one quantum, spread out in one layer on a flat surface. It was advisable to start with 1-quantum grains because the differences between the sensitivities measured by the wave theory and those determined by the quantum theory could be expected to be more pronounced with the one-quantic than with the many-quantic hypothesis of sensitivity. The only experimental evidence obtained, however, was found with X-ray exposures (4).

Because the measurements made with one-grain-layer coatings of experimental and commercial emulsions cannot be compared with the measurements made with multi-grain-layer coatings of these emulsions, the theory was extended so that the sensitivity of multi-grain-layer coatings could be measured, and the quantic sensitivity, r, be determined from 1 to ∞. The quantic sensitivity, r, denotes the minimum number of absorbed quanta required to make a grain developable. These quanta are called the "effective" quanta.

Definite indications of a minimum sensitivity were obtained empirically for certain groups of photographic materials. A general discussion will also be given of the conditions for approaching a maximum sensitivity of photographic emulsions.

THE EFFECTIVE SENSITIVITY

In the quantum theory of exposure, the sensitivity of a grain is defined as its probability (p) of becoming developable by the absorption of at least r quanta. From the beginning, p has represented the probability that a single grain of size, a, will absorb effectively a single quantum projected upon a plate coated with one layer of grains. This probability is written ϵa. No new assumption was introduced in the

* Communication No. 1243 from the Kodak Research Laboratories.

[1] Kodak Research Laboratories, Eastman Kodak Company, Rochester, N. Y.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

theory by this expression; the probability per unit projective area of the grain was merely denoted by ϵ. It was not assumed that ϵ was dependent upon the grain size, a.

It is clear that ϵ/r represents the probability of a single grain of size, a, becoming developable per quantum effectively absorbed. This holds true for single grains and also for an assemblage of grains of average grain size, $\bar{a}$, having constant r- and constant ϵ-values (5).

Quantic sensitivity measurements of a number of experimental photographic emulsions have shown that their fog-corrected characteristic curves can be represented by single-term or double-term equations. The single-term equation (2) is given by the general expression

$$D(y) = D_{\max} F(y), \tag{1}$$

in which D, $D_{\max}$, and y represent density, maximum density, and exposure, respectively.

THE MINIMUM SENSITIVITY

The first data investigated consisted of forty-two fog-corrected single-term characteristic curves of pure silver bromide emulsions, chosen at random from converted and non-converted, unsensitized and sensitized emulsions, developed in different developers for different times. The sensitivity constants ranged from $\epsilon/r \times 10^4 = 0.9$ to 14.4, and, correspondingly, from $10/i = 1.2$ to 280.0, in which i is the inertia point of the $D/\log E$ curve and $10/i$ the well-known inertia speed of photographic materials. The Bravais-Pearson correlation coefficient, k, between the ϵ/r- and $10/i$-values was found to be equal to 1.0. The linear equation which was calculated by the method of least squares in terms of $x = 10/i$ and $y = \epsilon/r \times 10^4$ is:

$$y = 0.0521x + 0.6,$$

which gives

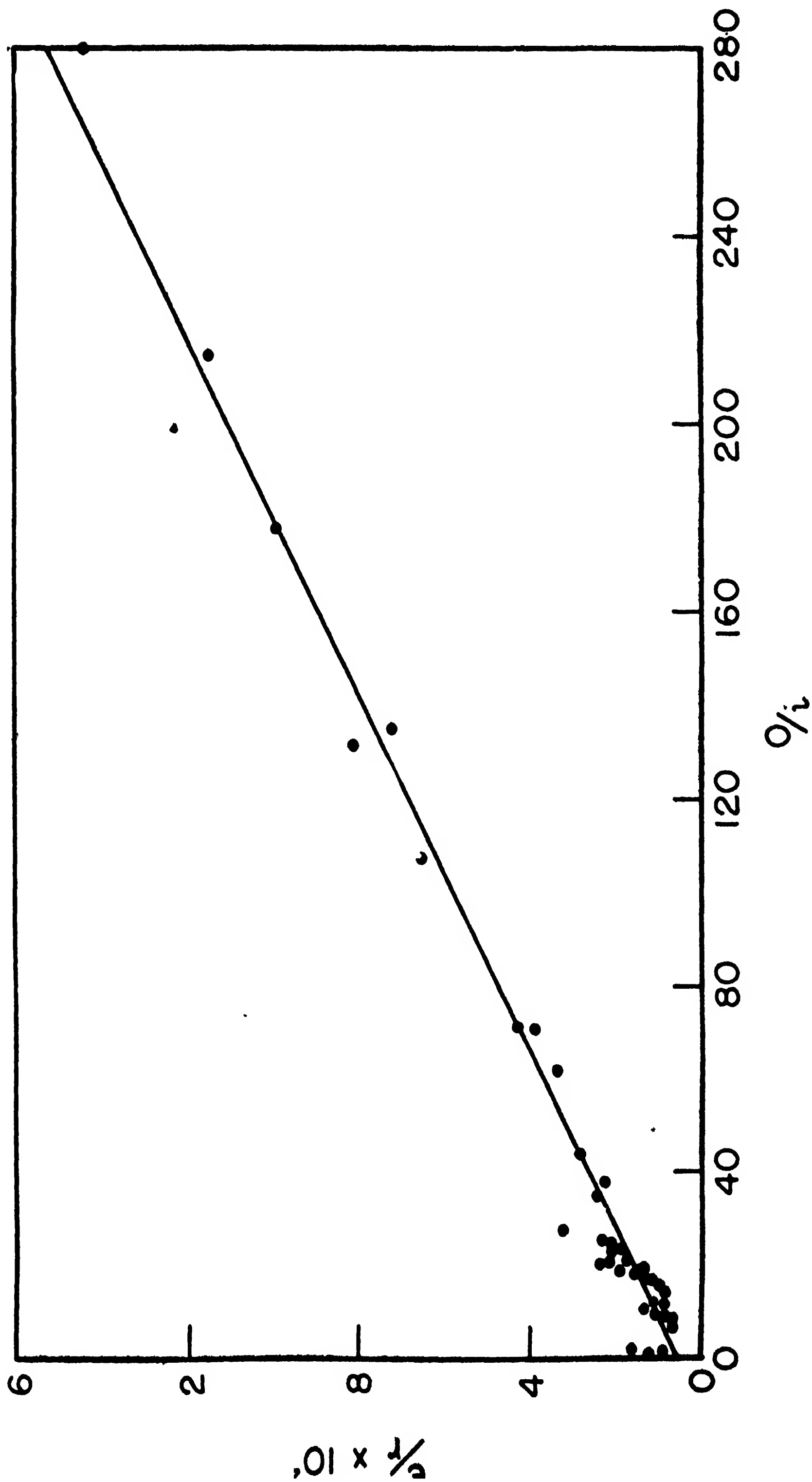
$$[\epsilon/r]_{\min} = 6 \times 10^{-5}.$$

A similar investigation was carried out with the fog-corrected characteristic curves of experimental silver bromoiodide emulsions containing 2.56 moles per cent of silver iodide which were representable by the same general equation (1). However, only a few emulsions of this type were available; not more than eleven curves could be collected. The sensitivity measurements ranged from $\epsilon/r \times 10^4 = 0.28$ to 11.70, and correspondingly, from $10/i = 6.2$ to 580.0. The correlation coefficient for $x = 10/i$ and $y = \epsilon/r \times 10^4$ is also 1.0. The linear relationship which was calculated by the method of least squares is:

$$y = 0.0207x + 0.4,$$

which gives

$$[\epsilon/r]_{\min} = 4 \times 10^{-5}.$$


0
40
80
120
160
200
240
280

The straight line calculated for $\epsilon/r \times 10^4$ and $10/i$ for the silver bromide emulsions, together with the observed data, is shown in the figure.

As $10/i$ approaches zero, ϵ/r approaches a positive minimum value of 6×10^{-5} with pure bromide emulsions, and a positive minimum value of 4×10^{-5} with bromoiodide emulsions.

The more complicated emulsions which are representable by two-term equations were found to be unsuitable for this investigation.

The expression, ϵa, can be regarded as representing a small part of the surface, a, which may be called the "effective area" of the grain that produces the latent image. From the minimum value of $\epsilon/r = 4 \times 10^{-5}$ for silver bromoiodide emulsions, the minimum effective area, ϵa, of the grains of a two-quantum high-speed emulsion was calculated. The average projective area per grain of such an emulsion was found to be 0.9×10^{-8} sq. cm., corresponding to a minimum effective area of 3.6×10^{-13} sq. cm., or 3600 square Å per quantum. If we suppose that this area is concentrated in a single square unit, the side of the square would be 60 Å., or, in terms of the atoms present in the grains, about 12 atoms. The total number of atoms in the effective surface is, therefore, of the order of a hundred. Because this emulsion has a two-quantic sensitivity, the effective surface will be twice as large.

It is interesting to compare this estimate of the minimum number of atoms in the effective surface with W. F. Berg's (6) estimate of the number of molecules of silver sulfide, about ten.

THE MAXIMUM SENSITIVITY

In 1926, Jones and Sandvik (7) published an investigation of the spectral distribution of the sensitivity of photographic materials. They showed that sensitivity gradually increases from the longer waves of the visible region of the spectrum to the far-ultraviolet region. These results are of interest in connection with the following considerations.

For the same frequency of radiation, the quantum sensitivity of a photographic emulsion layer increases with decreasing r, but the quantum energy increases proportionally to the frequency of the waves. In addition, as previously mentioned by the present author (8), ϵ also represents the effective absorption coefficient of the incident radiation during exposure. Its maximum value is limited by the absorption coefficient of the silver halide of the grain, which depends upon the wavelength of the incident radiation. Higher values of the absorption coefficient, therefore, will offer possibilities of higher ϵ-values. According to Eggert and Noddack (9), the absorption of silver bromide increases from 10 to 20 per cent between $\lambda = 436$ mμ and $\lambda = 365$ mμ. Measurements for shorter wavelengths are not known to the author. However, a series of photomicrographs of silver bromide grains at $2500 \times$ and $1250 \times$ enlargements which were made by the present

author (10) at $\lambda = 450, 365, 312$, and 275 mμ show beyond doubt a rapid increase of absorption with shorter wavelengths, indicating that this increase is much greater than the energy increase of the absorbed quanta at these different wavelengths.

The highest possible value of ϵ is one. This can be approached only if the absorption coefficient of silver halide is also one. Therefore, the highest values of ϵ can occur in the far-ultraviolet region. We may, therefore, expect that the maximum sensitivity of photographic materials occurs in the far-ultraviolet region with $r = 1$.

Jones and Sandvik (7) have found that gamma and maximum gamma increase with radiation of longer wavelengths. It appears, therefore, that the approach to the maximum sensitivity of photographic material may be connected with a disappearance of gamma. This is intensified by the greater scattering of radiations in turbid media with the shorter wavelength.

SUMMARY

1. It was found empirically that in emulsion coatings producing characteristic curves representable by single-term quantic equations, there is a straight-line relation between ϵ/r and $10/i$.

2. This relation shows a definite minimum value of $\epsilon/r = 6 \times 10^{-5}$ for silver bromide and of $\epsilon/r = 4 \times 10^{-6}$ for silver bromoiodide emulsion layers containing 2.56 moles per cent of silver iodide.

3. With the data obtained for simple emulsions, the minimum size of the effective surface of the silver bromoiodide grains of a commercial emulsion were calculated and found to be of the order of 100 to 200 atoms.

4. Maximum sensitivity is most likely to be reached in the far-ultraviolet region.

Acknowledgment

The author wishes to express his thanks to Miss Helen M. Menihan for her collaboration during this investigation, and to Dr. W. West for giving suggestions concerning the determination of the minimum size of the effective surface of the grains of the commercial emulsion used.

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Robot Announcer Keeps Factory Abreast of the News.—A robot "announcer," on duty 24 hours a day, is keeping management employes of the Westinghouse Electric Corporation, East Pittsburgh, Pa., right up to the minute on company news. By dialing a special number on their in-plant telephones supervisory employes can listen in while a continuous tape reels off the latest developments.

Designed to keep supervisors at all levels abreast of policy matters, labor relation matters, new orders secured, production loads, backlogs, outstanding services rendered customers, quality of Westinghouse products and its effect on increasing sales, the robot announcer is saving considerable time for plant executives, T. I. Phillips, Westinghouse vice president, reported. "It now is possible to pass on a greater volume of information without any individual having to repeat the same story over and over. It's accurate too because the robot never forgets a fact or a phase."

The system works as follows: A script is prepared on the subject to be discussed, then recorded on a standard tape recorder and later transferred to a circular metallic tape which runs continuously. A system of electrical circuits amplify the message and channel it into the plant's telephone system where 30 calls can be handled at the same time—more than that get a busy signal.

The average message runs from 3 to 5 minutes in duration and new recordings are made two or three times a week. At the end of the "broadcast" an announcement is made on the time a new message may be expected. Because it takes less than half an hour to make a recording, important news can be covered quickly.

Stearns Permanent Magnetic Pulley (*Rubber Age*, Vol. 65, No. 3).—A permanent magnetic pulley which incorporates a special design of Alnico metal poles, self-energized and requiring no electrical current, was recently announced by the Stearns Magnetic Manufacturing Co., Milwaukee, Wis. The pulley is applicable for problems involving automatic removal of tramp iron from raw or processed materials, for separation of magnetic and non-magnetic products, for reclaiming secondary metals, and protection of machinery grinders, pulverizers or similar equipment. It is designed for uniform high magnetic strength over the entire face of the pulley and because no wiring or electrical connections are needed the pulley can be operated under all atmospheric and temperature conditions. More than fifty sizes are available from 12-in. diameter by 12-in. width to 30 in. by 60 in.

New Electrical Laminate (*Modern Plastics*, Vol. 26, No. 11).—A series of heavy-duty Fiber-glas reinforced Glastic electrical insulating laminates having all the requirements of high-impact class B insulating plate materials, plus flame resistance, has been announced by Laminated Plastics, Inc., Cleveland, Ohio. The new materials possess a unique combination of high rigidity, impact strength, arc and flame resistance, heat resistance, and low moisture absorption, according to the manufacturer, and are said to have already found wide use in heavy-duty apparatus by several manufacturers of electrical power equipment. Glastic plate materials are available in standard thicknesses from $\frac{1}{32}$ to $1\frac{1}{2}$ in., and in standard sheets 36 by 48 in.

STRESS-STRAIN RELATIONS IN THE PLASTIC RANGE FOR BIAXIAL STRESSES*

BY

JOSEPH MARIN[1]

1. SUMMARY

This paper gives the results of an experimental investigation on thin-walled aluminum alloy tubes subjected to combined tension and torsion. A specially designed testing machine was constructed for obtaining plastic load-strain relations to rupture for various ratios of the principal stresses ranging from 0 to -1.0. During each test, the ratios of the principal stresses were maintained essentially constant. Nominal stress-strain diagrams for the elastic range and true stress-strain diagrams for the plastic range were plotted for each test and each stress ratio.

Values of yield strength, ultimate strength, fracture strength, ductility, and plastic stress-strain relations for the various biaxial stress ratios investigated were compared with theoretical values based upon uniaxial tension results. The yield strength values were found to be in good agreement with values predicted by the Von Mises-Hencky Distortion Energy Theory. The nominal ultimate strengths, fracture strengths, ductility, and plastic stress-strain relations were approximately in agreement with the values predicted by the distortion energy theory using St. Venant's plasticity relations.

2. INTRODUCTION

The need in machine and structural design for further information on biaxial plastic stress-strain relations has led, in recent years, to many experimental investigations. The majority of these studies have dealt with plastic stress-strain relations in which both the biaxial stresses have been tensile. There have been relatively few experimental investigations on biaxial tension-compression plastic stress-strain relations (1).[2] The present paper gives the results and interpretations of combined tension-compression tests on an aluminum alloy designated as Alcoa 24S-T. Biaxial tension-compression stresses were obtained by subjecting a thin-walled tubular specimen to combined axial tension and torsion. By means of these tension-torsion tests, the influence of the biaxial stress ratio on the yield strengths, ultimate strengths, fracture strengths, ductility, and plastic stress-strain relations was determined.

3. DESCRIPTION OF MATERIAL

The material tested was a fully heat-treated aluminum alloy (Alcoa 24S-T) supplied in tubular extrusions with an internal diameter of 1 in.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

and a wall thickness of $\frac{1}{4}$ in. The nominal chemical composition, in addition to aluminum and normal impurities, consisted of 4.4 per cent copper, 1.5 per cent magnesium, and 0.6 per cent manganese. The mechanical properties, as furnished by the manufacturer, were: tensile strength (nominal ultimate) 68,000 psi., yield strength (0.2 per cent offset) 44,000 psi., modulus of elasticity 10.6×10^6 psi., percentage elongation (in 2 in.) 14 per cent, and Poisson's ratio 0.33.

Tension tests were made on tubular specimens of the same dimensions as used for the combined stress tests (Fig. 1). The nominal and

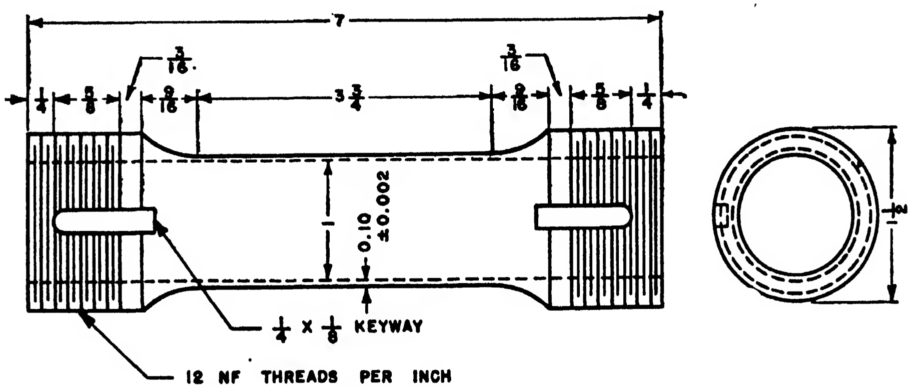

FIG. 1. Tension-torsion specimen.

true average values of the mechanical properties for three tension tests are given in Table I. In plotting the average true stress-strain diagram

TABLE I.—*Mechanical Properties of Alcoa 24S-T Tubing in Simple Tension.*

Nominal values	Yield strength (0.2% Offset)	48.1×10^3 psi.
	Ultimate strength	72.5×10^3 psi.
	Modulus of elasticity	10.7×10^6 psi.
	% Elongation (2″ gage length)	11.5
True values	Fracture strength	81.0×10^3 psi.
	Ductility	0.109
	Strength coefficient k	1.14×10^5 psi.
	Strain-hardening coefficient n	0.175

for tension, as shown in Fig. 4, the true stress and true strain were defined by the equations

$$\sigma = \frac{P}{A} = \frac{P}{A_0}(1 + e_0), \tag{1}$$

$$\epsilon = \log_e (1 + e_0), \tag{2}$$

where e_0 = the nominal strain,
P = the axial load, and
A_0 = the original cross-sectional area.

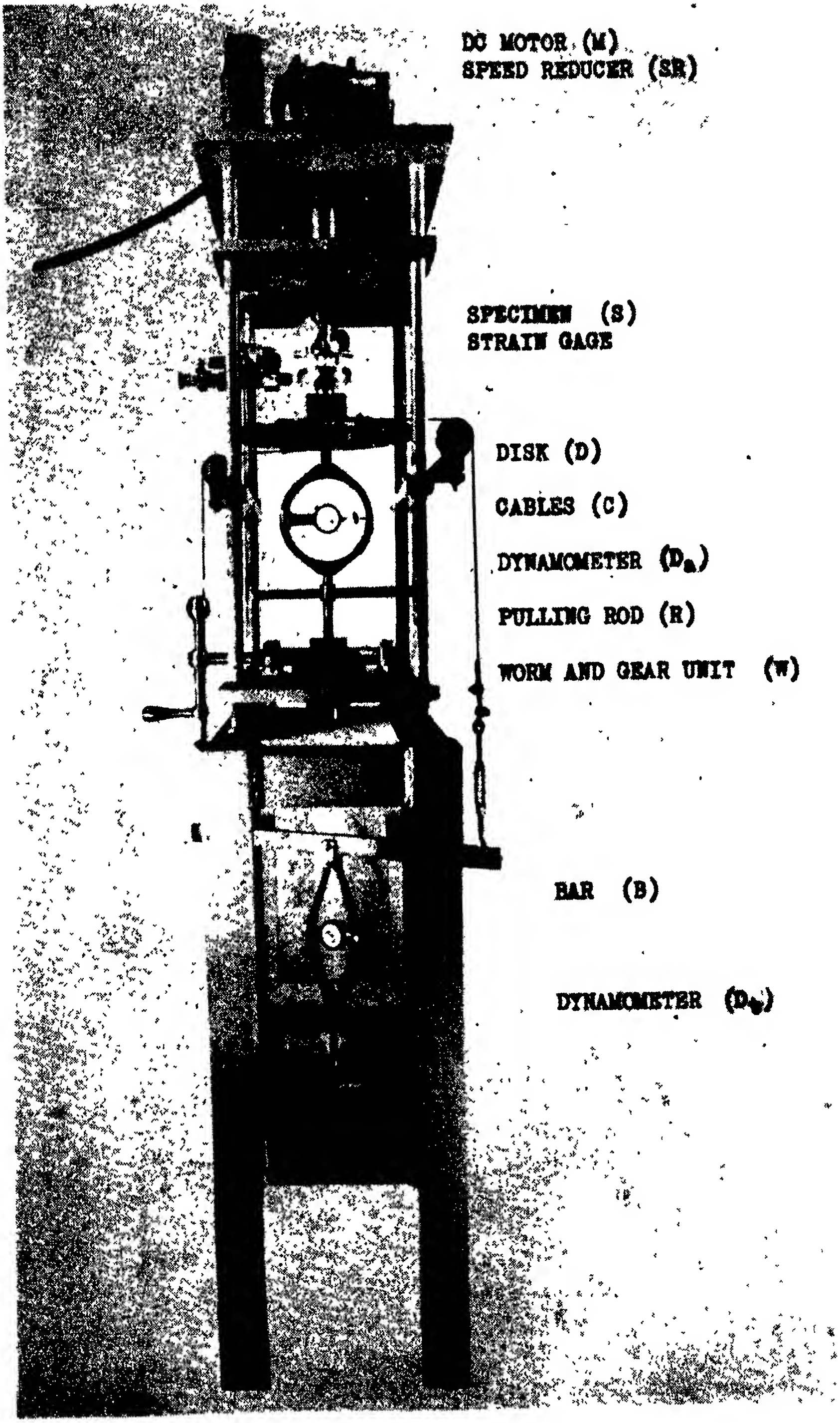

FIG. 2. Combined stress tension-torsion testing machine.

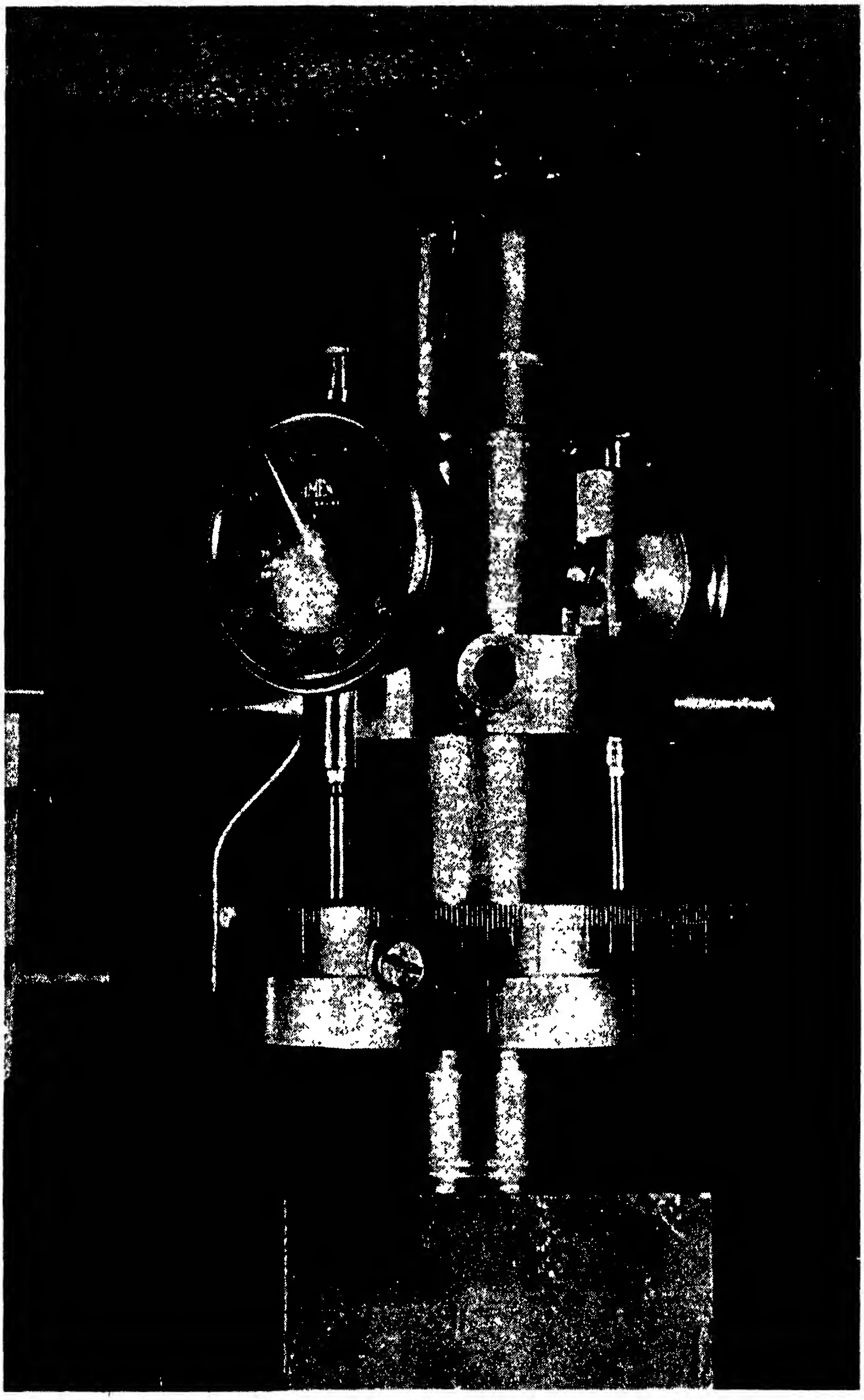

FIG. 3. Strain gage for measuring axial strain and angle of twist.

For the plastic range, many tension tests show that there is an approximate linear relation between the true stress and true strain when plotted on a log-log basis. That is, the true stress-true strain relation for ductile metals in simple tension is approximately defined by

$$\sigma = k\epsilon^n, \tag{3}$$

where k and n are experimental constants. The average values of k and n for the aluminum alloy tested are given in Table I.

4. COMBINED TENSION-TORSION TEST RESULTS

The *biaxial-stress test specimens* shown in Fig. 1 were machined from tubular sections. The dimensions were selected so that the stresses were essentially uniform throughout the wall and so that failure by buckling did not occur.

A *special testing machine* was designed for subjecting the tubular specimens to combined axial tension and torsion (Fig. 2). The axial tension load is applied to the specimen (S) by means of a worm and gear unit (W) through a pulling rod (R). The axial load is measured by a 10,000-lb. dynamometer (D_a). The twisting moment is applied by a $\frac{1}{4}$-hp. d-c. motor (M) and speed reducer (SR). Variation in speed of torque loading is provided by a rheostat of a motor-generator set. A disk (D) is attached to the lower part of the specimen to which cables are connected passing over frictionless pulleys to a bar (B). A 2000-lb. dynamometer (D_t) is attached to the bar (B) to measure the torque load.

For the *measurement of strains*, a special strain gage was designed, as shown in Fig. 3. This special gage measures both the axial strains and angles of twist in the elastic and plastic ranges. The axial strains were measured for a 2-in. gage length by two 0.0001-in. dials placed 180° apart. The angles of twist were measured for a 2-in. gage length by the twistmeter part of the gage shown in Fig. 3. Attachment of the strain gage to the specimen in the plastic range to rupture was maintained by rods which bear on the specimen at one end and are connected by preloaded springs to the strain gage at the other end. By the correct initial spring adjustment, attachment of the gage without slipping was maintained throughout the plastic range even with considerable reduction in diameter of the specimen.

The *test procedure* consisted essentially in applying torsion and tension loads of predetermined amounts corresponding to a selected value of the nominal principal stress ratio. At selected load intervals, readings of the axial strains and angles of twist were recorded for both the elastic and plastic range and up to fracture.

The test results for various principal stress ratios (σ_2/σ_1) from 0, or simple tension, to -1.0, or pure torsion, are represented by the nominal stress-strain curves in Figs. 4 and 5. In Fig. 4, the relations between the nominal axial stress σ_x' and nominal axial strain e_x are given

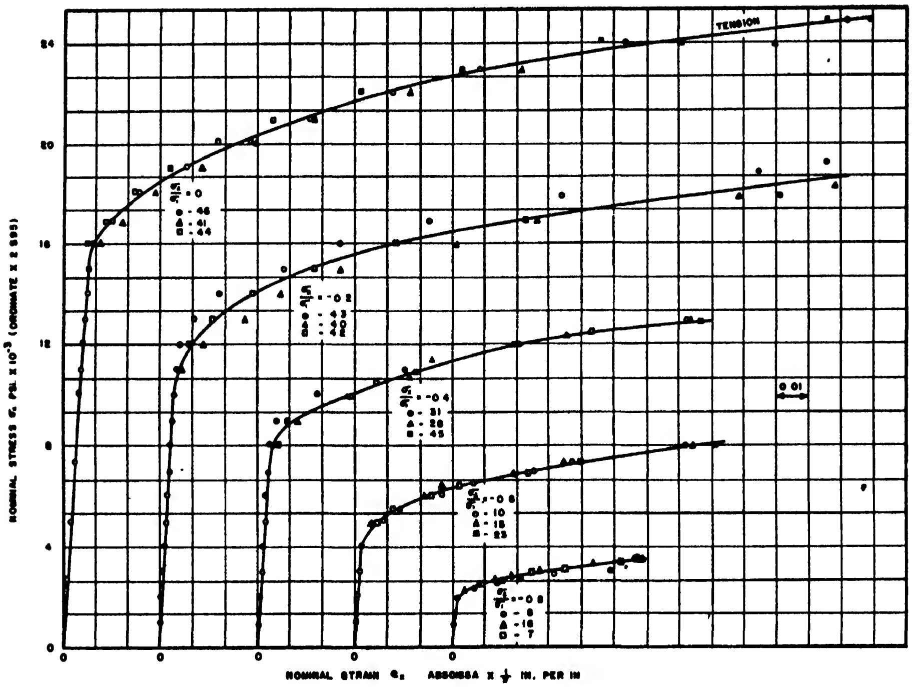

FIG. 4. Relation between normal nominal stress and strain.

NOTES: For all data in the elastic range the small circles are the average of three test results. Numerals opposite symbols are specimen numbers.

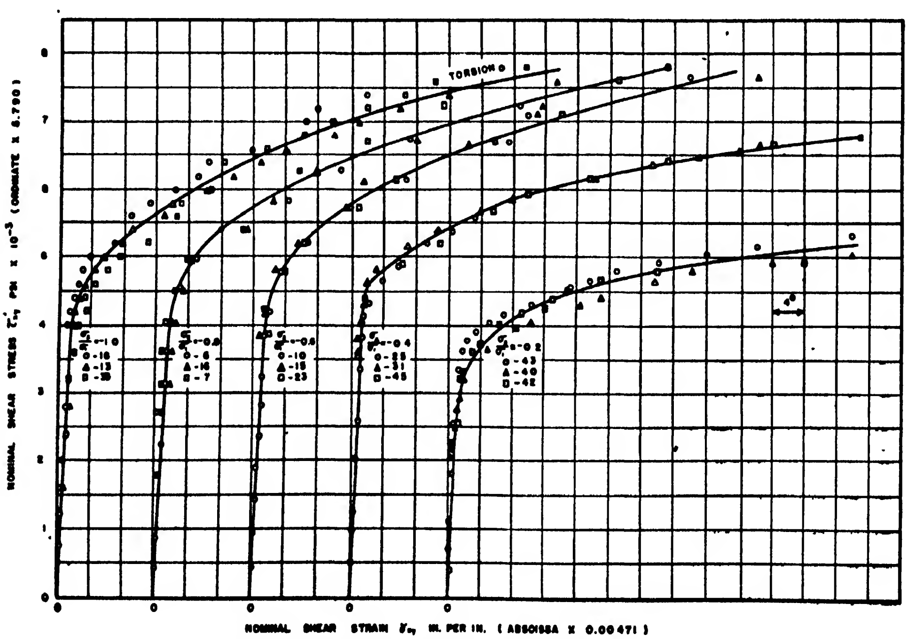

FIG. 5. Relation between nominal shear stress and nominal shear strain.

NOTES: For all data in the elastic range the small circles are an average of three test results. Numerals opposite symbols are specimen numbers.

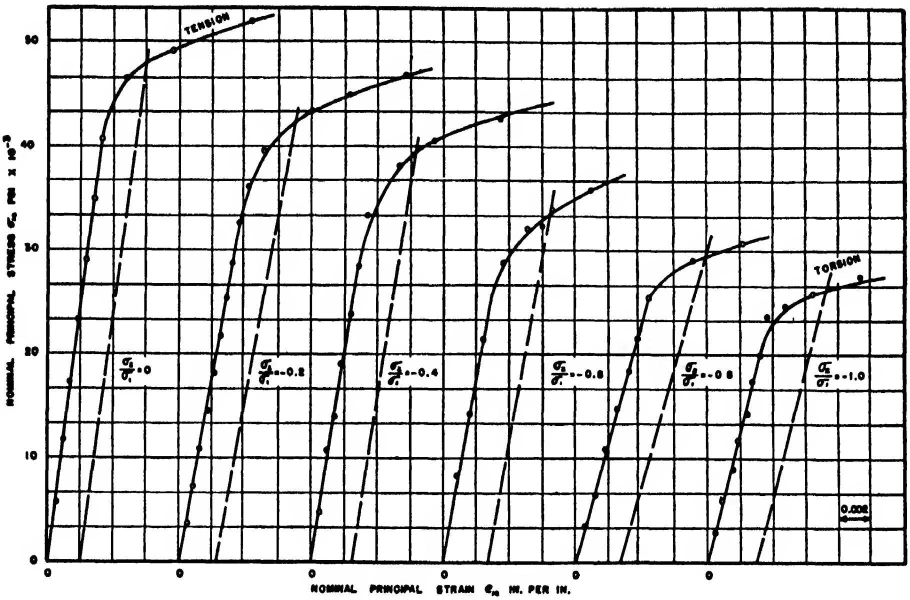

FIG. 6. Elastic stress-strain relations between principal stress σ_{1e} and principal strain e_{1e} for various biaxial stress ratios.

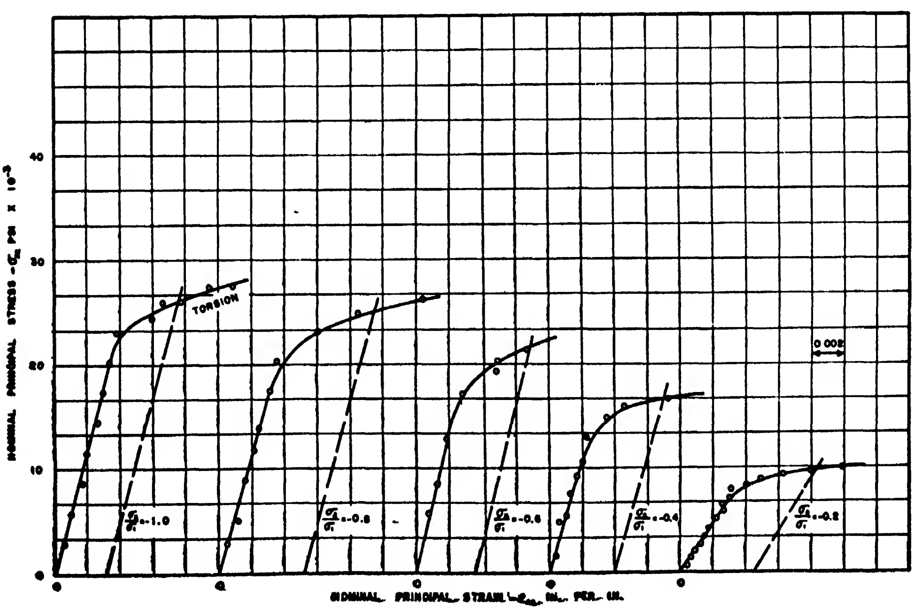

FIG. 7. Elastic stress-strain relations between principal stress $-\sigma_{2e}$ and principal strain $-e_{2e}$ for various biaxial stress ratios.

for various principal stress ratios. In Fig. 5, the relations between the nominal shear stress τ_{xy}' and nominal shear strain γ_{xy} are shown. The approximate values of the nominal axial and shearing stresses used in plotting Figs. 4 and 5 are, respectively,

$$\sigma_x' = \frac{P}{\pi t(d + t)}, \tag{4}$$

$$\tau_{xy}' = \frac{2M_t}{\pi dt(d + t)}, \tag{5}$$

where P = the axial load,
M_t = the twisting moment,
d = the internal diameter of the specimen, and
t = the wall thickness.

The nominal axial and shear strains, in terms of the measured strains are, respectively,

$$e_x = \frac{\delta_{1x} + \delta_{2x}}{2L_0}, \tag{6}$$

$$\gamma_{xy} = \frac{\pi}{720}(d + 2t)\rho, \tag{7}$$

where δ_{1x}, δ_{2x} = the measured axial strains on two sides of the specimen,
L_0 = the gage length, and
ρ = the angle of twist in degrees.

5. ANALYSIS AND INTERPRETATION OF TEST RESULTS

Elastic Stress-Strain Relations

To determine the stresses defining elastic failure and to compare with values predicted by the theories of failure, values of the nominal principal stresses and principal strains are plotted as shown in Figs. 6 and 7. The values in Figs. 6 and 7 are based on the stress and strain components shown in the initial part of the curves in Figs. 4 and 5. The values of the nominal principal stresses and strains have been shown to be, respectively,

$$\left.\begin{matrix}\sigma_{1e}\\ \overline{\sigma_{2e}}\end{matrix}\right\} = \frac{\sigma_x'}{2} \pm \sqrt{\left(\frac{\sigma_x'}{2}\right)^2 + (\tau_{xy}')^2}, \tag{8}$$

$$\left.\begin{matrix}e_{1e}\\ \overline{e_{2e}}\end{matrix}\right\} = \frac{e_x + e_y}{2} \pm \tfrac{1}{2}\sqrt{(e_x - e_y)^2 + \gamma_{xy}^2}. \tag{9}$$

To define elastic failure or yielding, the yield strength in simple tension will be based on an offset strain of 0.002 in. per in., as shown in Fig. 6. For combined stresses an equivalent offset strain value (2),

equivalent to the value of 0.002 in. per in. for simple tension, will be used. The equivalent offset strain e_{01} and e_{02} corresponding to the principal strains e_{1e} and e_{2e} are (2)

$$e_{01} = \frac{\sigma_y'}{E}\left(1 - \frac{1 - m\alpha}{\sqrt{1 - \alpha + \alpha^2}}\right) + e_t, \tag{10a}$$

$$e_{02} = \frac{\sigma_y'}{E}\left(1 - \frac{\alpha - m}{\sqrt{1 - \alpha + \alpha^2}}\right) + e_t, \tag{10b}$$

where σ_y' = the yield strength in simple tension based on an offset strain e_t = 0.002 in. per in.,
E = the modulus of elasticity in tension,
m = Poisson's ratio, and
α = the stress ratio.

By Eqs. 10a and 10b the offset strains e_{01} and e_{02} can be calculated for each principal stress ratio α. The offset strains so calculated are used to determine the yield stresses σ_{2y} and σ_{1y} as shown in Figs. 6 and 7. The values of these yield stresses, representing the average of three tests, are given in Table II. To compare the biaxial yield strength values with values predicted by the theories of failure, the principal stress ratios $x = \sigma_{1y}/\sigma_y'$ and $y = \sigma_{2y}/\sigma_y'$ are plotted as shown in Fig. 8. Figure 8 also shows the relation between the stress ratios x and y, as defined by the shear, distortion energy, and stress theories. A comparison between the test results and the theories of failure shows that the test results are in closest agreement with the distortion energy theory.

TABLE II.—*Biaxial Yield, Ultimate and Fracture Strengths for Various Biaxial Stress Ratios.*

NOTE: Each value recorded is based on an average of three tests. Stress values given in table are in psi. $\times 10^{-3}$. Stress ratios $x = \sigma_1/\sigma$ and $y = \sigma_2/\sigma$.

$\alpha = \sigma_2/\sigma_1$	Yield Strength				Nominal Ultimate Strength				True Fracture Strength			
			Stress Ratios				Stress Ratios				Stress Ratios	
	σ_{1y}	σ_{2y}	x	y	σ_{1u}	σ_{2u}	x	y	σ_{1r}	σ_{2r}	x	y
0 (Simple tension)	48.1	0	1.0	0	72.5	0	1.0	0	81.0	0	1.0	0
−0.2	42.4	− 9.9	0.88	−0.21	67.0	−13.4	0.93	−0.18	67.9	−14.0	0.84	−0.17
−0.4	39.4	−16.8	0.82	−0.35	59.8	−23.5	0.83	−0.32	63.5	−25.4	0.79	−0.31
−0.6	33.5	−21.7	0.70	−0.45	55.8	−33.4	0.77	−0.46	59.4	−35.8	0.73	−0.44
−0.8	29.6	−24.3	0.62	−0.51	50.6	−40.4	0.70	−0.56	51.0	−40.9	0.63	−0.51
−1.0 (Pure torsion)	26.3	−26.3	0.55	−0.55	44.8	−44.8	0.62	−0.62	44.8	−44.8	0.55	−0.55

Plastic Stress-Strain Relations

To determine the plastic stress-strain relations, the changes in the dimensions of the specimens must be considered. In the plastic range these changes in dimensions are appreciable and the stresses and strains can no longer be based on the original dimensions and gage length.

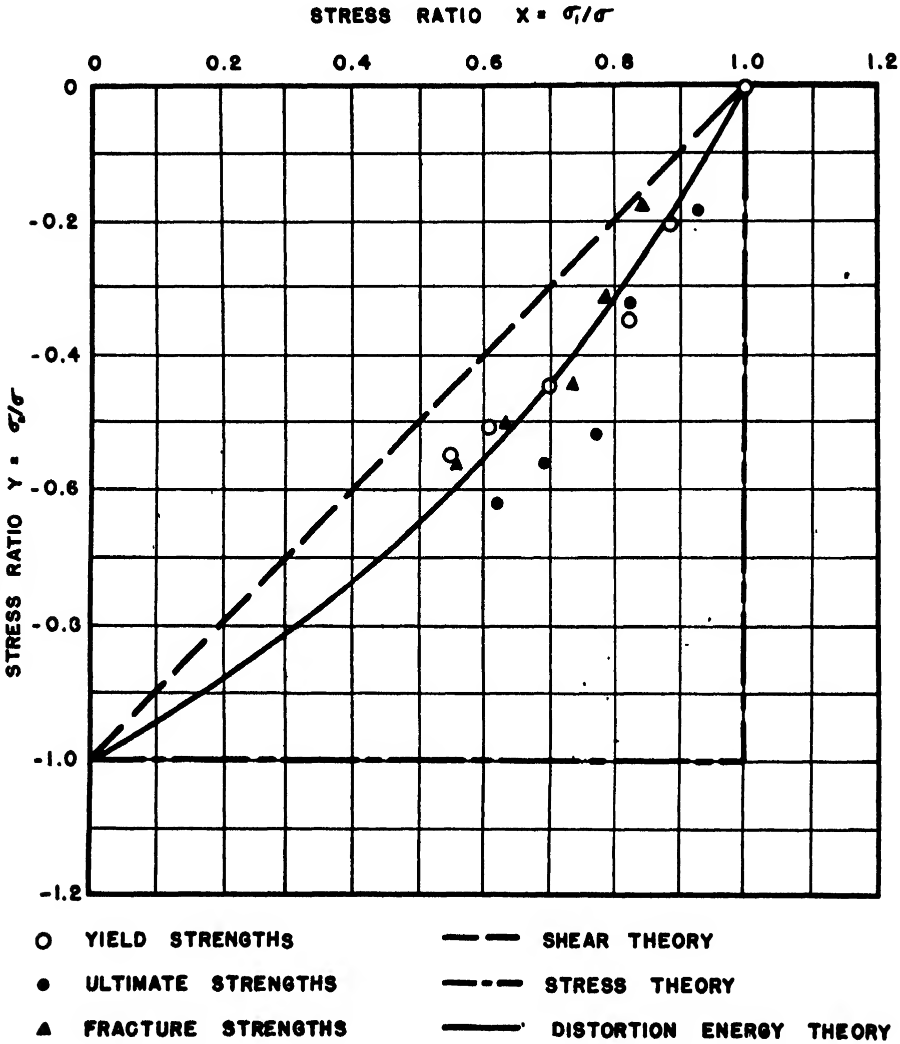

FIG. 8. Comparison of actual and theoretical strengths for various ratios of the principal stresses from 0 to −1.0.

True stresses and strains based on change in dimensions must now be used in place of the nominal values given by Eqs. 8 and 9.

The true stress components σ_x and τ_{xy} can still be defined by Eqs. 4 and 5 provided the original values of d and t in these equations are replaced by their true values.

To determine the true value of wall thickness (t_p) we can write, using Eq. 2, the true strain in the direction of the wall thickness as

$$\epsilon_z = \log_e (1 + e_z) = \log_e \frac{t_p}{t}. \qquad (a)$$

If the volume is assumed to remain constant in the plastic range, $\epsilon_x + \epsilon_y + \epsilon_z = 0$. Since $\sigma_y = 0$ and $\sigma_2 = 0$, then $\epsilon_y = - \epsilon_z$ or

$$\epsilon_z = - \frac{\epsilon_x}{2} = - \tfrac{1}{2} \log_e (1 + e_x). \qquad (b)$$

From Eqs. *a* and *b* the wall thickness t_p in terms of the original wall thickness is

$$t_p = t/(1 + e_x)^{\frac{1}{2}}. \qquad (c)$$

The *true value of the diameter* (d_p) is given by

$$\epsilon_z = \log_e d_p/d. \qquad (d)$$

From Eqs. *b* and *d*,

$$d_p = d/(1 + e_x)^{\frac{1}{2}}. \qquad (e)$$

Replacing the values of *d* and *t* in Eqs. 4 and 5 with d_p and t_p from Eqs. *c* and *e*, the true stress components σ_x and τ_{xy} are

$$\sigma_x = \frac{P}{\pi t(d + t)} (1 + e_x) = \sigma_x'(1 + e_x), \qquad (11)$$

$$\tau_{xy} = \frac{2M_t}{\pi dt(d + t)} (1 + e_x)^{\frac{3}{2}} = \tau_{xy}'(1 + e_x)^{\frac{3}{2}}. \qquad (12)$$

The *true principal stresses* corresponding to the true stress components σ_x and τ_{xy} are

$$\left.\begin{matrix}\sigma_1 \\ \sigma_2\end{matrix}\right\} = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}. \qquad (13)$$

The directions of the planes of principal stresses are defined by the values

$$\left.\begin{aligned}\theta_1 &= \frac{1}{2}\left[\tan^{-1}\left(\frac{-2\tau_{xy}}{\sigma_x}\right)\right] \\ \theta_2 &= \frac{\pi}{2} + \theta_1.\end{aligned}\right\} \qquad (14)$$

The values of θ_1 and θ_2 determine the direction of the planes perpendicular, respectively, to the stresses σ_1 and σ_2.

The *true principal strains* will be defined as the strains in the direction of the true principal stresses. The values of the principal strains in terms of the nominal strains e_x and γ_{xy} are defined by Eqs. 24 and 22 given in Appendix 1. In calculating the true principal stresses and strains, values of the nominal stress and strain components are selected

from the average curves shown in Figs. 4 and 5. The values of the true principal stresses and strains computed in this manner are plotted in Figs. 9 and 10. The principal stress ratios shown on the curves of Figs. 8 and 9 represent approximate values, since these ratios are based

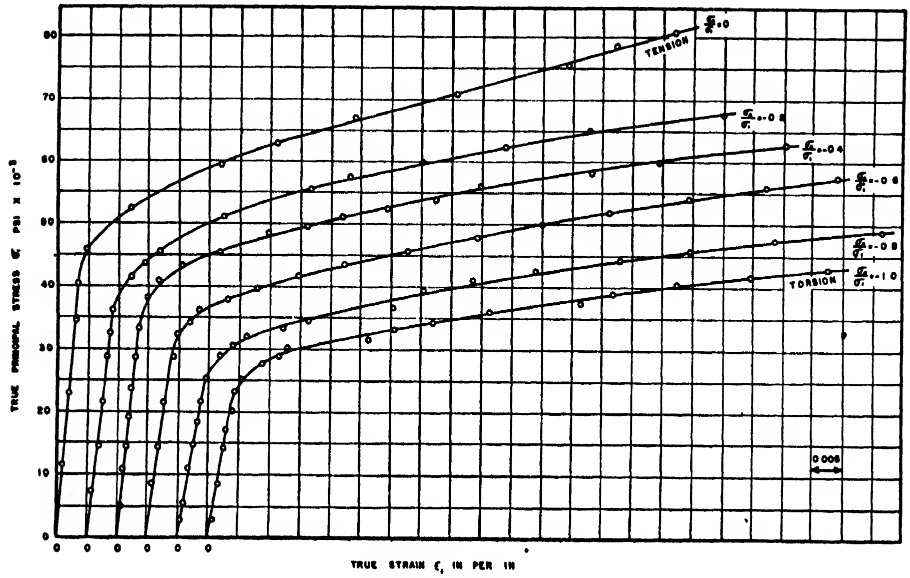

FIG. 9. Relation between true principal stress σ_1 and true strain ϵ_1 for various biaxial stress ratios.

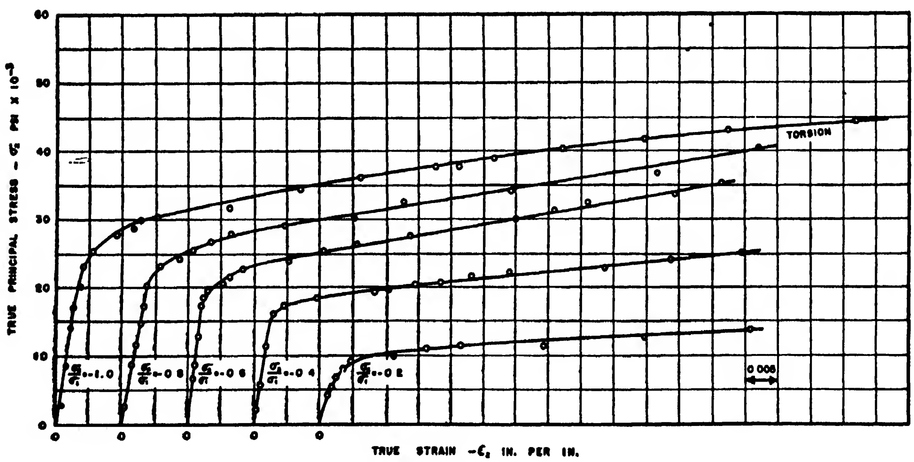

FIG. 10. Relation between true principal stress $-\sigma_2$ and true strain $-\epsilon_2$ for various biaxial stress ratios.

on nominal stress calculations. With the test results in the form of true principal stresses and strains, it is now possible to compare the test results with theoretical values.

Several theories have been proposed to define the true plastic combined stress-strain relations in terms of the stress-strain relation for

simple tension. These theories are of two types, either flow theories or deformation theories, as discussed in several papers including those by Ilyushin (3), Handelman, Lin and Prager (4), and Prager (5, 6). In the loading condition considered in this investigation, and for tests in which the ratio of the principal stresses remains essentially constant, the flow and deformation theories agree. Most of the recent experimental investigations on plastic combined stress-strain relations have used the deformation type theory for interpretation of the test results.

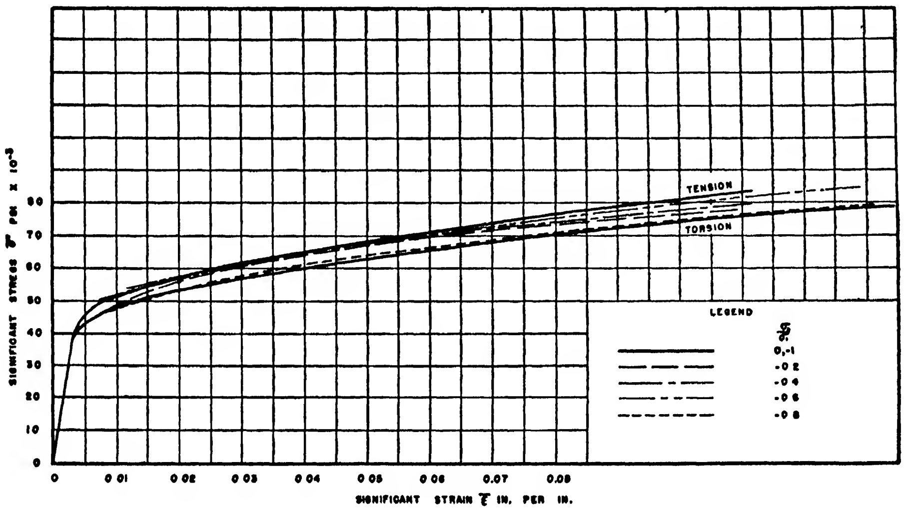

FIG. 11. Comparison of significant stress-significant strain relations for various biaxial stress ratios.

The following interpretation of the plastic stress-strain relations will utilize the deformation type theory based on distortion energy since this theory was found to be in better agreement with the test results than the shear theory (9). The theory used defines a stress and strain, called the significant stress and strain (7, 8, 9).[3] For biaxial stresses the values of the significant stress and strain have been shown to be (10)

$$\bar{\sigma} = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}, \tag{15}$$

$$\bar{\epsilon} = \sqrt{\frac{4}{3}(\epsilon_1^2 + \epsilon_1\epsilon_2 + \epsilon_2^2)}. \tag{16}$$

Using the values of the principal stresses (σ_1, σ_2) and strains (ϵ_1, ϵ_2) shown in Figs. 9 and 10 in Eqs. 15 and 16, the significant stresses and strains can be determined and the values plotted as shown in Fig. 11. For simple tension, by Eqs. 15 and 16, $\bar{\sigma} = \sigma_t$ and $\bar{\epsilon} = \epsilon_t$. That is, by this theory, the significant stress-strain relation for simple tension should coincide with the true stress-strain diagram for simple tension.

[3] The significant stress and strain are equivalent to the effective stress-strain and octahedral shear stress-strain as defined in various reports.

Figure 11 shows that there is an approximate agreement between the significant stress-strain relations for the various biaxial stress ratios and the true stress-strain relation for simple tension. That is, in terms of the stress, the maximum difference between the significant stress values as defined by the distortion energy theory and by test results is about 6 per cent.

Biaxial Nominal Ultimate Strengths

Table II gives the values of the principal nominal ultimate stresses for various biaxial stress combinations. The values of the stress ratios x and y based on the nominal ultimate stresses, as given in Table II, are plotted in Fig. 8 for purposes of comparison with the shear distortion energy, and stress theories of failure.

Biaxial True Fracture Strengths

Table II also gives the approximate values of the principal true fracture stresses and stress ratios x and y for the various biaxial stress combinations. The stress ratios x and y are compared with the theories of failure in Fig. 8.

Biaxial Ductility

Approximate values of the measured nominal and true ductilities are listed in Table III. These values are the strains in the direction of the

TABLE III.—*Nominal and True Ductilty Values for Various Ratios of Biaxial Stresses.*

Biaxial Stress Ratio	Measured Nominal Ductility* in. per in.	True Ductility (ϵ_1 Values)		
		Measured in. per in.	Theoretical in. per in.	% Diff.
0 (Long. tension)	0.088	0.084	0.096	+14
−0.2	0.111	0.105	0.100	− 5
−0.4	0.118	0.112	0.119	+ 6
−0.6	0.134	0.126	0.149	+18
−0.8	0.149	0.139	0.117	−16
−1.0 (Pure torsion)	0.148	0.122	0.096	−21

* Principal strain value e_1 prior to fracture.

maximum principal stresses and are based on the measured strains just prior to fracture. The theoretical value of the true ductility based on the distortion energy theory has been shown to be (10)

$$\epsilon_{1r} \doteq \left(\frac{\sigma_r}{k}\right)^{1/n} (\alpha^2 - \alpha + 1)^{(1-n)/2n} \left(1 - \frac{\alpha}{2}\right), \tag{17}$$

where $\alpha = \sigma_2/\sigma_1$ = the principal stress ratio,
σ_r = true fracture strength in simple tension, and
k and n = material constants for simple tension.

With average values of k and n as given in Table I and the average value of σ_r based on three tests, the theoretical values of the true ductilities ϵ_{1r} were calculated by Eq. 17 for various values of the principal stress ratio α. These values are shown in Table III and are compared with the actual true ductilities. In view of the inaccuracies and difficulties involved in obtaining the values of both the theoretical and experimental ductilities, the agreement between these values is satisfactory.

CONCLUSIONS

For the aluminum alloy tested and for tension-compression biaxial stresses, the actual values of the yield strengths, true fracture strengths, and plastic stress-strain relations are in satisfactory agreement with the distortion energy deformation theory, while the true ductilities and nominal ultimate strengths are in approximate agreement with this theory.

Acknowledgments

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APPENDIX I

Determination of True Principal Strains

The plastic deformation of the element in Fig. 12(*a*) is shown in Fig. 12(*b*). The torsional stress results in motion of B toward B_1 and the axial stress in movement of B to B_3. If the element has a unit length $\overline{AC}$, then the shear strain $= \tan \overline{BGB_1} = \tan \gamma_{xy}$. Also, $\overline{B_3J} = e_x =$ the nominal axial strain and $B_1J = (e_y)(\overline{AG})$ where $e_y =$ the nominal lateral strain. In this case, $\sigma_y = 0$, and for constancy of volume, $e_y = \frac{1}{2}e_x$. Referring again to Fig. 12(*b*), a line $\overline{AB}$ before the stresses are applied moves to $\overline{AB_3}$ so that the nominal strain for $\overline{AB}$ will be defined by

$$e' = \frac{\overline{EB_3}}{\overline{AB}}. \qquad (a)$$

It should be noted that the definition of the strain given by Eq. *a* is an arbitrary one since the reference line $\overline{AB}$ changes in direction as the stresses are applied. However, it is expected that the errors produced by the foregoing assumption are small. In any case, the procedure used may be considered as approximate.

By a consideration of triangles ABE and BEB_3,

$$\overline{EB_3} = \overline{AB}[\cos(\beta - \alpha) + \sin(\beta - \alpha)\cot\gamma - 1]. \qquad (b)$$

Placing the value of $\overline{EB_3}$ from Eq. *b* in Eq. *a*,

$$e' = \cos(\beta - \alpha) + \sin(\beta - \alpha)\cot\gamma - 1. \qquad (c)$$

Applying the Sine Law to triangle ABB_3,

$$\sin\gamma = \frac{\overline{AB}}{\overline{BB_3}}\sin(\beta - \alpha). \qquad (d)$$

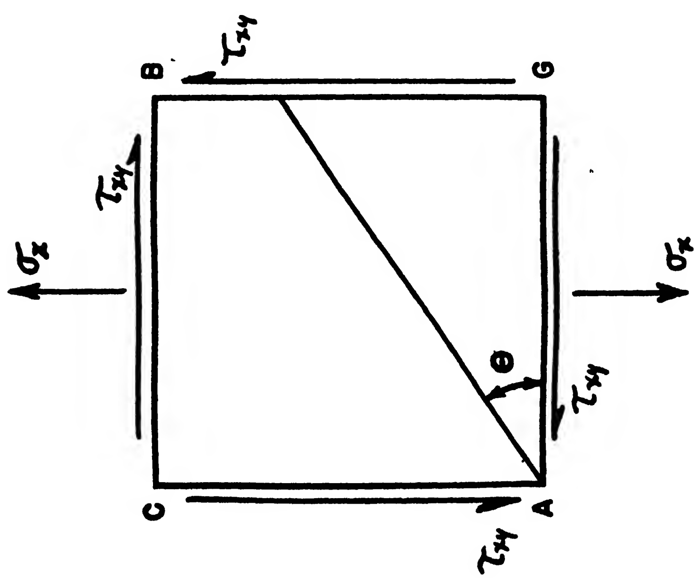

FIG. 12(*a*). Stress components.

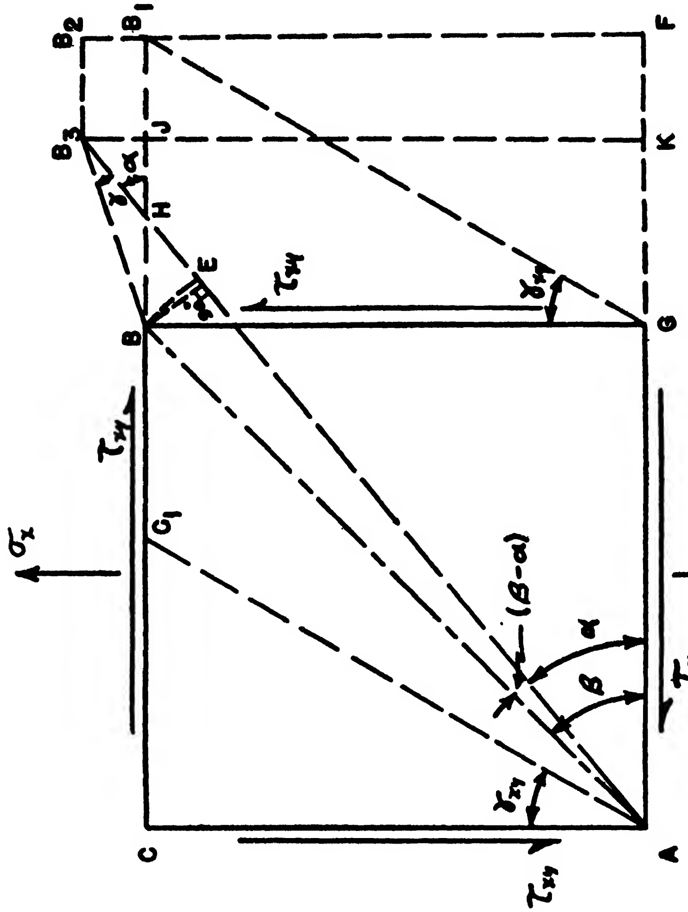

FIG. 12(*b*). Deformation of element subjected to axial stress and shear stress.

Considering triangle BB_3J,

$$\overline{BB_3} = \overline{AC}\sqrt{(\tan \gamma_{xy} - e_y \cot \beta)^2 + e_x^2}. \tag{e}$$

From Eqs. d and e,

$$\sin \gamma = \frac{\sin (\beta - \alpha)}{\sin \beta \sqrt{(\tan \gamma_{xy} - e_y \cot \beta)^2 + e_x^2}} \tag{f}$$

or

$$\cot \gamma = \sqrt{\frac{[e_x^2 - (\tan \gamma_{xy} - e_y \cot \beta)^2] \sin^2 \beta - \sin^2 (\beta - \alpha)}{\sin^2 (\beta - \alpha)}}. \tag{g}$$

Substituting the value of cot γ from Eq. g in Eq. c,

$$e' = \sqrt{[e_x^2 + (\tan \gamma_{xy}) - e_y \cot \beta)^2] \sin^2 \beta - \sin^2 (\beta - \alpha)} + \cos (\beta - \alpha) - 1. \tag{18}$$

The strain e_β would be completely defined by Eq. 18 if α were known. It can be shown that the angle α is defined by

$$\tan \alpha = \frac{\overline{B_3k}}{\overline{Ak}} = \frac{e_x + 1}{\cot \beta + \tan \gamma_{xy} - \frac{e_x}{2} \cot \beta}. \tag{19}$$

That is, the nominal strain e' in any direction β is completely defined by Eq. 18 where the value of the angle is determined by Eq. 19.

The *true strain* ϵ' corresponding to the nominal strain e' is, by Eq. 2,

$$\epsilon' = \log_e (1 + e'). \tag{20}$$

The *true principal strains* will be defined as those strains in the direction of the principal stresses. The values of these strains can be determined by Eqs. 18, 19, and 20 provided the values of β are taken as

$$\left.\begin{aligned} \beta_1 &= \left(\theta_1 + \frac{\pi}{2}\right) \\ \beta_2 &= \left(\theta_2 + \frac{\pi}{2}\right) \end{aligned}\right\}, \tag{21}$$

where θ_1 and θ_2 are the directions of the planes of principal stress as given by Eqs. 14. From Eqs. 18 and 19 the nominal principal strains are

$$e_1 = \sqrt{[e_x^2 + (\tan \gamma_{xy} - \frac{e_x}{2} \cot \beta_1)^2] \sin^2 \beta_1 - \sin^2 (\beta_1 - \alpha_1)} + \cos (\beta_1 - \alpha_1) - 1,$$

$$e_2 = \sqrt{[e_x^2 + (\tan \gamma_{xy} - \frac{e_x}{2} \cot \beta_2)^2] \sin^2 \beta_2 - \sin^2 (\beta_2 - \alpha_2)} + \cos (\beta_2 - \alpha_2) - 1, \tag{22}$$

where

$$\left.\begin{aligned}\alpha_1 &= \tan^{-1}\left[\frac{e_x + 1}{\cot\beta_1 + \tan\gamma_{xy} - \frac{e_x}{2}\cot\beta_1}\right]\\ \alpha_2 &= \tan^{-1}\left[\frac{e_x + 1}{\cot\beta_2 + \tan\gamma_{xy} - \frac{e_x}{2}\cot\beta_2}\right]\end{aligned}\right\}, \tag{23}$$

where β_1 and β_2 are defined by Eqs. 21.

With values of e_1 and e_2 defined by Eqs. 22, 23, and 21, the true principal strains are, by Eq. 20,

$$\left.\begin{aligned}\epsilon_1 &= \log_e(1 + e_1)\\ \epsilon_2 &= \log_e(1 + e_2)\\ \epsilon_3 &= -(\epsilon_1 + \epsilon_2)\end{aligned}\right\}. \tag{24}$$

That is, the principal strains can be determined by Eqs. 24 when e_1 and e_2 are calculated by Eqs. 22 with the α and β values in Eqs. 22 determined by Eqs. 23 and 21.

Portable Dark Room.—The Camera Specialty Company has been appointed exclusive distributors for the new Bendick Portable Dark Room. This new unit now makes it possible for amateur and professional photographers to process their film, load holders, make prints in brilliant sunlight or anywhere, any time of day with absolute safety.

The Bendick Portable Dark Room replaces the old changing bag. Now the photographer can get right into this portable dark room, switch on the light and prepare his accessories for processing. The light can be snapped off in a hurry and complete finished pictures can be made without waiting to take them to the studio to complete the job. Summertime is no handicap as there is a removable cooling system if needed; uses ice cubes or dry ice in a non-rust cylinder.

The Bendick Portable Dark Room folds up smaller than a rain coat. It weighs 3½ lb. and is only 12 in. long. It is absolutely light tight and is made of high grade porous material.

Westinghouse Precipitron Aids Technicolor in Keeping Film Free of Defects.—Although cleanliness of air is of great importance in many industrial processes, few plants have as great a need for clean-as-humanly-possible air as the celebrated Technicolor plant in Hollywood. In the processing of *motion picture film in color*, any microscopic speck on the postage-stamp-sized frame becomes huge when the image is projected on a theater screen, at a magnification varying from 200 to 300 times.

Furthermore, the film *consists of a gelatin coating on a plastic base*, and *it therefore tends* to hold an electrical charge. This characteristic, in turn, *causes the film to attract* and hold any particle—however microscopic—of dust or foreign matter in the air *around it.*

As a result, the Technicolor Motion Picture Corporation must necessarily keep its premises—particularly the areas where film is handled and processed—spotlessly clean. This task has recently been considerably facilitated in these areas by an extensive installation of Westinghouse Precipitron apparatus, which is in various air intakes of the building's air-conditioning system. Every minute, 133,000 cubic feet of air pass through the Precipitron apparatus. Sizes of the Precipitron vary from 24,000 CFM capacity down to 4,800 CFM.

Although the Precipitron installation is so new that no accurate figures on its performance are available as yet, "it has already had a very noticeable effect on our building maintenance, by making it easier to keep our film-processing areas clean," according to Dr. Herbert T. Kalmus, President of the Technicolor Motion Picture Corporation.

PREPARATION OF VANADIUM PENTOXIDE SOLS BY ION EXCHANGE

BY

JAMES A. DEVLIN,[1] WALLACE M. McNABB[1] AND FRED HAZEL[1]

The most widely used method for the preparation of vanadium pentoxide sols involves addition of an acid to ammonium vanadate, washing the precipitated oxide until it begins to peptize and then suspending in water until dispersion is complete (**1**).[2] The sols contain a variable amount of molecular vanadium pentoxide, depending on their age (**5**). The molecular fraction passes through dialyzing membranes and, since it is in equilibrium with the colloidal particles, purification of the system by dialysis is accompanied by continuous loss of vanadium. The preparation of a sol of low electrolyte content has been reported by Prandtl and Hess (**4**). The method consisted of hydrolyzing the tertiary butyl ester of orthovanadic acid followed by distillation of the alcohol.

This report describes the preparation of a relatively pure vanadic acid system from ammonium vanadate by the method of ion exchange. A batch procedure has been employed because of the low solubility of the starting material.

EXPERIMENTAL

The exchange resins, Amberlite IR-100H and Dowex-50, were conditioned by three sodium-hydrogen cycles and used in the hydrogen form.

Sol I.

A weighed amount of C.P. ammonium vanadate, 1.0479 g., was mixed with 10 ml. of water and 15.0 g. of air-dried Amberlite IR-100H. The paste was stirred until it acquired a deep red color. This occurred in about five minutes. One hundred ninety ml. of water was added with rapid stirring. The system had the deep color characteristic of V_2O_5 sols. It was filtered and analyzed for ammonia (**6**) and vanadium (**3**).

Sol II.

A weighed amount of ammonium vanadate, 21.1688 g., was mixed

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

with one liter of water and 300 ml. of wet Dowex-50. The system was stirred one hour, filtered and the sol analyzed.

RESULTS

Analyses of the Sols

TABLE I.

Weights on a Liter Basis	Sol I	Sol II
Wt. of NH_4VO_3 taken/liter	4.1916 g.	21.1688 g.
Wt. of V taken	1.8255	9.2189
Amount resin used	60.0	300 ml. (ca. 138 g.)
Wt. V in sol, total	1.1780	5.3060
Wt. V as V_2O_5 in sol	1.1289	5.0134
Wt. V as NH_4VO_3 in sol	0.0491	0.2926
Per cent conversion to V_2O_5	61.84	54.38
pH of sol	2.8	2.4

Clouding Values

Clouding values (7), indicating the concentrations of electrolytes required to coagulate the colloidal system, were determined by adding 2.0 ml. of sol to 3.0 ml. of electrolyte and observing after 24 hours. The data, recorded in Table II, reveal that the particles were negatively charged.

TABLE II.

Electrolyte	Clouding Value, m.eq. per liter	Remarks
NH_4Cl	8.0	Weak gel
NH_4VO_3	13.6	Weak gel
$BaCl_2 \cdot 2H_2O$	0.64	
$LaCl_3 \cdot 7H_2O$	0.60	

DISCUSSION

The reaction occurring in the preparation of the sol may be expressed as follows:

$$\begin{array}{l} \underset{\text{solid}}{NH_4VO_3} \\ \quad\downarrow \\ \underset{\text{dissolved}}{NH_4^+ + VO_3^-} + R\cdot H \rightleftharpoons R\cdot NH_4 + HVO_3 \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad \upharpoonleft\!\downharpoonright \\ \qquad\qquad\qquad\qquad\qquad\qquad\quad \underset{\text{colloidal}}{V_2O_5\cdot xH_2O} \end{array}$$

The formation of a colloidal product shifts the reaction to the right.

It was found that an excess of resin was required for the production of a stable sol. Calculated amounts of reactants yielded a precipitate because of the coagulating action of ammonium vanadate present in the equilibrium system. Results indicate that excess resin, however, contributed to the low yield of V_2O_5 in the sol. It was shown by quali-

tative tests that the resin adsorbed pentavalent vanadium. A mechanism similar to the following may be suggested for the adsorption.

$$V_2O_5 + 2H^+ + 3H_2O \rightleftharpoons 2V(OH)_4^+$$
$$V(OH)_4^+ + R\cdot H \rightleftharpoons R\cdot V(OH)_4 + H^+.$$

The lower yield of V_2O_5 in Sol II may be attributed to the above reactions and the use of the resin with the greater exchange capacity, Dowex-50, in the preparation of this system.

The Amberlite resin caused partial reduction of vanadate to vanadyl ion, which then competed with the ammonium ion for exchange on the resin.

$$VO^{++} + 2R\cdot H \rightleftharpoons R\cdot VO + 2H^+$$

On regenerating the resin with hydrochloric acid, the reaction was shifted to the left and the solution acquired the blue color of the vanadyl ion. The reducing property of this resin has been reported previously (2). Dowex-50 proved to be more stable in the presence of vanadate. However, on contact with the solution for several days, this resin showed some reducing action.

The chemical properties of vanadic acid prepared by ion exchange are being investigated.

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NOTES FROM THE NATIONAL BUREAU OF STANDARDS*

PANEL-TYPE ACCELEROMETER FOR AIRCRAFT

A novel type of accelerometer[1] which gives both an instantaneous indication and record of the acceleration of an airplane in flight has been developed by A. S. Iberall of the National Bureau of Standards for the U. S. Air Force. Outstanding features of the device are compactness, which permits mounting on an instrument panel, and the use of spark recording to obtain a readily available record on a paper chart. The details of the design are such that small panel-type recorders of other physical quantities—temperature, pressure, or voltage, for example—can be similarly constructed to occupy a space not much larger than an ordinary indicating instrument.

The immediate application of the new accelerometer is in the training of aviators for combat flying, where sharp turns must be made at high speeds. Practice in making such turns can be more effectively obtained if the instrument panel of the plane is provided with an accelerometer that gives an instantaneous visual indication of acceleration while the maneuver is being executed. At the same time, it is desirable to have a record of acceleration for quick reference after a flight has been completed. However, the accelerometers that have been available are either too large for instrument-panel mounting or, if small enough for panel use, do not provide an automatic record of acceleration. Moreover, as most recording accelerometers require photographic equipment, interpretation of the trace must usually wait until the film has been developed. The Air Force therefore requested the National Bureau of Standards to undertake the development of a small, compact indicating and recording accelerometer that would meet the requirements of flight training use.

A sliding bob mounted on a shaft and restrained by a helical spring serves as the acceleration-sensitive element of the device. Acceleration of the aircraft along the axis of the shaft displaces the bob by an amount proportional to the acceleration, at the same time compressing the spring. The varying displacement of the bob from its rest position is permanently recorded on a paper chart by a repetitive high-voltage spark occurring at the position of the bob. An instantaneous indication of acceleration is also obtained since the record paper is visible through the transparent front of the accelerometer case immediately after puncture.

* Communicated by the Director.

[1] For more complete details, see "A Novel Recording Accelerometer," by A. S. Iberall, *Rev. Sci. Inst.*, Vol. 20, p. 304 (1949).

The accelerometer constructed at the Bureau is built into a rate-of-climb indicator case of the standard aircraft-panel type. By means of the zero-setting shaft of the rate-of-climb indicator, power from a small synchronous timing motor for driving the record chart is transmitted from the back to the front of the case without obstructing the available space. Sprocketed rollers mounted just behind the front of the case are thus made to pull the recording paper across the inner face of the cover and push it out the side of the case, where it may be torn off. As the recording paper passes across the face of the instrument, it is punctured by sparks at a 60-cycle repetition rate. The writing electrodes consist of a nickel tip projecting perpendicularly from the bob, and the edge of a narrow strip mounted on the inside of the cover directly under the tip. The bob and nickel tip are made the high-voltage electrode while the plate and other parts are grounded to permit removal of the cover without risk of electrical shock. The high voltage is provided by a small model-airplane spark coil mounted behind the motor. Current is brought to the bob through insulated banana plugs so that removal of the cover for loading the record will cut off the voltage to the electrodes.

The linear speed of the record chart is about 6 inches per minute. The cartridge for storage of the record chart is mounted in a semi-cylindrical depression in the outside of the transparent plastic cover, which is slotted to received the tangent lip of the cartridge.

A 35-mm. single-weight perforated photographic paper with no emulsion was chosen for the record chart. As the thickness of the paper (0.005 in.) required more energy to form an easily visible trace than was available from the small spark coil, a simple preliminary chemical treatment was developed for the paper to intensify the trace as it formed.

The necessity of operating the accelerometer aboard a plane from a 28-volt source of direct current also required the development of a special power supply. A 60-cycle vibrator is employed in the usual way to produce 60-cycle alternating current for the motor. However, the power dissipation of the small spark coil makes operation by a repetitive voltage "pip" most desirable. This pip is supplied by the inductive decay of the vibrator coil on the "break" of its main reed contact.

In laboratory performance tests at the National Bureau of Standards, the accelerometer successfully withstood shock and vibration of at least 15 *g* units of acceleration. Paper speed was found uniform to within a few per cent. The record obtained was easily interpreted and adequate for resolving detail of amplitude not less than 2 per cent of the total record width.

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Abstract of **An A-c. System of Remote Indication and Its Application to the Measurement of Fluid Pressure and Flow.**[1]—HOWARD D. WARSHAW.[2] A moving-vane indicator in circuit with a two-coil transmitter forms a system of remote indication relatively free from errors due to variations in ambient temperature, supply voltage, and supply frequency. The transmitter is simple and should withstand severe vibration and extreme temperatures. The electrical elements of the transmitter are completely isolated from the mechanical components so that measurements in the presence of corrosive or explosive fluids may be made with safety. Applications are presented in which the transmitter measures oil pressure in the crankcase of a gasoline engine and the flowmeter measures fuel flow. This system of remote indication has been developed for use in aircraft, but its potentialities for industrial applications will become evident. The experimental models were designed to operate from 400-cycle aircraft power sources; larger components would be required for 60-cycle operation.

The transmitter contains a movable core within a set of coils which are connected so that their magnetic fields are in opposition. The core is displaced by a force derived from the fluid under measurement, and its position is established when the fluid force is balanced against the opposing force of a coil spring. Three conductors serve to transmit two electrical signals from the transmitter to the indicator.

The indicator contains a vane of highly permeable magnetic material located within a set of crossed coils. The transmitter voltages are applied to the indicator coils through a bridge circuit, and their resultant magnetic field positions the vane which is fastened to the pointer shaft. The indicator contains resistors for sensitivity and end-scale adjustments, and suitable assemblies for damping and pointer zero return.

One application of this system is to the electrical remote indication of aircraft engine oil pressure. There is a need for an aircraft remote-reading oil-pressure gage with a transmitting element suitable for direct mounting on the aircraft engine. The coagulation, at low temperatures, of the engine oil in the line from the crankcase to the transmitter of any of the remote-indicating oil-pressure gages in present use makes these gages inoperative during a part of the engine warm-up period. The lack of oil pressure indication at this time frequently leads to serious engine damage which would not otherwise occur. The device herein described is capable of offering a satisfactory solution to this problem.

Models of both transmitter and indicator for measuring oil pressure are shown in Fig. 1. Figure 2 shows the same transmitter partly disassembled.

Another application of this system is to the remote indication of fluid rate-of-flow through tubing such as the fuel supply line of an aircraft engine. One model of the flowmeter transmitter is shown in Figs. 3, 4.

Electromagnetically, the operation of the flowmeter transmitter is identical to that of the pressure transmitter. The difference lies in the hydraulic system used to derive the force necessary to displace the core. In this application, the magnetic core is tubular, and is designed to support firmly an orifice plate that restricts the flow of fluid. The restriction sets up

[1] Paper published in *Instruments*, Vol. 22, p. 402 (1949).

[2] Research Engineer; The Franklin Institute Laboratories for Research and Development, Philadelphia, Pa.

a pressure differential across the orifice plate that positions the core against the restraining force of a coil spring. Figure 4 shows the core with one orifice plate installed, and Fig. 3 shows two replacement plates that are used for different ranges of rate-of-flow.

The pressure gage was tested to determine the inherent inaccuracies of the electrical system.

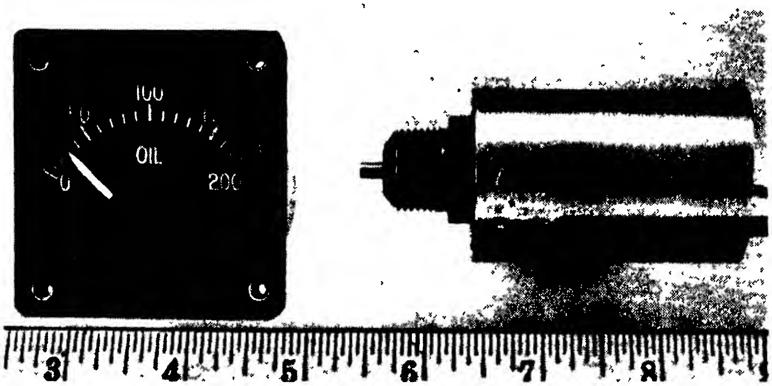

FIG. 1. Pressure transmitter and indicator.

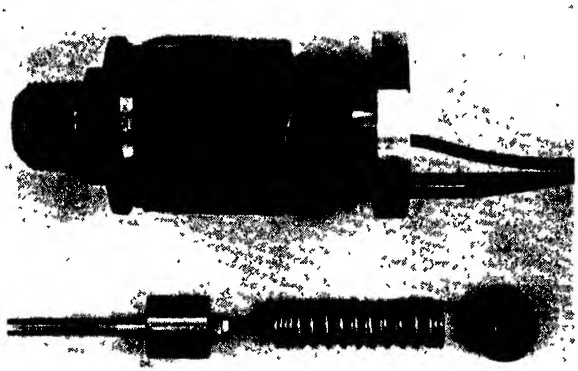

FIG. 2. Pressure transmitter disassembled.

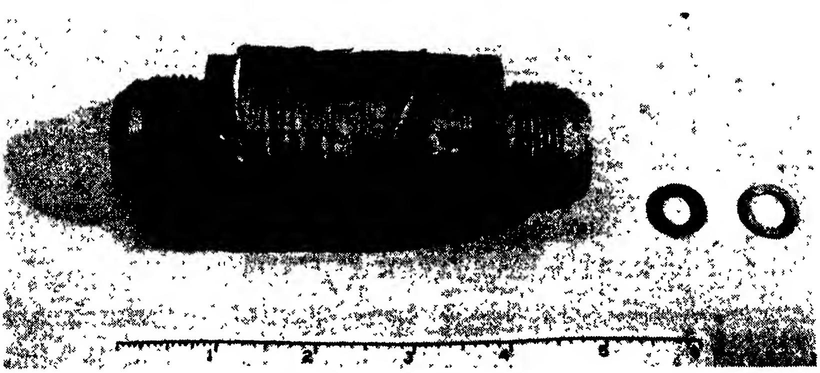

FIG. 3. Flowmeter transmitter.

With the indicator at room temperature (25 deg. C.) the transmitter temperature was varied from minus 55 deg. C. to plus 70 deg. C. The maximum error in indication was approximately plus-or-minus two per cent of full scale. When the supply voltage was varied from 80 to 120

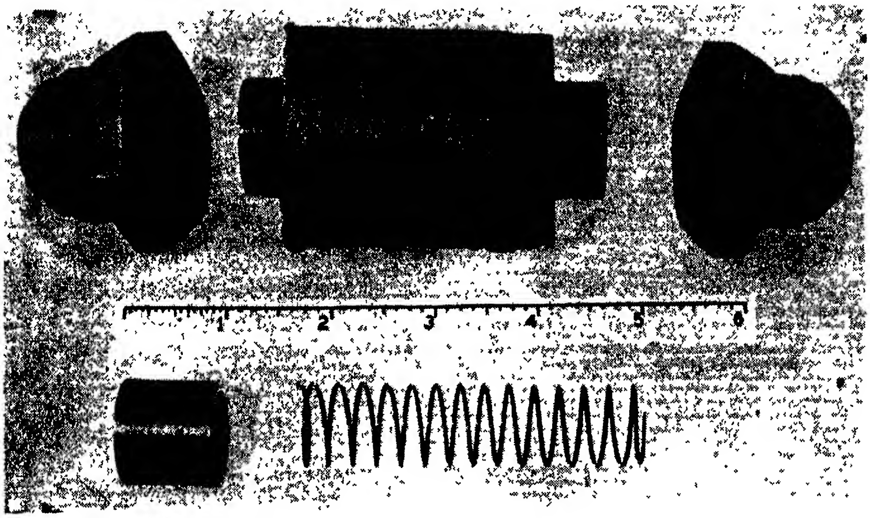

FIG. 4. Flowmeter transmitter disassembled.

volts, the maximum errors were plus 0.3 per cent and minus zero per cent of full scale. Variations in supply frequency of plus-or-minus 5 per cent produced maximum errors in indication of plus-or-minus 1 per cent of full scale.

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in the JOURNAL within the next few months:

COULSON, THOMAS: The Story of Aids to Navigation.

FANO, ROBERT M.: Theoretical Limitations on the Broadband Matching of Arbitrary Impedances.

FANO, ROBERT M.: A Note on the Solution of Certain Approximation Problems in Network Synthesis.

LUCKIESH, MATTHEW, A. H. TAYLOR, THOMAS KNOWLES AND E. T. LEPPELMEIER: Inactivation of Molds by Germicidal Ultraviolet Energy.

MECHLER, E. A., J. B. RUSSELL AND M. G. PRESTON: The Basis for the Optimum Aided-Tracking Time Constant.

JACOBSEN, LYDIK, R. L. EVALDSON AND R. S. AYRE: Response of an Elastically Non-Linear System to Transient Disturbances.

YU, Y. P.: Applications of Network Theorems in Transient Analysis.

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GRIFFITH, ROY T.: Minimotion Typewriter Keyboard.

SILBERSTEIN, LUDVIK: Developable and Developed Silver Halide Grains.

BARTOL RESEARCH FOUNDATION

Abstracts of three papers by staff members, presented at the I.R.E. Conference on Electron Devices held at Princeton, June 20–21, 1949.

Decay and Recovery of Emission from Sintered Thoria Cathodes in High-Vacuum Diodes, by DANIEL L. GOLDWATER. A variety of phenomena involving time-changes of

emission from sintered thoria cathodes have been explored. The results were discussed with reference to separation into three components: one steady state and two changing. Order of magnitude values of various parameters were presented and discussed briefly with reference to possible applications of this type of cathode.

Rate of Evaporation of Thoria, by EDWARD M. SHAPIRO. The rate of evaporation of thoria from thoria-coated filaments as a function of temperature was measured in the range 1700–1950° C_{Br}. Vapor pressures calculated from the evaporation rates were compared with vapor pressures measured by the effusion method. In addition some studies of evaporation rates with emission were discussed.

Radiation Transfer Considerations in the Heating of a Cathode Sleeve, by W. E. DANFORTH. The theory of radiation heat transfer was briefly developed with particular reference to cathode heating. Experiments were described in which metal and ceramic sleeves were heated by an internal cylindrical radiation. The observed temperatures were discussed in relation to the thermal emissivities of the several surfaces.

Abstract published in the *Bulletin* of the American Physical Society's meeting in Cambridge, Mass, June 16–18, 1949.

Radiations from Indium (114) and Barium (140),* by C. E. MANDEVILLE[1] AND E. SHAPIRO.[1] The 72-Second In^{114} activity, in equilibrium with its 48-day isomeric parent, was induced in metallic indium irradiated by slow neutrons in the Oak Ridge pile. Chemical separations were performed for the removal of Ag, Ca, Cu, Fe, Ni, and Pb as possible impurities. The beta-rays have a maximum energy of 1.89 Mev as measured by aluminum absorption, and the maximum gamma-ray energy is 0.90 Mev as determined by coincidence absorption. A lead absorption curve gives quantum energies of 0.15 Mev and 0.70 Mev. Beta-gamma coincidence data show that the harder gamma-rays are associated with less than 5 per cent of the nuclear beta-rays. A gamma-Gamma-coincidence rate of 0.25×10^{-3} coincidence per gamma-ray was noted. Fission-produced Ba^{140} emits beta-rays of maximum energy 0.91 Mev. Lead absorption curves observed continuously for 90 hours during growth of La^{140} reveal the presence of 0.14 Mev and 0.6 Mev quanta emitted by Ba^{140}. Beta-gamma coincidence measurements show that the beta-spectrum of Ba^{140} is complex.

* Assisted by the joint program of ONR and AEC.

[1] Bartol Research Foundation of The Franklin Institute, Swarthmore, Pa.

The Maximum Range of High Energy Electrons in Matter.—F. L. HEREFORD [1] AND C. P. SWANN.[1] Investigations of the penetration of high energy electrons in matter are of considerable interest because of the widespread use of absorption measurements in the determination of beta and gamma ray energies. Information concerning electron scattering also finds ready application in this connection. In the absence of experimental data for electrons of energy exceeding 3 Mev. we have undertaken an investigation of these matters in the energy range from 3 to 12 Mev. The beta rays from B^{12} are being employed as a source, the production of B^{12} being attained through the B^{11} (d,p) B^{12} reaction with the Bartol Vân de Graaff machine.

An electron beam homogeneous in energy is obtained by analysis of the B^{12} beta ray spectrum with a 90° magnetic spectrometer. Using a triple coincidence train of Geiger-Mueller counters absorption curves of this beam have been determined at various energies. Extrapolation of the linear portion of the curves through the zero intensity axis yields range values which may be compared with existing computed values,[2] the range so determined corresponding to the so-called "practical maximum range." Results obtained thus far with aluminum absorbers[3] have indicated experimental values somewhat less than the computed values.

It seems probable that this discrepancy can be due to multiple scattering effects which would yield actual ranges greater than the measured absorber thicknesses. Energy loss through

radiation is also effective at these energies. However, for moderate energies (60 Mev.) in aluminum most electrons will not experience radiative collisions; hence, their maximum range will be unaffected.

Since both radiation and scattering depend heavily upon the atomic number of the absorber, it is hoped that measurements employing heavier elements will throw some light on the question. Preliminary results obtained with copper have indicated that maximum range values determined by the method employed are definitely dependent upon scattering effects (and hence counter geometry) to a sufficient extent to account for the discrepancy mentioned above. Further work is continuing with the aim of obtaining experimentally defined maximum range values which may be of use in absorption measurements in the high energy region.

[1] Bartol Research Foundation of The Franklin Institute, Swarthmore, Pa.

[2] W. A. FOWLER, C. C. LAURITSEN AND T. LAURITSEN, *Rev. Mod. Phys.*, 20, 237 (1948).

[3] *Phys. Rev.*, Aug. 15, 1949.

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MUSEUM

"Is that a demonstration of perpetual motion?" ask scores of museum visitors who peer down the main stairwell at the gently swinging 1800-lb. bob of the Foucault pendulum. Anyone who witnessed the extraordinary parade which heralded the installation of the pendulum might have been justified in supposing it had some such momentous purpose. Surrounded by a mounted police escort, eleven sturdy men were strung out over a length of 85 feet, each holding on to a slender bronze wire only one quarter inch in diameter. Not that the wire was any great weight, it weighed only nine pounds, but it was essential that no kinks form in the wire.

The pendulum is an impressive sight, the 1800-lb. shot-filled ball swinging at the end of an 85-ft. suspension. Its periodical swing is a visual demonstration of the Earth's daily rotation.

The issue of the JOURNAL for May, 1851, contains the translation of a paper by Jean Bernard Leon Foucault, the eminent French physicist, who had hung a similar pendulum from the dome of the Pantheon in Paris. In his paper Foucault describes the purpose of the pendulum, which was to furnish a physical demonstration of the Earth's rotation without any reference to a point beyond the Earth. The significance of the paper appears to have been realized

by the members of the Institute, because the following six months produced communications from three of them relating to the pendulum.

The pendulum is set swinging in a north-south direction each day at a predetermined time. An observer will perceive that at the end of an hour the compass dial beneath the swinging bob has rotated nearly ten degrees. Each successive hour reveals that a similar change has taken place. The impression gained is that the pendulum itself changes the direction of its motion, but this is not true.

The physical property of a swinging pendulum is that once it has been started swinging in a particular plane, the pendulum will maintain that plane, and so it might be considered an invariable plane of reference.

The Earth is a spinning body which rotates once in twenty-four hours. Consequently, a body on the equator of the Earth is carried around with a linear speed of about 1000 miles an hour. In our latitudes the linear speed of a body is about 750 miles an hour, while at the north pole the linear speed is zero. As a result, in the northern hemisphere a body south of us has a higher linear speed than we have and, likewise, we have a higher speed than a body north of us. Thus, if we are in a room the south end of the room will travel faster than the north end—the room will rotate.

Combine the two motions of the pendulum in a plane and the rotation of the room due to the rotation of the Earth and we see the compass dial turning under the pendulum.

To appreciate how the pendulum maintains its uniform motion, independent of the roof from which it is suspended, it is advisable to descend to the stair foot where a model of the pendulum and its unique suspension is displayed and may be operated.

The principle may be visualized further by thinking of a merry-go-round in which the animals represent the compass points. Imagine a pendulum suspended at its center. While the merry-go-round stands at rest the pendulum is set swinging in the direction of a stationary spectator. Then the merry-go-round is set turning. The pendulum will continue to swing in the direction of the spectator. But one after another the animals will pass it in succession, and to a rider on the merry-go-round the pendulum will appear to be changing the direction of its swing.

If it were possible to suspend a pendulum immediately over the north pole it would act as in the merry-go-round analogy, it would change its apparent direction 15 degrees every hour. On the other hand, at the equator the Earth turns in a plane perpendicular to that at the pole and the Foucault pendulum would show no change in direction because the Earth is not actually turning under it. The degree of motion increases the farther north one proceeds. At the latitude of Philadelphia the Earth turns through 9.6 degrees every hour.

The Institute's pendulum does not have the longest suspension which has been used to demonstrate this principle. Experiments in Rheims and Amiens cathedrals (the latter with a suspension of 165 ft.) exceeded its length. But the entire setting, the gleaming 23-in. steel shell filled with small shot, the simple yet beautiful compass dial, and the convincing model, constitute a notable tribute to the public spirit of the donors, Mr. and Mrs. Richard L. Binder.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ABSORPTION OF ULTRA-VIOLET LIGHT BY LIVING CELLS *

BY

J. O. ELY AND M. H. ROSS

Nucleic acids, because of the presence of purines and pyrimidines in the molecule, absorb ultra-violet light with a maximum absorption at 2600 Å. Recently, Larionow and Brumberg (1) suggested, on the basis of ultra-violet light photomicrographic studies, that desoxyribonucleic acid, as it exists in the nuclei of living cells, does not absorb ultra-violet light of wave-lengths near 2600 Å. These investigators contended that absorption develops as a result of injury or death of the cell.

Ultra-violet light photomicrographs of living cells made in this laboratory show absorption of light of wave-length 2654 Å. Cells of the Walker Carcinoma No. 256 of the rat were used. Cell suspensions

Ultra-violet light (2654 Å) photomicrograph of Walker Carcinoma No. 256 of the rat, magnification 900 ×.

were prepared immediately after removal of tumours from the host by mincing tissue finely with scissors in Ringer's solution and straining through a thin layer of cotton to remove coarse aggregates. Cells prepared in this manner and kept for much longer periods of time than was required in these experiments produced tumours when implanted into rats. The cells, in Ringer's solution, were placed on a quartz slide and covered with a quartz coverglass and sealed with paraffin.

Ultra-violet light photomicrographs were made with a quartz optical system microscope by the method previously described (2), with a 2.5 mm. objective, N. A. 0.85 and a 10 × ocular, magnification 240 ×. Condenser diaphragms were closed to small aperture.

* Reprinted from *Nature*, Vol. 163, p. 906, June 11, 1949.

Focusing was performed by means of a fluorescent screen and a magnifier with 2654 Å light. The focus was obtained on one area of the field; all light was then barred from the microscope, and the slide was moved to a distant field. In this procedure the cells were exposed to only the amount of ultra-violet light necessary to produce the image on the film. Panatomic X (Eastman Kodak Co.) was used. Photomicrographs made in this manner (see illustration) show the same absorption pattern as those made after the cells have been exposed to several times the amount of ultra-violet light necessary for photomicrographs.

Nuclear and cytoplasmic absorption varied considerably from cell to cell, depending on the metabolic state. Chromosomes, where present, absorbed intensely. In the resting cells the nucleolus usually absorbed much more intensely than did the remainder of the nucleus.

Visual observations of fluorescent images of cells illuminated by ultra-violet light made by rapidly moving the slide across the microscopic field revealed absorption by cells as they came into view; under these conditions exposure to ultra-violet light was less than one second. Chicken erythrocytes in whole blood when observed in this manner within three to five minutes after drawing the blood showed intense nuclear absorption. These observations, which have been repeated many times, indicate that the nucleic acids as they exist in the living cells absorb ultra-violet light. This contention is supported by the work of Wyckoff (3), who made successive ultra-violet (2750 Å) photographs of a living grasshopper spermatocyte and showed absorption by the chromosomes throughout division.

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BOOK REVIEWS

Industrial Rheology and Rheological Structures, by Henry Green. 311 pages, illustrations, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1949. Price, $5.50.

This book is neither a reference book nor a textbook. It is a highly readable distillation of the late Henry Green's experiences as a rheologist, microscopist, and industrial researcher. The book is intended to guide "Bill," an industrial investigator, into the field of rheology, and its primary purpose is to present Bill with a workable rheological system based on accepted principles, rigorously demonstrated. The organization of the book is intended to further this purpose. Part I, A Rheological System, defines Newtonian, Bingham and "pseudoplastic" flow, and discusses the measurement, calculation and meaning of viscosity, yield value, plastic viscosity and thixotropy. It describes the various types of flow and the various systems which have been proposed to measure them. Based on this very clear review of rheological science, Mr. Green constructs a rheological system for the evaluation of Newtonian liquids and Bingham bodies. Part II, The System Applied to Laboratory Measurements, is most complete in its description of the calibration and use of the rotational viscometer and the tackmeter. Some other types of viscometers are mentioned but not described in detail. Further conceptions regarding the types of flow exhibited by various materials are presented and materials are described whose behavior illustrates the principles stated in Part I.

Part III, The Particle—The Basis of Rheological Structure, discusses the role of suspended particles in determining the flow properties of a material. It is chiefly concerned, however, with the measurement of particle sizes with the white light, ultraviolet and electron microscope. This relatively short section (40 pages) is well worth the attention of anyone who desires to examine particles in a liquid suspension. The Journal of The Franklin Institute published two of Mr. Green's original papers on the microscopy of pigment particles, in 1921 and 1927.

Part IV, Particle Groups and Rheological Structures, integrates the conceptions of Parts I and III to explain the causes and formation of structure in suspensions and colloids. This section includes the following types of materials: Newtonian, Pseudoplastic, Dilatant, Plastic, Brittle, Thixotropic, Two-Dimensional Flocculates, and Artifacts. The inclusion of photomicrographic and optical evidence to account for flow properties is a unique and valuable feature of this presentation. Structure is explained in terms of the flocculation of the particles present.

The book contains nine appendices which include supplementary information on instrument corrections and use, and advice to Bill and to Bill's boss, the Laboratory Director. A glossary, bibliography and indices are included.

This book, the fruit of thirty years of investigation by one of this country's outstanding rheologists and microscopists, presents for the first time, to quote the jacket, "the unique scientific correlation of rheological with microscopical investigations." This relatively novel approach commends the book to practicing rheologists, even though the book also successfully fulfills its primary purpose of giving Bill a practical rheological system. The rheological system that Mr. Green presents reflects his special interest in printing inks, paints, varnishes, oils and other low-consistency materials, and an investigator in these fields can use Mr. Green's system readily.

The system is not intended to be useful for the investigation of high-consistency materials such as resins, asphalts, rubber and industrial plastics. In fact, Mr. Green considers that no "industrially-useful" system for such materials can be recommended at present because no way is yet known for describing their flow properties independently of such experimental variables as rate of shear or shearing force. The various power equations that have been introduced for evaluating the flow of such materials are dismissed as mathematical methods of curve-fitting, devoid of real physical significance. The evaluation of elasticity, so necessary

for characterizing high-consistency materials, is barely mentioned. The possible role of selective adsorption in the formation of liquid layers around the solid particles is not mentioned, even though this conception is useful in explaining thixotropy, aging and adhesion of high-consistency heterogeneous materials.

In summary, this book is an important compilation and correlation of rheological principles and microscopic phenomena. It presents a rheological system that can be applied readily to investigations of the flow properties of low-consistency materials. Although it presents much which is of interest to workers with high-consistency materials, such investigators will have to look elsewhere for methods and conceptions unique to their problems or useful in their solution.

E. THELEN

MAGNETIC RECORDING, by S. J. Begun. 242 pages, drawings and photographs, 15 × 23 cm. New York, Murray Hill Books, Inc., 1949. Price, $5.00.

This is one of the first full-length books devoted exclusively to magnetic recording. It includes in an elementary fashion a large variety of material dealing with the history and theory of magnetic recording, properties of magnetic materials and examples of much modern apparatus. The volume is profusely illustrated with photographs of commercial magnetic recording equipment manufactured by all of the major producers. By providing this survey of equipment made to-day it shows the design manifestations of the engineering problems peculiar to magnetic recording techniques. Circuit diagrams showing values are provided for most of the sets.

The equipment and diagrams occupy approximately one half of the book. The other half is devoted to the presentation of simple conceptions of sound, magnetism, magnetic materials and induction principles. The treatment is similar to that used by J. F. Rider, the technical content being at the radio serviceman's level.

The elementary nature of the book is emphasized by the care taken to describe what is meant by such terms as "wavelength," "period," etc. The style of writing is informal. Each chapter is followed by a liberal bibliography. It is also interesting to note that a list of patents is given, and this may be extremely valuable to certain readers.

The lack of fundamental information such as might be derived from a thorough research program on the many phases of magnetic recording is surprising in the light of the author's affiliation (Vice-President and Chief Engineer, The Brush Development Co.). Such information must have been available to him, and it is certain that, unless the competitive nature of the industry prevented his publishing such data, the book could have been far less dated and infinitely more useful in the engineering field. Owing to the broad applicability of magnetic recording to communications and especially to the process of information storage in large-scale, high-speed computing equipment, the subject has far greater significance than "a challenge to the phonograph."

C. W. HARGENS

OUR SUN, by Donald H. Menzel. 326 pages, illustrations, 15 × 22 cm. Philadelphia, The Blakiston Co., 1949. Price, $4.50.

This is the long-awaited, eighth volume of the Harvard Books on Astronomy, and is in every way worthy to rank with the other seven. The author explains that its publication was prevented during the war years and until now by military censorship, because of the chapters on Solar Chemistry and Atomic Energy.

The style of the writing is very clear and easily understood. Difficult mathematics is largely omitted, and where necessary, is explained by drawings and simple illustrations. The wealth of photographs is one of the most delightful parts of the book.

The chapters on Sunspots, Prominences and Fine Details of the Solar Surface give more information than we have ever had before. The chapter on Eclipses gives abundant details of the phenomena and the methods of observing them.

The author is particularly pleasing in his brief summary of the origin of life and the course of evolution in its relation to solar energy, and his last chapter, called The Sun and You, gives

serious but practical thought to the future when the sources of energy that we use now will be exhausted and to the possibility of using the direct energy of sunlight.

Altogether the book is fine, and it will be enthusiastically welcomed by readers of the former volumes. It ought to make new friends for the whole series.

WILLIAM L. FISHER

PHOTOELASTICITY AND ITS APPLICATION, by V. K. Zworykin and E. G. Ramberg. 494 pages, illustrations, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1949. Price, $7.50.

This volume replaces the earlier work by Zworykin and Wilson entitled "Photocells and Their Application," the second edition of which was published at a time when many of our currently useful devices were still in their infancy. In common with the previous work, the authors once again authoritatively present practical data as well as the physical properties of modern photoelectric devices, with a minimum of detailed physical theory and mathematical development.

The twenty chapters of this book can be roughly divided into two major groups, those pertaining to the principles and preparation of photo-sensitive devices and those pertaining to the application of such devices. An historical introduction acquaints the reader with the three common photoelectric effects: photovoltaic, photoconductive, and photoemissive; and the early experiments which led to their discovery. This chapter is followed by one which discusses the general theory of radiant and thermal energy and the basic principles of the photoemissive effect.

The special characteristics looked for in phototubes are dependent upon the physical and chemical treatment given to the emissive surfaces, which are usually derived from thin alkali films on a metal base. These are described, again with reference to the latest experiments, with numerous graphs showing the sensitivities of a wide variety of surfaces.

The modern materials, apparatus, and practical methods followed in making phototubes are described for the cases of the: vacuum, gas-filled, multiplier, and image phototubes, and photoconductive and photovoltaic cells. The physical and electrical characteristics of each of these classes of tubes are described in a logical and continuous manner, starting with the simplest type and going on to the more complex means of amplification. Detailed explanations of recent basic findings supplant discussions of their operating characteristics.

The remaining nine chapters are devoted to applications of photoelectric devices, from the common and simple trigger relay to the more complex and physically more interesting field of infra-red detection. Basic circuits for light measurement and light activation are examined and many practical auxiliary electronic circuits are described utilizing a wide variety of available phototubes. When small photo currents are to be measured, consideration must be given to the many types of noise which ultimately limit the smallest light signal which can successfully be detected. One chapter is devoted to this interesting subject.

Descriptions of many industrial applications which use one form or another of the phototube as a scientific tool include the following: photometry, reflectometers, spectrophotometers, sound reproduction, television picture transmission including facsimile, flying spot and ultrafax, television camera tubes, light beam signaling, infra-red detection, industrial process controls, and others.

The broad fields considered by the authors allow the reader who specializes in any one of many scientific branches to apply the practical information to his own experimental work. While not written as a formal textbook, both practicing engineers and students should profit from the book as a useful tool, not only because of its own content, but because of the hundreds of references, listed throughout the work. It should find its way into the technical libraries of many serious investigators.

S. CHARP

ERRATUM: The August JOURNAL incorrectly listed the publisher of "Principles of Electricity and Electromagnetism," by Gaylord P. Harnwell. The book is published by McGraw-Hill Book Co., New York.

BOOK NOTES

ELECTRIC RESISTANCE STRAIN GAUGES, by W. B. Dobie and P. C. G. Isaac. 114 pages, illustrations, diagrams, 14 × 23 cm. London, English Universities Press, Limited, 1948. Price, $3.50 from Macmillan Company, N. Y.

Among the newer tools which have proved their value to the research engineer are electric resistance strain gauges. Messrs. Dobie and Isaac have written this compact summary on the subject, based on a wide perusal of the pertinent literature. The various types of gauges are described and a long table gives the characteristics of the various commercial types. The determination of static strain, and stress analysis are both treated at some length. Other chapters provide the necessary elementary electronics and discuss electronic bridge circuits. Numerous applications of strain gauges have been located by the authors and are described briefly. Extensive bibliographical references are given for those who wish to refer back to the original sources.

DIFFERENTIAL EQUATIONS, by Harry W. Reddick. Second edition, 288 pages, figures, 14 × 21 cm. New York, John Wiley & Sons, Inc., 1949. Price, $3.00.

This textbook, limited to ordinary differential equations, appears now in a revised second edition. The author has taken the opportunity of rewriting parts of the text as well as adding a new chapter on the linear equation of second order. Additional problems have been added and answers are provided for all.

CONVEYORS AND RELATED EQUIPMENT, by Wilbur E. Hudson. Second edition, 468 pages, illustrations, tables, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1949. Price, $7.00.

Newer methods of handling materials are reflected in this revised edition of Hudson's work. The use of pallets and skids with trucks and tractors is discussed, while additional data have been supplied on the subjects of pneumatic conveyors and belts. Two new chapters have been added, one dealing with special problems and their solutions, and the other with the increasing number of dust-explosion hazards that are encountered in materials handling. With the revisions it continues to be a useful work in the field.

DESIGN OF MACHINE ELEMENTS, by M. F. Spotts. 402 pages, diagrams, tables, 15 × 23 cm. New York, Prentice-Hall, Inc., 1948. Price, $6.65.

In the art of machine design it is necessary to know the fundamental principles involved in the design of the various elements of which a machine is composed. This textbook is limited to a consideration of these machine elements, the material being so developed that the chapters are independent of each other and may be studied in any order. A preliminary chapter considers fundamental principles while concluding chapters discuss Dimensioning and Details, and Engineering Materials. Problems, bibliographies and tables of properties of engineering materials are features of the book.

TABLE OF SINES AND COSINES TO FIFTEEN DECIMAL PLACES AT HUNDREDTHS OF A DEGREE, by National Bureau of Standards. 95 pages, 20 × 29 cm. Applied Mathematics Series 5. Washington, D. C., Government Printing Office, 1949. Price, $0.40 (paper).

This new table of sines and cosines with decimal divisions of a degree has been issued to meet the need for a high-accuracy table of this type. The columns of sines and cosines are arranged side by side for the benefit of those needing both functions for the same argument. Another of the many valuable tables being prepared by the Computation Laboratory of the National Applied Mathematics Laboratories.

CURRENT TOPICS

Seek Cosmic Ray Secrets in Far North Research.—An expedition of American scientists is seeking new clues to the secrets of cosmic ray activity at 20-mile altitudes, near "the top of the atmosphere," over the Canadian outpost settlement of Churchill on the shore of Hudson Bay.

The project is being sponsored jointly by the National Geographic Society and the Bartol Research Foundation of The Franklin Institute, Swarthmore, Pennsylvania. The National Defense Board of Canada is cooperating in the undertaking. It is part of a continuing program of cosmic ray research instituted in 1946 by the two sponsors and supported by the Air Force and the Office of Naval Research.

A major objective of the expedition is to develop experimental evidence whether the sun is surrounded by a constant magnetic field similar to that possessed by the earth. There is considerable scientific controversy on this subject.

Test For Theory

Dr. Martin A. Pomerantz of the Bartol Foundation, who heads the expedition, believes that a study of cosmic ray behavior at high altitudes in the Far North may lead to the answer for this riddle.

Data pointing to the importance of the project resulted from previous experiments conducted as far north as 52 degrees geomagnetic latitude and at altitudes of 30,000 ft., utilizing specially equipped B-29's. The geomagnetic latitude of Churchill, Manitoba, is roughly 69 degrees north (ten degrees greater than the more familiar geographic latitude), and this research program is being conducted at heights in excess of 100,000 ft., or more than three times the altitudes possible in the B-29 tests.

The theory under test is that cosmic rays should increase in intensity as the North Pole is approached, since one is moving farther away from the equator where the earth's magnetic field is the strongest.

However, if the sun possesses a magnetic field of sufficient magnitude, its effect would be superimposed on that of the earth's magnetic field. In that event, the intensity of these primary rays would remain constant north of a particular latitude.

To Use Geiger Counters

Geiger counters, the now familiar atomic age instrument for reporting radioactivity, are doing the high altitude detective work for the scientists in a number of free-balloon flights.

Each flight takes aloft a four-fold coincidence arrangement of Geiger counters. This arrangement, Dr. Pomerantz explained, acts something like a telescope in the sense that it produces a signal whenever a cosmic ray particle goes through in a certain direction.

All data are continuously transmitted by radio to a receiving station on the ground. Information regarding the atmospheric pressure, and hence the

altitude, the temperature within the apparatus, and cosmic ray intensity, likewise are received throughout the duration of the flights and recorded on a moving paper tape. A special trailer serves as the party's mobile laboratory.

Weather A Factor

In directing the program, Dr. Pomerantz will be assisted by two colleagues from the Bartol Foundation, Robert J. Kerr, and Robert C. Pfeiffer, both of Swarthmore, Pennsylvania. Dominion authorities promise that additional assistance will be provided by Canadian meteorological personnel stationed at Churchill, now the location of a defense installation.

Dr. Pomerantz and his party left for Churchill about August 1, and hoped to complete their program of field work in a month.

The August period was selected because it usually has the most favorable weather conditions for the work at the Canadian outpost—minimum surface winds at sunrise when the instrument-laden balloons will be launched, and very light precipitation.

Waning Rays Noted

Previous research has established that primary cosmic ray particles (those impinging upon the top of the earth's atmosphere) are affected by the earth's magnetic field when they arrive from an as yet undetermined source or sources in outer space. In fact, the earth acts like a huge mass spectrograph in separating particles of different energies.

This phenomenon gives rise to what is called the latitude effect. As one proceeds toward the equator, strongest zone in the earth's magnetic field, the rate at which cosmic particles arrive is observed to diminish. This is caused by the exclusion of rays of lower energy.

On the other hand, as the point of observation approaches the magnetic North Pole, it is expected the rate of the arrival of the primary cosmic rays might continue to increase with increasing latitude—unless there exists a solar magnetic field which interferes.

Diamant Manufacturing Corp. Announces New Photographic Equipment.—Jiffy Lens-Ex is a soft lint-free tissue especially created for all precision optical surfaces. It is packed in 100-sheet booklets in a handy 3 × 5 in. size, priced at 24¢ a booklet.

Jiffy Lens Coat is a superb cleaner for all precision optical surfaces designed especially for cleaning coated lenses. The handy 2-oz. bottle lists for 29¢.

The first of its kind, the Jiffy Brilliant Multi-Purpose Enlarger was created especially to fill the ever growing demand for an enlarging tool that will make evenly illuminated, ***brilliant*** enlargements from both 8 mm., 16 mm., moving picture films and from the new, popular, miniature cameras using 16 mm. film, as well as from 35 mm. films. Designed for professional ease in handling, the Jiffy Brilliant Multi-Purpose Enlarger, despite its extremely compact size, is a versatile instrument that will make enlargements to any size desired from either positive or negative films in both color and black and white.

Electronic Scale Developed to Weigh Livestock.—An electronic scale for weighing livestock at public markets, permitting greater accuracy and speed

and eliminating nearly all possibility of error or incorrect weights, has been developed under the direction of the Production and Marketing Administration, U. S. Department of Agriculture.

The new scale measures the weight of livestock through electrical impulses and records pressures electrically. When certain buttons are pressed, it prints automatically the weight, number, and type of animals, names of the weigher and selling agency, and the date and time of weighing. It is so constructed that it is impossible to register anything other than the weight of the load actually on the scale platform, and it is accurate to within five pounds on loads up to 32,000 pounds.

The device was developed by an aircraft instrument company, working under contract with the Department under the Research and Marketing Act of 1946. The work was supervised by the Livestock Branch of PMA. The company previously had produced electrical strain-gage cells which are a basic feature of the device and had used them in scales for weighing aircraft. Much modification of the aircraft scale was required to adapt it to use for weighing livestock, and extensive tests and trials at markets were necessary to assure dependability and accuracy under all weather conditions of the changed mechanisms.

Errors of several kinds are possible with the conventional weighbeam type of scale. Dirt accumulations, wear and corrosion tend to reduce accuracy. Dirt and manure on the scale platform are reflected in the weights recorded, unless the scale is frequently rebalanced. Rodents interfering with the weight beam also may cause errors amounting to several hundred pounds. In addition, carelessness or intentional manipulation of the scale by the weigher could produce an erroneous weight record, unless the scale is frequently rebalanced. Weight values indicated on the weighbeam ordinarily can be read only at a very short distance.

With the new scale, the weight is indicated on a large dial, easily readable from ten feet away. Rebalancing of the scale to allow for accumulations on the platform can be accomplished simply by pushing a button.

The electronic scale costs more than conventional scales, but its far greater speed in weighing animals, its higher accuracy, and other points of superiority are believed to more than counterbalance the extra cost. Some stockyard managements already are planning to install scales of this type. The Livestock Branch reports that the new scale undoubtedly will be applicable also to other industries using large-capacity scales.

A detailed report on the scale is being prepared.

New Drug for Parkinson's Disease.—To the thousands who suffer from Parkinson's disease, with its accompanying pathetic palsy and melancholy moods, there's a bright ray of hope for relief through the use of a new drug ARTANE* Trihexyphenidyl Tablets *Lederle* recently developed by Lederle Laboratories Division, American Cyanamid Company at the huge research and manufacturing plant at Pearl River, N. Y.

Without experiencing the side reactions that previous medication for this malady produced, patients taking ARTANE find that the tremors of the

* Reg. U. S. Pat. Off.

hands, the involuntary muscular spasms and the constant drooling are markedly reduced almost immediately after they start taking the tablets.

Furthermore, it has been found that in a high percentage of the cases where ARTANE is used, the mental outlook of the patients is considerably brightened and they are then more easily persuaded to take part in activities that may have additional therapeutic effects.

Physicians who have given ARTANE in clinical trials agree that it is the best palliative for Parkinson's disease that is available at the present time. In addition, they consider the new Lederle drug remarkable in that it is effective about 90% of the time against a condition as severe as Parkinsonism and in cases where heretofore the patients have had constant treatment for many years with little success. ARTANE is now available throughout the country for physicians to prescribe.

The tablets come in two sizes, 2 milligram and 5 milligram, and Lederle packages both in bottles of 100 and 1,000 ARTANE tablets.

New Film Illustrates on-the-Job Fabricating and Erecting Techniques for Low-Cost Trussed Rafter.—Builders of multi-unit dwelling projects now have an opportunity to follow, through motion pictures, actual on-the-job truss fabricating techniques which are permitting builders to reduce roof framing material and labor as much as 25 per cent.

The Timber Engineering Company, Washington, D. C., has just announced the availability, for free use by architectural and building groups, of a 16-mm. motion picture covering the mass manufacture and assembly of 3,500 timber connector equipped trussed rafters used in the Bonhaven Apartments, Richmond, Va. The picture is a single reel, black and white silent film which requires 18 minutes for projection.

The new film depicts in detail the quantity production methods used to fabricate, assemble and erect the new low-cost trussed rafter which has been used in over 17,000 housing units in the last three years. The rafter itself is a simple, four-member assembly composed of standard 2×4 and 2×6 lumber. It is designed to cover spans up to 32 feet.

The member sizes in the trussed rafter are small because the Teco Timber Connectors in the joints permit use of the full allowable working stress for the members.

Because of their extreme simplicity and economy of material, the trusses have been fabricated and erected in quantity at costs as low as $9.89 each in place. The basic design, which is available in four pitches, requires no left or right hand members and makes turning of the truss during assembly unnecessary.

Distribution of the film is being handled directly by the Timber Engineering Company. Copies may be secured by interested groups by writing the company at 1319-18th Street, N. W., Washington, D. C.

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EDITOR, HENRY BUTLER ALLEN, MET.E., D.Sc., Sc.D.

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THE STORY OF AIDS TO NAVIGATION

BY

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"There be three things too wonderful for me, yea, four which I know not: the way of an eagle in the air; the way of a serpent on a rock; the way of a ship in the midst of the sea; and the way of a man with a maid."

When the wise Solomon put the way of a ship in the category of things beyond his comprehension he must have been pondering over the tales of those seafarers who had brought the gold and ivory, the apes and the peacocks from far distant lands to adorn the great king's court. Their tales of finding a way over the trackless waters must have savored of necromancy to the landsman. His wonderment must be shared by every reflective person who has given thought to that magic.

The science of navigation has advanced to that stage of refinement, with mechanical aids and complex apparatus, that we are prone to forget there was a time when men sailed over wide stretches of ocean without aid from any of these devices. Vessels were accustomed to travel over rivers, inland seas, and short coast-wise voyages long before they deliberately ventured out of sight of land. But the time arrived when the traders of Asia and the bold islanders of Polynesia took advantage of the monsoons to travel across the Indian Ocean and the China Sea without benefit of instruments. We have no written record of these voyages and we can only surmise at the methods adopted by the pilots.

Coming to recorded history, we find the Arabs, the Phoenicians, and other Mediterranean nations engaged in a lively intercourse, both for peace and for war, which must have taken their vessels out of sight of land for short periods. The hardy sea-rovers of the North developed a means of finding their way over the sea and back to their home ports.

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From the very earliest records of these voyages and from later contributions to our knowledge of the sea, we catch glimpses of their navigational methods. It should be remembered that the sights of nature would be observed even more closely in those days than they are by the modern mariner, because they were the principal aids to following a course.

The stars were used by the early navigators to guide them on their courses. There are many references in classic literature to the use of stars as aids in navigation, as in that passage in the *Odyssey* where Homer writes: "The Bear which they likewise call the Wain, which turneth ever in one place and keepeth watch upon Orion, and alone hath no part in the baths of ocean. This star Calypso the fair goddess bade him keep ever on the left as he traversed the deep," when Ulysses departed from her island home. A less known but more significant passage relating to the practice of holding a certain star on the rigging, not unknown to helmsmen today, occurs in Lucan's *Pharsalia:* "Here always the Lesser Bear . . . stands on the ropes of the main yard; then do we look towards the Bosphorus."

Quotations on the subject could be multiplied, and quite naturally, for astronomy was the first of the sciences to be developed. The sky and celestial phenomena were more closely observed by the ordinary man in those days, and a knowledge of the stars, visible to the unaided eye, was more widespread than it now is.

Another aid to navigation was the study of the flight of birds, especially in their migrations. Then, we have authentic records of the use of shore-sighting birds which, when released, made their way to land, in the manner of Noah's dove. An early mention of birds being employed for navigation purposes occurs in the *Landnamabok* (Book of the Colonization of Iceland). Describing the start of Floki's voyage from the Shetland Islands to Iceland, it says: "Floki, son of Vilgerd, instituted a great sacrifice and consecrated three ravens which would show him the way; for at the time no man sailing the seas had lodestones up in the northern lands." A few days out from the Shetlands, one of the ravens was released and, after circling, saw the land astern and flew toward it. Floki then had a back bearing on his point of departure. Several days later, another bird was released but, as it failed to sight land, it returned to the ship. At the third attempt, the bird took a forward course and led them to land.

The sea-ravens of Viking times were the Cormorants or Shags of today. The Frigate or "Man-of-war" bird has been used by the Polynesians as a shore-sighting bird.

When the great voyages of Columbus' time are studied, it is seen that the navigators had many visual indications of the near proximity of land. Those navigators used acutely developed senses to discern cloud formations, the color of the water, turbulences caused by land

configuration, etc., to aid them. They learned to sniff the breeze and to interpret it to read the message it conveyed. It was only by cultivating his senses and utilizing his instincts that man learned how to overcome the immense difficulties involved in conquering the waste of waters. The literature of Greece and Rome contains frequent expressions of the awe in which the ancients held the unknown sea, and the dread with which they approached a voyage. As a picture to be looked upon from a comfortable place on shore, the sea was a thing of beauty, but to be compelled to trust one's self on a long voyage was a misfortune from which they shrank. "The flung spray and the blown spume, and the sea-gulls crying," had little allure to them. "Heart of oak and triple bronze had he who first committed his frail craft to the savage sea,'' wrote Horace, in admiring tribute.

There are three things essential to safe and comfortable navigation on the high seas, according to John Davis, who says in his *Seaman's Secrets*, published in the year 1607: "the Sea Compasse, Chart, and Crosse staffe are instruments sufficient for the seaman's use." The equipment of the modern ship has grown considerably more varied and complex than this, but the coastal vessel creeping cautiously from port to port, and rarely out of sight of land, had no need of instruments. Once the mariner pointed the prow of his ship into the open sea with intention of remaining out of sight of land, he had need of more than instinct and observation of nature to enable him to find his way back to port. In the time Davis wrote, the compass, chart, and cross staff were the essential instruments, but as man's knowledge increased he made use of whatever instruments the state of his civilization provided for ease of mind and safety.

The sea remains unalterable and unimpressionable, its capacity for service neither increases nor diminishes. We are still, with all our modern equipment, unable to whistle up a breeze or to quell the turbulence created by a gale. But in many other respects, great advances have been made in the manner by which man has found his way from one point to another on the surface of the globe. In the growth of knowledge there has been a natural progress in navigation, extremely slow in some periods and fast in others, in the development of the mariner's methods of following the sea's highways and byways. It is this development we now propose to follow, piece by piece. Stand by to get under way!

CHARTS

A chart is the representation on paper of a portion of the navigable waters of the world. Since most of our early records of sea voyaging are based upon records relating to navigation in the Mediterranean Sea, it is but natural that our earliest charts should relate to this tract of water. The explanation that we have no earlier charts resides in the

simple fact that earlier navigators had no need for position finding, they cruised within constant sight of land. We do not learn of them ever having found use for the sun, moon, and stars except to steer by them to maintain direction. They felt their way from port to port, and for this purpose required nothing but the simplest "sailing directions." These were contained in a *periplus*. The word signifies a voyage, or a "sailing around," and came to be applied to the book containing distances between ports, directions how to reach other harbors, dangers that lay in between, and other matters of interest to the mariner.

The *periplus* was the ancestor of our current *Coast Pilot*. Although there are many intervening steps there has been remarkably little change in either purpose or method, even the language employed is akin.

The "sailing around" books may very well have been supplemented by crude charts (although it is right to say that none has come down to us) since map-making for land purposes had already begun. Long before the time of Herodotus, the first real historian, maps of a sort were in use, for he speaks of a map of the world being made in Greece. The first actual sea charts, attributed to Marinus of Tyre, date from the first century of the Christian era. They were probably compiled from fragments of information furnished by mariners. Claudius Ptolemy discussed them, and adopted their system of projection for his better known land maps.

The method of nautical chart making was little better than the graphic presentation of notes compiled by shipmasters for their own use. The result could have been a fairly large scale chart of a short section of coast produced from the notes of men who required a considerable degree of accuracy in their information. The reports of more casual travellers were valueless to men whose lives and wealth depended upon their safe navigation. So few copies of this type of chart have survived that it is doubtful whether the making of large scale maps was a general practice.

An impenetrable mist conceals the methods adopted by the map-makers of the Dark Ages, but that some progress was made seems clear from the next type of chart we encounter. These charts were drawn to illustrate the "sailing around" books and, while the earlier ones are not accurate, they attained a degree of excellence in ornamentation which shows a genuine improvement in draftsmanship. Color was introduced with fine effect.

Portolan charts, as these were called, reached their peak of delicate drawing and rich ornamentation during the fifteenth and sixteenth centuries. They may not compare in minute exactness with the charts of today, but they are works of art that give a greater joy to the observer. Networks of fine lines drawn in colored inks radiate in the direction of the principal winds from various centers, known as

"wind roses." The principal, half, and quarter winds each had an individual color, and many another gracious note of color was added by the map-maker. It might be noted that these "wind roses" reappear on the cards of the early compasses.

The portolan charts were the maps used by the great explorers. Naturally, they were of no great assistance to the explorer who sailed an unknown sea to new lands. But the importance to which map making had arisen is reflected in the promptitude with which newly discovered countries appeared on published maps. There was as yet no improvement embodied in the compilation or projection of these maps over those of more ancient times. Care was devoted to greater accuracy in regard to distances, but every map was still a representation of the earth as a plane surface, ignoring the fact that it was, in reality, a sphere.

It was a strange world in which the early navigator found himself. A universe of phantoms floated around medieval Europe in which that continent saw itself as a small oasis, where life was normal, while mystery and magic veiled everything beyond.

To the west surged the ocean which no man had crossed, enigmatic and impassable as infinity; to the east and south all trailed away into the outer dark. The earth went on, men knew, but none could say just how. Strange stories filtered through from time to time like echoes from another planet, strange apparitions broke through the wall of ignorance, as when a man of an unknown race from the world's end was seen on the confines of Christendom, or else some greatly daring traveller went into the shades and disappeared.

The Crusades served to raise a corner of the veil. The crusaders touched the fringe of the unknown, heard and saw many new things. Thus Europe was made aware of Far Cathay, the Old Man of the Mountains, and Prester John, the Christian king who ruled a heathen continent. When medieval dreams gave place to the pursuit of facts, men set out to seek reality.

A prince of Portugal led the way. Henry the Navigator stood between two worlds—the dying Middle Ages and the dawn of the Renaissance. He was a crusader and man of science rolled into one. He established himself on the Cape of Sarges, the last point in Europe, where the Mediterranean Sea and the Great Ocean met, and set himself to solve the riddle of finding the way to India.

Dom Henry surrounded himself with mathematicians, astronomers, cartographers, makers of precision instruments, and builders of ships, collected from far and wide. With the men of theory came men of action, seamen, pilots, and adventurers out to see the world and to seek a fortune. Continually, he sent forth ships with orders to sail farther and farther south until they found the end of Africa.

They sailed cautiously at first, but soon with reckless ease when they found that the seas did not boil at the tropics, as those who were supposed to know affirmed they did. When they learned they could not sail home again against the current which swept down the coast, they did not hesitate to sail out of sight of land by making a wide sweep out into the ocean.

Thus, the Great Ocean began to lose its terrors for Henry's navigators. Not only did they build forts and trading posts along the African coast, but their caravels learned to sweep wider and wider toward the unknown west. Whether they reached the Antilles forty years in advance of Columbus, or Brazil fifty years before Cabral, is a question historians are still debating.

When Prince Henry died in 1460 the problems he had set himself were yet partially unsolved. The dream of a sea route to India had not been substantiated, but he had changed the world. He had swept away the phantoms with which men's minds peopled the Atlantic; the terror of the ocean had passed. No longer was a sea voyage a haphazard adventure, but a problem to be worked out by the rules of exact science. As a result, the world wore a wholly different aspect to the map-maker.

The mapping of the American continent proceeded as the successors of Columbus exploited his discovery of the new continent. Most of them started under the impression that somewhere west of Europe was a short cut to the treasures of the East, and they were not surprised to find islands in the ocean. For land had been represented on their maps and globes before Columbus' time. The existence of these islands had no foundation in recorded fact, but were remnants of ancient beliefs that mythical islands, like Atlantis, were located in the oceans. Having once found America, it was not long before the explorers realized the new lands formed a great continent dividing two oceans.

The idea had not taken root when the name America first appeared on a map published by Martin Waldseemuller in 1507. Waldseemuller was a theoretical map-maker, not a traveller, and he constructed his map from the dubious accounts of Amerigo Vespucci's travels. The map-maker himself evidently came to doubt the authenticity of Amerigo's claim to fame, for he changed the name of the new land to Terra Incognita in his map of 1513, but it was too late to change the habits of a people who had come to designate the land by the name bestowed upon it by a map-maker of high repute, and America it remained.

Balboa added considerably to knowledge of the mainland and was the first white man to obtain a glimpse of the Pacific Ocean from the American side, but it was not until Magellan made his famous voyage in 1520–1521 down the east coast of South America, through the Straits which now bear his name, and out into the Pacific Ocean, that the truth came to light that a great continental barrier lay athwart the way to

the Orient. Magellan died on the voyage, but a handful of his followers pushed across the ocean and, after terrible hardships, rounded the Cape of Good Hope to complete the first circumnavigation of the globe.

From this time onward it became a matter of filling in the detail of the maps of the world, a task which is not entirely completed. A new difficulty arose in presenting the curved surface of the globe on a flat piece of paper. Even at this very early stage, dissatisfaction was expressed at the errors which arose from this difficulty, and suggestions were being advanced for their remedy. The method of "great circle" sailing, that is following the shortest distance between two points on a spherical surface, was being practiced. Great circle sailing is like following the line of vision were it possible to see the point of destination. Because the surface of the earth is curved, it is necessary to change constantly the compass course. On a Mercator chart a great circle course must be shown by a compensated curved line. This commonplace of modern navigation and the deficiencies of the portolan charts to encourage its application were set forth in print by Pedro Nunes in a treatise on the sphere published in 1537.

The defects in the majority of charts were not grave because the areas they covered were small, and the errors were proportionately trifling, but when it became necessary to find space on a relatively small map for a representation of the expanded world, the errors became serious.

Sailors and scholars who were conscious of the existing defects were unable to advance an immediate remedy. Their problem was to flatten a portion of the earth's spherical surface in a plane, to level it out so that the parallels and meridians would retain their correct relationships. In the year 1569, Gerhardus Mercator made an attempt to overcome the difficulty when he published his map of the world for the use of navigators, which was thought to provide a happy solution to the problem.

Mercator's method of projection was to conceive of the world globe as expanded on the inner surface of a cylinder. The parallels fell into place as horizontal lines and the meridians crossed them vertically at right angles. When the cylinder was flattened out (as one opens a roll of paper) it was laid in a rectangular plane. The distances and proportions near the equator are correct, but the earth's surface north and south of the equator is progressively distorted as one nears the poles. These charts will be recognized by the straight meridians which, if prolonged, will not meet, when, in fact, all meridians meet at the poles.

Thirty years after Mercator published his map of the world which, by the way, did not at first appeal to mariners, an English mathematician named Edward Wright came forward with a set of rules and tables to be applied in an improved construction of Mercator's projection for the correction of some errors. With Wright's work, the task of modern chart making had begun.

Later efforts to arrive at strict accuracy have related to stretching the map over a cone instead of a cylinder. This form of projection is used mostly for representations of continents and smaller areas because some parts of the globe can be shown in conic projection with relatively small distortion. Variations of the conic projection have been advanced, notably the polyconic (many cones) method which was employed by the International Map Committee in 1909, for the construction of their map of the world. The method adopted by the Coast and Geodetic Survey of the United States is known as the Lambert Conformal Conic Projection. Essentially, this consists of small sectional maps and information derived from aeronautical surveys being fitted together in one large map.

The modern nautical chart bears much information of interest to the mariner, such as depth soundings, lighthouse locations, permanent buoy positions, etc., in addition to topographical information. All small but important corrections occurring subsequent to the date of publication are issued in the *Notice to Mariners*, and should be entered by hand upon the chart immediatley after receipt of the notification.

The accuracy of these charts is proverbial, as Hillaire Belloc affirms when he writes: "I pick up my charts, I read my various 'Pilots' . . . and the truth comes out, august, white-robed, with level brows, contemptuous of advocacy. The documents of this great Department please me like the Creed. Their level voice is the voice of doom."

SOUNDINGS

Although the modern charts give information relating to soundings, no navigating officer would be held guiltless in the grounding of his ship unless he nad been taking frequent soundings. A knowledge of the depth of water under the keel of the ship is a necessity to safe navigation when moving into or out of port, or when approaching an anchorage, shoal water, etc.

Lead and line were used for this purpose at a very early date, as we know from references in Herodotus and the Bible (*Acts*, xxvii, 29). The soundings taken up to the time of the Great Voyagers rarely exceed two hundred fathoms. The apparatus used was a simple weight on the end of a line. As navigation began to expand, many ingenious minds were applied to the problem of making deeper soundings and of escaping the arduous task of hauling in the heavy weight after taking a reading. One of the devices produced was a weight without a line which detached a float when it reached bottom. It was proposed to calculate the depth of the water by measuring the time required for the float to reach the surface.

The first deep sea soundings were not made until 1773, when a depth of 683 fathoms [2] was recorded in the Norwegian Sea, but thereafter

[2] A fathom is six feet.

Polar explorers were conspicuous for their efforts to make deep soundings. A great impetus was given to this form of deep sea investigation by Captain Mathew Fontaine Maury, U. S. N., who is regarded as the father of oceanography.

Depth of water is not the only information furnished by the lead. As Kipling's Dan, in *Captains Courageous*, said: "'taint soundin's dad wants. Its samples. Grease her up good, Harve." Harvey followed his instructions by putting a plentiful supply of tallow in the shallow cup on the end of the lead. This enabled him to draw up sand, shell, or whatever might be on the sea bed. Disko carefully fingered and smelled the product before giving his judgment upon the vessel's location.

The end of the lead is hollowed to form a cup into which is variously placed tallow, heavy grease, or yellow soap. The original purpose was to enable the seaman heaving the lead to determine that it had actually touched bottom. The nature of the material clinging to the grease reveals the character of the sea's bottom at that point. Sailors acquire such familiarity with what the lead recovers that those on an often repeated voyage will know their location and plan their next moves like a blindfolded chess player.

The hand lead is still indispensable on many merchant ships up to twenty fathoms. It is simply a narrow block of lead weighing from seven to fourteen pounds attached to a marked line. The line is not always marked in the same fashion but the commonest way is to mark it at 2, 3, 5, 7, 10, 13, 15, 17, 20, and every subsequent five fathoms. The marks are unique. They consist of two strips of leather, three strips of leather, a white rag, a red rag, a leather with a hole in it to mark the depths up to ten fathoms. Those for 13, 15, and 17 fathoms are the same as for 3, 5, and 7, but from twenty fathoms on the line is marked by short pieces of line with knots in it.

For greater accuracy in shoal water or when surveying, a line marked in feet is used.

Several quaint calls and actions arise from the heaving of the lead. It is customary to report a sounding which agrees with one of the fathom markers with the call: "By the mark three, or five, etc." and from the common call in the shallow waters of the Mississippi, "Mark Twain," our popular novelist, took his pen name.

The deep sea (or "dipsey") lead, usually weighing thirty to one hundred pounds, is handled in similar fashion, except that it is too heavy to be manipulated by a single man. Instead, the lead is dropped from the bow, and men are stationed at intervals along the deck, each with ten or twenty fathoms of a continuous rope coiled in his hand. If the lead has not touched bottom when the sailor's rope has paid out, he calls the traditional "Watch-o-watch" as he abandons the rope to the next man in line. The sailor who feels the rope slacken as the lead

touches bottom reports the sounding. All hands then turn to the arduous task of hauling in the lump of lead and its heavy line.

The deep sea lead has been largely replaced by the sounding machine, of which there are several types. Nearly all are designed to make use of the principle that water pressures increase with depth at a regular rate. The lead, weighing twenty-five to fifty pounds, has a long thin hollow handle which acts as a protection for a glass tube, open at one end. The glass tube may be ground glass or ordinary glass with an inner coating of chemicals. Either tube will discolor to indicate the height to which the water rose within the tube, and from this indication of pressure, the depth may be calculated.

This lead and its tube are lowered at the end of a long piano wire coiled on a reel, operated by hand or by a small electric motor. A dial on the reel shows the amount of wire which has been paid out, and this record is used to check the depth indicated by the glass tube. Records made with the sounding machine are frequently liable to a considerable margin of error when the ship is underway or when other conditions are unfavorable.

The most efficient type of depth sounding device is the Fathometer which is based upon the length of time required by sound to travel through water. This instrument comprises three essential elements: (1) an oscillator to produce the sound, (2) a hydrophone for receiving the echo, and (3) a suitable device for measuring the interval between the emission of the sound and receipt of the echo, thereby indicating the distance the sound has travelled.

In general these instruments emit a sound near the keel. The sound wave travels vertically down until it strikes the bottom and is reflected. It is picked up on its return by the hydrophone, located in the skin of the ship, and the time elapsed between emission and reception is measured. As sound travels 4800 feet per second in salt water at 60° F., half the distance travelled by the sound wave (which is water depth) can easily be calculated automatically in the instrument. The instruments vary in their manner of recording depth. All show the soundings by light flashes on a dial, and some keep a permanent record on a strip chart.

These echo sounding machines possess distinct advantages over all other methods of measuring the depth of water because they give instantaneous readings, and measurements can be made as frequently as the navigator desires. The most striking instance of their accuracy was in locating the wrecked *Lusitania*. One type of instrument is so sensitive that it is claimed it can be used in locating the depths of shoals of fish.

Buoys

Although buoys are not used for soundings, they are so frequently used in navigation to indicate a safe channel that they are dealt with

here for want of a better place. Buoys are used for a variety of purposes along the coasts and in harbors. In the United States, on approaching a channel from seaward, red buoys are held on the starboard hand in passing, and black buoys mark the port side of the channel. As a rule, starboard buoys are nuns (conical) and port buoys are cans (cylindrical).

Buoys painted with horizontal bands of black and red mark obstructions; those with black and white perpendicular stripes mark the center of the channel and must be passed closeby in order to avoid danger. Light buoys mark both sides of a channel, or a danger off the coast. Red lights mark the right hand (starboard) side of the channel; white lights mark the port (left) side. Spar buoys may take the place of either nuns or cans. Whistling or bell buoys are usually placed at the entrance of a channel or off the coast. White buoys mark anchorages.

THE COMPASS

Although it is generally supposed that the nautical compass was one of China's many contributions to the benefit of mankind, this attribution of its original source is now being disputed. Historians of science appear unable to affirm definitely by whom and at what time it was first learned that a needle touched with magnetite or lodestone, pointed to the Pole. It is true, however, that men were using these compasses before they ascertained that the needle pointed to the magnetic north and not to the true north. It is equally true that the Arabs, the Etruscans, the Greeks, the Vikings, and the Italians, as well as the Chinese, were aware of the pole-pointing property of the magnetic needle long before it was applied to nautical purposes.

From whatever source the first compass came, its use on shipboard appears to have been in the crude form of a magnetic needle "six yaches long, or longer, on a pinne" that was thrust crosswise through a straw or a splinter of wood, and floated on the surface of water in a bowl. Such a compass is mentioned in the work of the Augustinian abbot Alexander Neckham of St. Albans (*circa* 1180) who further says that it was used by mariners who were unable to see the Pole Star by reason of obscurity. This is probably the first reference we have to the mariner's compass in any language. There are several more references to the "water compass" before the first pivoted compass is recorded.

This occurs in the *Epistola de Magnete* of Pierre de Maricourt, commonly called Petrus Peregrinus, written in 1269.[3] His account is not only the first description of a pivoted compass but also of the surrounding graduated circle with 360 degrees, and an external rule or alidade for taking sights. From this device the mariner's compass as we know it today was gradually evolved.

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[3] A translation of this rare letter will be found in the Elihu Thomson Collection in the library of The Franklin Institute. The part relating to the pivoted compass is on p. 28 of this translation.

Refinements came from many quarters. The pivoting of the compass card so that it turned with the needle (called a "fly") is mentioned by the Italian writer de Buti as in operation in 1380. He says that sailors use a compass "at the middle of which is pivoted a wheel of light paper to turn on its pivot, on which wheel the needle is fixed and the star (windrose) painted." Only one symbol used on these earlier compasses remains in use today, that is the conventional *fleur de lys* used to mark the north point. This design developed from the windrose which was used on old maps and charts before the compass card had come into general use. The eight points of the compass were represented by the eight principal winds of the Mediterranean—Tramontano, Greco, Levante, Scirocco, Ostro, Ponente, and Maestro—as delineated in the Temple of the Winds in Athens. The north point was represented by a spear or arrow head combined with the letter T, for Tramontano, and it was from this combination that the *fleur de lys* evolved about the year 1492. The east point was generally marked by a cross instead of the L for Levante at about the same time, and will be found on old English compasses down to the year 1700.

The eight compass points were expanded to thirty-two. Chaucer, writing in his treatise on the Astrolabe in 1391, says: "ship men rikne thilke partiez in xxxii."

With this compass to guide them mariners could now abandon their costal sailing, and the triumph of oceanic exploration may well be claimed for this invaluable instrument, for without the confidence inspired by its guidance Vasco de Gama, Columbus, and Magellan would hardly have dared venture so far into unknown waters. The voyage of Columbus was to re-act upon our knowledge of terrestrial magnetism in leading to the discovery of magnetic variation.

The card was fixed and placed at the bottom of the box below the needle by compass makers of Nuremberg and Bruges toward the end of the sixteenth century, and very shortly afterwards the gimbals, or suspension ring for counter-acting the rolling and pitching of the ship, were introduced. Thereafter improvements came slowly, although there are frequent mentions in nautical literature of the unsatisfactory performance of the compasses in use. By the year 1750 the needles of merchant ship's compasses were made of two pieces of steel bent in the middle and united in the shape of a rhomb, or spinning top. Godwin Knight of Oxford substituted straight steel bars of small breadth, hardened throughout and suspended edgewise. He demonstrated that this Chinese method of suspension conduced to sensitivity. Knight long kept his invention secret, and another man received public credit for the idea in 1751 when the Royal Society awarded the Copley Medal to Canton for this very invention.

After Knight's death compass makers either lost the art of making good needles or they became uncommonly careless for, in 1820, an

English mathematician called Peter Barlow informed the Admiralty that "half the compasses in the British navy were mere lumber and ought to be destroyed." He offered suggestions for compensating compass errors by introducing discs of soft iron to correct the ship's own magnetic effect, for iron was becoming more generally used in construction. His work was substantially the only major contribution to the improvement of the mariner's compass until the genius of Lord Kelvin was accidentally directed to the subject, and he produced a greatly improved instrument.

The compasses in use were sluggish in action and would sometimes stick entirely. Their construction was ill adapted to naval use because the concussion of gun-fire would throw them out of service. When a vessel rolled in stormy weather, the needle was inclined to oscillate so much that it made readings difficult. Some of these defects arose from the weight of the moving part, which Kelvin proposed to reduce.

He made a thorough investigation of the whole subject of the mariner's compass, and was able to advance suggestions for the remedy of most errors. To overcome the greatest defect, the permanent magnetism of the ship itself, he placed permanent magnets beneath the compass bowl, and to correct for any temporary induced magnetism of the ship's hull, he placed two globes of soft iron, one on each side and on a level with the compass card. Instead of a single needle, eight small needles of thin wire were fixed like the rungs of a ladder on two parallel silk threads and slung from an aluminum ring. The frictional error of this light moving part was thereby reduced to ¼ degree.

The Kelvin compass is now known as the "dry" compass because it was found to be useful enough on merchant ships but had limitations under battle conditions. It was superseded on naval vessels by the "liquid" compass. In the latter, the compass floats on a liquid (water mixed with alcohol to prevent freezing), and bears only slightly on the pivot.

As the magnetic compass needle points to a pole which is several hundred miles from the geographic pole, all readings on its card are subject to variations that differ widely at various points on the earth's surface. Then, additional compensation must be made for deviation due to magnetism hammered into the vessel during building and by magnetism induced in the soft iron of the ship's construction by the earth's magnetic field. Variation and deviation could become grave hindrances to safe navigation unless constantly observed and adjustments made.

About 1910, Elmer Ambrose Sperry applied the principle of the earth's rotation to a new device which he called the "gyro" compass. This is operated by a spinning gyroscope and is quite independent of the earth's magnetism or the magnetic attraction in the iron and steel of the ship's structure. Moreover, the needle points unswervingly to the

true north, so that corrections need be made for neither variation nor deviation.

The master gyro compass is located below decks. It has four essential features: the gyroscopic system or sensitive element, the phantom ring in which the former is supported, the mercury ballistic to overcome the effect of rolling, and the spider, in which the phantom ring is supported.

The gyroscope is a heavy wheel rotated by electric current at a speed of 6000 revolutions per minute. This form of compass relies upon two properties of this spinning wheel: (1) gyroscopic inertia which is shown by a considerable resistance opposed by the gyroscope to any couple tending to turn its spinning axle into a new direction, and (2) precession, which is the name given to the movement of the axle which occurs when such a couple is applied. The phantom ring is so connected that it must follow all the movements of the sensitive element about the vertical axis. The compass card is mounted on the top of the phantom ring. The mercury ballistic is a system by which mercury flows from one cup to another to make compensation for the ship's rolling. It is the equivalent of an ordinary pendulum, but working in the opposite direction. The spider element supports the phantom ring and serves to drive it to follow all movements of the sensitive element about its vertical axis. It also carries a transmitter which conveys the compass bearings to "repeater" instruments situated at various parts of the ship where bearings require to be known.

A course recorder is operated electrically by the master compass. This is a printed chart on which is recorded all changes of the ship's course and the times they take place. These course recorders are valuable permanent records. At the conclusion of a voyage they are usually detached and entered as part of the ship's log. They are admitted in court as legal evidence. In cases involving collision it is frequently impossible for the ship's officers to remember accurately the sequence of changes of course leading up to the collision. The course recorder does this very accurately, often marking the point of impact with an indication when the pen was roughly jogged.

The latest development of these devices permits the vessel's course to be laid upon a chosen bearing, and the ship's head is held automatically to that bearing by the gyro-mechanism, familiarly known as "Metal Mike." This form of automatic control is not used, of course, in narrow, busy, or winding channels, where frequent adjustments of the helm are required. It is reserved for use only when the ship's course remains constant over a long distance. On these occasions it serves a very useful purpose, for steering a ship by compass is a monotonous task and the best of human helmsmen will allow the ship's head to wander after the lapse of some time. The minimum use of the helm means a maximum use of the ship's propulsive power which the engines

are delivering. Human frailty and fatigue being what they are, it became wise to turn over control when possible to a mechanism which would instantly detect any deviation from the course, and apply the necessary adjustment to the rudder to correct it. This is done by the gyro-pilot.

THE PELORUS

In order to secure a wider field of vision around the horizon when smokestacks, masts, and ventilators interfere with the view from the

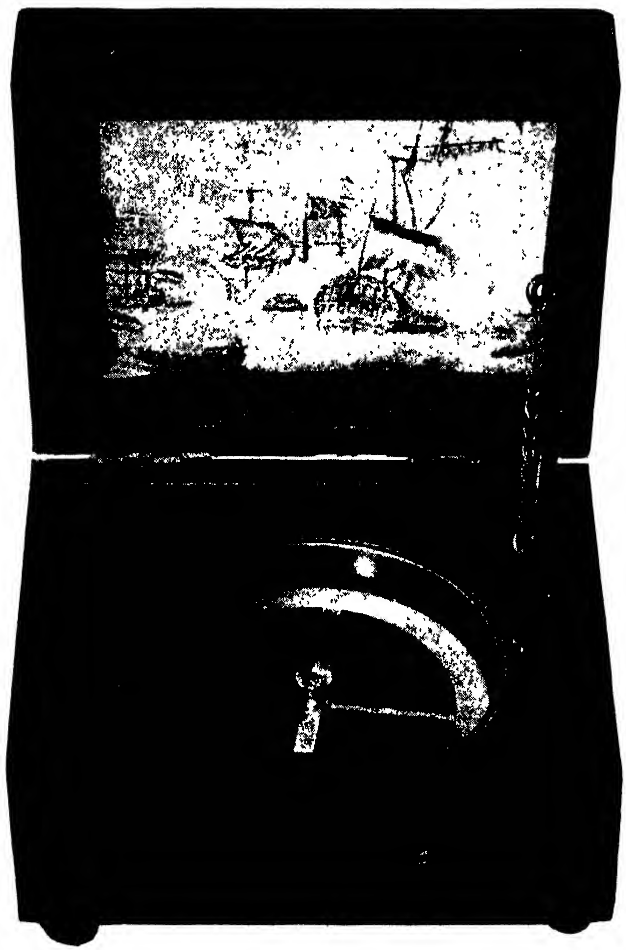

FIG. 1. Small pelorus, with colored print of Dutch men-of-war in lid.

standard compass, auxiliary means of taking bearings must be provided on the wings of the bridge. These take the form of gyro-repeaters if the ship is fitted with a gyro-compass; or a pelorus, if the ship carries only a magnetic compass. A pelorus is a dummy compass card without magnets. The card can be rotated by hand until any desired degree or point of the card is in line with the lubber's line, which is set to coincide

with the ship's keel. Sighting vanes are then used to take a bearing exactly as they would be on the standard magnetic compass. However, the dummy card must be set to read the true, magnetic, or compass course, and the necessary adjustments made in calculation.

RADIO BEARINGS

Although in no way connected with the use of the compass, the modern method of locating a position at sea by radio direction finding cannot be omitted. Government radio beacon stations located on important lighthouses and lightships transmit identifying signals which are picked up by means of an especially designed antenna carried by the ships. The operator turns this antenna until the signal he is receiving reaches its maximum volume. He then observes the angle through which the antenna has been turned. By a process of triangulation from two or more such bearings, the operator secures a very accurate location of the ship. Such bearings are necessarily relative bearings, and not magnetic bearings, that is they refer to the ship's head unless a gyro-repeater is installed in conjunction with the radio direction finder.

Another refinement of location finding is the sonic system of submarine signals transmitted from these lighthouses and lightships. These underwater signals travel with greater certainty from interference than do radio signals, which are subject to atmospheric distortion. A cruising vessel furnished with one of the underwater receiving apparatus may determine the bearing of the transmitting station. When the vessel is fitted with both radio and underwater receiving apparatus it is possible to determine distance with considerable accuracy as the radio and submarine signals are transmitted simultaneously, and the difference in the times required for the signals to travel through the two media is the basis for calculating the distance travelled. Sound travels through water at the speed of 4800 feet per second, while the rate for radio signals is practically instantaneous.

For aerial navigation, radio beacons are placed at convenient spots, such as airfields, and send out continuous signals forming a "beam" along which the pilot conducts his airplane.

Radar

During the progress of World War II scientists presented navigators with two aids employing principles which were entirely new, and which removed navigation from all dependence upon optical systems. The first of these was radar, used in nearly all phases of combat operations.

A radar set on a ship provides the captain and pilot with complete information about the objects around the ship. It will show distance and position of other vessels, buoys, islands, or land masses so that a safe course can be steered. The radar set is entirely self-contained, furnishing information to the operator without contacts with other

ships or shore bases. This feature is of importance under weather conditions that render radio communication difficult or impossible. Equally important is the fact that radar furnishes its information instantaneously.

In a matter of a few seconds a radar set is able to produce a map-like display of the objects around it. This display is produced on a screen resembling those used in television sets. If the radar set is on a ship in motion the display changes continuously as the vessel moves, and moving objects within range are seen to move on the viewing screen.

A radar set includes a special radio-type transmitter, an antenna, a receiver, and a viewing screen or indicator. Objects are detected by the transmitter sending out a pulse, or short spurt, of radio waves. The transmitter produces the energy which is sent out from the antenna in a narrow beam. The antenna is rotated so that the beam sweeps the area within range. Objects which lie within the path of the beam reflect back to the receiver a portion of the energy transmitted. The receiver is tuned to pick up the reflected energy, just as a radio receiver picks up the direct signals from a broadcasting station. The radar receiver passes the reflected signals through special electronic circuits to the visual viewing screen where they form the map-like picture which may be "read" by the operator.

Human vision is assisted by telescopes in daytime and searchlights at night but fog can render these instruments relatively useless. Radar sets, employing short radio waves, will penetrate the fog and will be reflected.

Although the radar viewing screen does not present a picture with the same clarity, it may be likened to a motion picture. Island and coast-lines are represented with close approximations to their true forms, while smaller objects, such as ships and buoys, are shown as specks. Since all such objects, whether moving or stationary, must be avoided by the navigator, their appearance on the screen as a speck of indefinite outline is sufficient warning to ensure caution.

Loran

While radar was quickly adopted as an invaluable aid to navigators it was restricted as to range. The principle of using radio signals was modified and combined with time measuring techniques to produce Loran, the long range navigation device. In the employment of Loran, two ground stations transmit a series of pulses in such a manner that the pulses of one are distinguishable from those of the other. The interval from the emission of a pulse from one station to the emission of a pulse from the other has a fixed, known value. The interval between these pulses as they are received by the navigator depends on his location. It will be equal to the fixed value if the vessel is equally distant from both transmitting stations. It will be greater than the

fixed value if he is nearer to station A, for then the pulse from station B, having farther to travel than the pulse from station A, will fall behind it in time. It will be less than the fixed value if the ship is nearer to station B, for then the pulse gains by its shorter journey. Every distinguished interval characterizes a different hyperbola of position, which is a fixed line on the earth's surface and may be computed and drawn on a chart.

Two pairs of stations (which may be three stations grouped in different pairs) define two intersecting lines of position, forming a Loran grid. Two observed time intervals, one from each pair of stations, define an intersection or point. The navigator is furnished with a receiver having a cathode-ray tube, on the face of which are displayed the successive incoming pulses. The time intervals are accurately read on numbered dials. The navigator enters the chart with the numbers as read, and finds by inspection the corresponding point among the grid lines of the system. Fixes of position may be obtained at distances of 1400 nautical miles by using reflections of the signals from the lower ionosphere.

QUADRANTS, SEXTANTS, ETC.

For their short coastal voyages, the ancient mariners found little need for instruments to aid their navigation, but when trade between the South and North of Europe through the Strait of Gibraltar began to develop, navigators experienced the need of a reliable means of locating their position at sea. The simple need became a matter of pressing urgency when the great voyagers, turning the prows of their ships into the south, lost sight of their one constant guide, the Pole Star.

The use of astronomical instruments of observation for position finding at sea is predicated upon the existence of tables which show the declination (distance north or south) of the sun from the equinoctial (celestial equator), and of star positions. Latitude was ascertained by measuring the sun's angle of altitude at its meridian height, plus or minus the angle of declination for the day of observation. Tables of the sun's declination had been compiled before the time of the Ptolemies, but it is difficult to say when they were introduced for nautical purposes. Manuscript copies of these tables were to be found in the observatories and other places of learning, but it is improbable that many ship captains possessed them before the time of the great voyages.

The first tables to appear in print were restricted in scope to a period of one or two years. The most comprehensive set was published in 1496 by Abraham Zacuto in his *Almanach Perpetuum*. This work was known to Columbus in his later voyages. But not until 1509 was there a set of tables which could be called satisfactory to the unlettered shipmaster. This came with the publication in Portuguese of the *Regimento do Estrolabio*. From this time onward there was less difficulty about the determination of latitude from the sun's declination.

Other tables of star positions quickly followed to meet the needs of those who were compelled to rely upon astronomical observations to find their positions at sea.

The *astrolabe* was the first mechanical device used for measuring angles. It had been in use for many years (it was known to Hipparchus in 150 B.C.) but in the service of astronomers it had acquired a degree

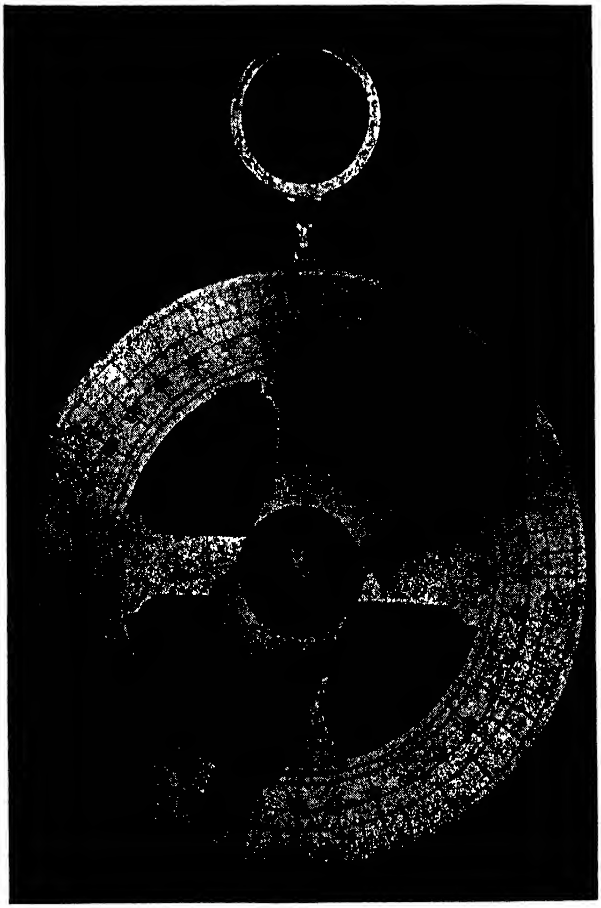

FIG. 2. The astrolabe, assumed to have been invented by Hipparchus in Greece in about 150 B.C. and used by Persians, Arabians and Moors during the Middle Ages and later by Europeans.

of complexity which rendered it unsuitable for the less scientifically minded mariner. By the year 1300, however, a simplified form had been devised which came into maritime use.

In this simple form, the astrolabe consisted of a graduated circular disc, six to eight inches in diameter, with a revolving sight bar pivoted at the center of the circle. The observer held the instrument on his

thumb freely suspended from a ring, and directed the sight bar to the celestial body whose angle he wished to measure. The angle between the angle of the sight bar and the horizontal base line on the astrolabe was then read on a graduated scale.

The astrolabe was never an accurate instrument in the hands of a seaman on the swaying deck of a ship, and it is not improbable that its principal function was to check the hour by determining the mid-day position of the sun. It remained in use until the eighteenth century, and was held in esteem by Italian seamen long after it had been abandoned by other sailors.

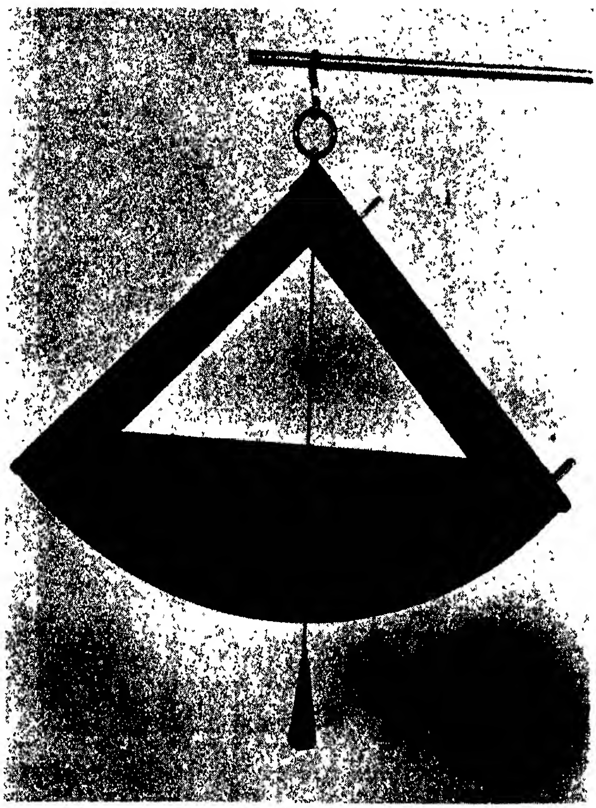

FIG. 3. The quadrant, used at the time of Christopher Columbus.

The *quadrant* had also been in use on land before it was adapted to nautical use. From an entry in Columbus' log we learn that his quadrant was a clumsy affair, only an arc and a plumb line. The arc was a graduated quarter circle and the ends were joined by two wooden bars forming a right angle. A weighted cord was suspended from the vertex of the angle so that it fell across the graduated scale of the arc. Peep

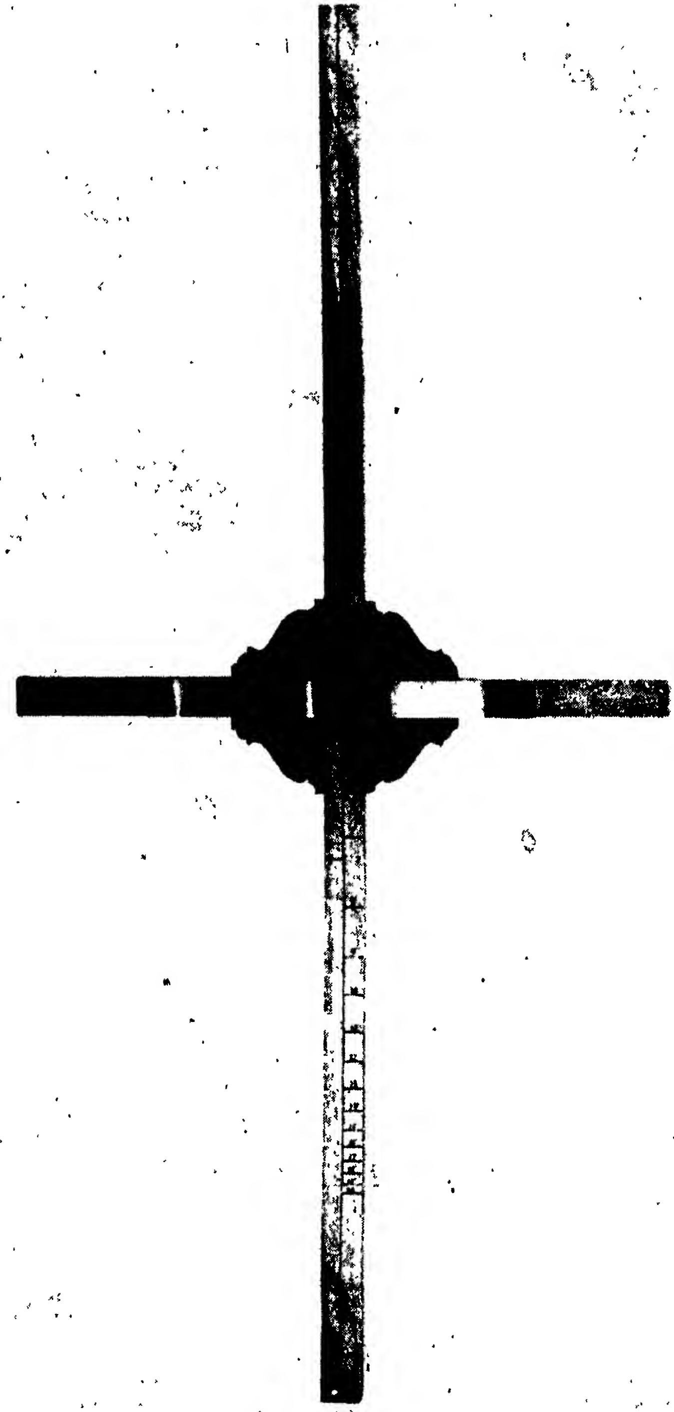

FIG. 4. The cross staff, used in the 16th and 17th centuries. The observer had to look directly at the sun.

sights were later added at the ends of one straight edge, and when these were sighted on the sun or another star, the cord registered the height of the object in degrees.

Like its predecessor the astrolabe, the quadrant swung too freely when used on the moving deck of a ship, and its readings were rarely accurate. Both instruments were inconvenient because the face on which the scale was marked could not be read by the observer while he held the instrument to his eye. However, the quadrant was less cumbersome and expensive and was rather more accurate in a rough and ready way than the astrolabe, but because of its defects it need cause no wonderment that much study was devoted to its improvement.

The *cross-staff*, fore-staff, or Jacob's staff, is probably an evolution from Ptolemy's parallactic ruler. It had not appeared in general use in Europe when Da Gama, having rounded the Cape of Good Hope, came in contact with Arab sailors who were familiar with some such instrument. Although it came to bear the very undignified name of "the hog's yoke" when it became popular among English seamen, its proper name was reverently derived from the staff of Saint James, patron saint of pilgrims, because of the resemblance in shape. While deceptively simple in construction and appearance, the cross-staff was not less learned in its mathematics than was the astrolabe.

Briefly, it consisted of a light staff about twenty-seven inches long, crossed at right angles by a shorter arm (sometimes by two arms) that could be adjusted. Upon the longer shaft was a scale indicating ninety degrees of angular measurement. Sighting along the shaft with the adjustable bar in a vertical position, the observer moved the bar back and forth until one end rested on the horizon (the only "fixed" point on the open seas) and the other upon the sun. The figure upon the scale at which the adjustable bar had come to rest gave the required angle.

When the sun stood well overhead at noon, as it does in the lower latitudes, it was difficult to use the cross-staff to measure its angle. For this reason, the great Portuguese and Spanish discoverers, who did much of their exploration in these latitudes, held the astrolabe in higher esteem than did the English.

By using the instrument with the cross arms in a horizontal position it was possible to measure the angle between two points on land, and by laying out the angle on a chart, the shipmaster could fix his position quite accurately.

The *nocturnal* was an auxiliary instrument used to supplement the readings made with the astrolabe, quadrant, or cross-staff. This instrument comprised two metal circles, one graduated to correspond to the months of the year, while the inner circle was graduated to correspond to the twenty-four hours. Observations were made by setting the circles to the correct date, and peering at the Pole Star through a hole at the common center of the two circles while moving a long arm,

pivoted at the peep-hole, until its free end pointed to the two stars in the Little Bear farthest from the Pole Star. The reading made by this bar against the scale of the inner circle gave the approximate hour of the night.

The *backstaff* was a vast improvement over any of its predecessors. It was developed by Captain John Davis, a celebrated English navigator, from an earlier instrument he had devised in the year 1594. In all previous instruments the observer had been compelled to look in two directions at the same time when making a reading. Moreover, he had been forced to look directly at the sun, and was thus blinded by its glare unless it was hidden by a cloud.

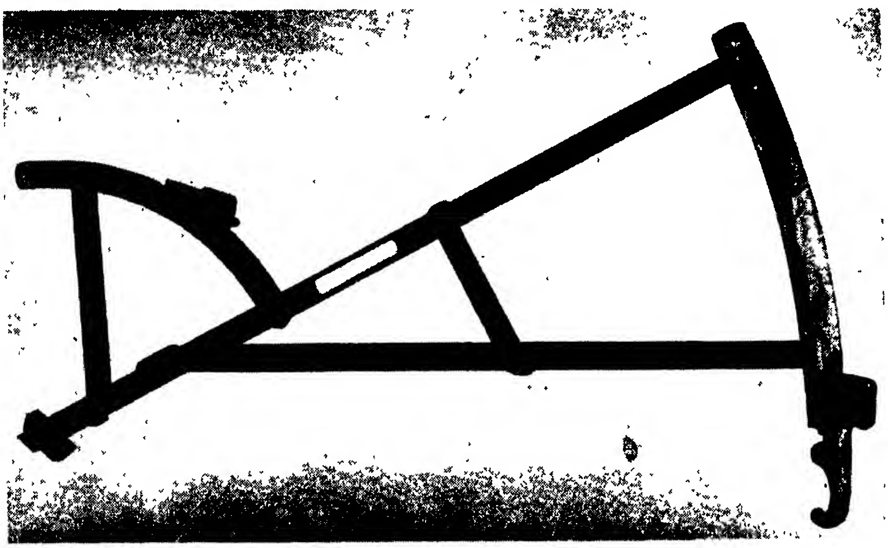

FIG. 5. The backstaff, improved by Captain John Davis in 1590 and which can be used with the sun behind the observer.

Davis' backstaff was a quadrant divided into two arcs, one of sixty degrees of about six inches radius, and the other thirty degrees of two feet radius. The observer stood with his back to the sun and sighted the horizon through a small hole in the thirty-degree arc and a slit in a vane at the other end of the instrument. When the horizon was correctly sighted, the observer moved a shade vane working on the sixty-degree arc, until he brought the sun's shadow into coincidence with the slit through which the horizon was sighted.

The instrument was constructed entirely of wood, including the sighting and shadow vanes. It acquired a considerable popularity under a variety of names, as double quadrant, English quadrant, etc., and was the favored instrument of American navigators well through the eighteenth century.

Meanwhile, several refinements which were to add greatly to the perfection of the sextant when it arrived, had come into use. The

Vernier scale had been introduced for greater accuracy in readings; Gascoigne had applied the telescope to the quadrant; and Hevelius had invented the tangent screw to provide slow and steadier adjustment.

FIG. 6. The improved quadrant, a late 18th century instrument made by Thomas Helmsley, London.

The *sextant* may be said to have had its origin in a suggestion offered by Robert Hooke, the Oxford scientist, in 1666. He suggested that by using a reflecting mirror in the quadrant it would be possible for the observer to see the two objects between which the angular distance had to be measured, and to see one superimposed upon the other. Shortly afterwards, Newton recognized the value of such an instrument for navigation. He designed one, but no sextant made by or for him has come to light.

The modern reflecting sextant was invented independently in America and England. Thomas Godfrey of Philadelphia (in whose house Benjamin Franklin lived for some time) was a glazier by trade but a mathematician of no mean caliber. He constructed a sextant in 1730 which was used by Joshua Fisher in his survey of the Delaware Bay, and later by George Stewart, mate of the sloop *Truman* (George Cox master) on voyages to Jamaica and Newfoundland. This sextant (an improvement of an earlier one of uncertain date) was described in a letter by James Logan which was not published until 1734.[4] The Royal Society had received a similar instrument from an English country gentleman named Hadley, who had been working independently upon the instrument. Being unable to determine which of the two contestants for the honor of priority was entitled to it, the Society awarded both a prize equivalent to $1000. In Godfrey's case this took the form of household furniture, since the Society had evidently been informed that he had a weakness for alcoholic liquors.

In spite of the simultaneous invention, the sextant became universally known as Hadley's sextant. Actually, it was an octant, that is to say, its arc covered 45 degrees. Hadley's principal superiority over Godfrey's instrument lay in the use of a telescope along the observing radius. The sextant is held vertically in the hand to measure the altitude of an object, or horizontally in the plane passing through two objects to measure angular distance. The objective glass was only half silvered so that the observer could sight directly upon one object while bringing the reflected image of the other into coincidence.

Originally sextants and octants were constructed of wood, the arms being from 19 to 22 inches in length, as compared to the modern instrument's 6 or 8 inches. Brass and ivory fittings were added for their embellishment, as were shade glasses, Vernier scales, etc. The wooden construction proved to be unsatisfactory, for in damp climates the material would warp and bend. To overcome this defect, the metal construction was introduced in the middle of the nineteenth century. These were larger still than the modern sextant. Reduction in size was effected during that century, when sextants of brass and bronze frames with silver arcs became uniform. They were true sextants of 60 degrees, with the arc divided into 120 graduations. The index arm

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[4] *Philosophical Transactions of the Royal Society*, Vol. vii, p. 672.

had a Vernier scale furnished with a magnifying glass for fine reading. The better instruments are fitted with other refinements to meet varying needs and conditions.

As it is now made, the sextant combines the greatest simplicity with the greatest precision. It is probably the most perfect appliance for hand manipulation that has been invented. No one who has not held a sextant in his hand and seen how local time can be determined to a second, or the latitude can be fixed to within a few hundred yards, can realize the beauty of this instrument, nor the sense of power it gives to the user.

Difficulties have arisen in taking altitudes of celestial bodies when the horizon was invisible or indistinct, as in hazy weather, so that a new instrument was developed which provides an artificial horizon mark. This instrument is called a *bubble sextant* to distinguish it from the conventional instrument. A floating bubble serves the purpose of the horizon and, in taking a sight, the instrument is adjusted until the reflection of the celestial body is brought to rest in the center of the bubble. The bubble is difficult to control owing to the motion of the ship or airplane so that the error in a measured altitude of a celestial body is prone to be greater than when the visible horizon is used.

Since the sextant is an instrument for measuring angles, mention might be made here of the *stadimeter*, which is another instrument used for measuring small angles to calculate distances. Its principal use is to enable a vessel moving in formation to maintain its allotted position.

Neither the great navigators of the Age of Discovery nor any others for two more centuries had the means of determining their longitudinal position with any degree of accuracy. The best they could do was to keep a record of their compass course and to make an estimate of the distance covered on each bearing. When these were computed and transferred to the chart, the navigator could calculate his position east or west of the point of departure. This method was called "dead reckoning," and it will be realized that it was no easy matter to calculate absolute distance when the vessel had to tack frequently in contrary winds, or was buffetted by storms. One early writer speaks of longitude as determined by the mariner of his day as the "imaginative position," and this was probably more nearly true than the writer suspected.

The instruments available before the eighteenth century were incapable of rendering accurate results. When Champlain wrote his treatise on navigation in 1632, he expressed the pious conviction that the good Lord did not intend that man should so penetrate the mysteries of creation as to be able to determine longitude at sea. Philip of Spain adopted a more optimistic attitude (at least as regards navigation), for he offered a rich reward to the man who discovered the means of determining longitude. Later, the British Parliament offered a still higher reward, and the search for the missing link in navigation was diligently pursued.

The method of dead reckoning with inadequate instruments not infrequently resulted in shipwreck. In at least one instance the course of empire was affected by an error "in the logg" of 45 minutes. Eight British ships of war were wrecked in the mouth of the St. Lawrence in the year 1711, when a joint expedition from New England and the parent country was frustrated through the loss of ships and men due to faulty navigation. Another paralyzing loss occurred when dead reckoning showed an open sea ahead of Sir Cloudesley Shovel's squadron when, in reality, they were close in to the shores of the Scilly Isles, where they were wrecked.

The Log

In order that the distance travelled might be calculated the navigator had to have some apparatus for measuring time and speed. It was the inadequacy of the equipment which handicapped the earlier navigators. A rough and ready method of calculating speed employed some 300 years ago was to walk aft, keeping pace with bubbles or some floating object close alongside, chanting a certain ditty as one walked. When the words of the ditty were ended the distance walked along the deck was measured and a unit of measure was thus obtained. More accurate means were available at the time had they been developed. Writing shortly before the Christian era, Vitruvius describes a device then in use on boats propelled by oars. This employed a rotating wheel. Succeeding this, we find no mention of floating "logs" for several centuries. None of the great navigators appears to have used one except Magellan. He appears to have employed some form of apparatus when he sailed from Spain in 1520, but he makes no mention of its operation. Vespucci also mentions the determination of longitude by instruments but gives no clue to the nature of those employed.

The word "log" bears a literal relationship to the nature of the apparatus first used by seamen, for the wooden object they threw overboard was intended to behave precisely like a log of wood. It was intended to float without perceptible progress on the surface of the water, remaining in one place like a marker, while a measured line was paid out from the ship. Newer mechanical logs do not serve as markers; they move with the ship.

Toward the sixteenth century, a practical device was introduced in England. It is described as a small boat-like construction containing a wheel and apparatus for turning pointers on a dial to register fathoms, leagues, and scores of leagues. This was the forerunner of the Harpoon Log which was discontinued because of the inconvenience of having to draw the log inboard to make a reading. Numerous improvements were made to this device until the familiar common log acquired its shape and method of handling. It comprised four essential parts: the chip (or log-ship), the line, the reel, and the glass. The chip was a wooden quadrant weighted on its arc so that it floated in an upright position.

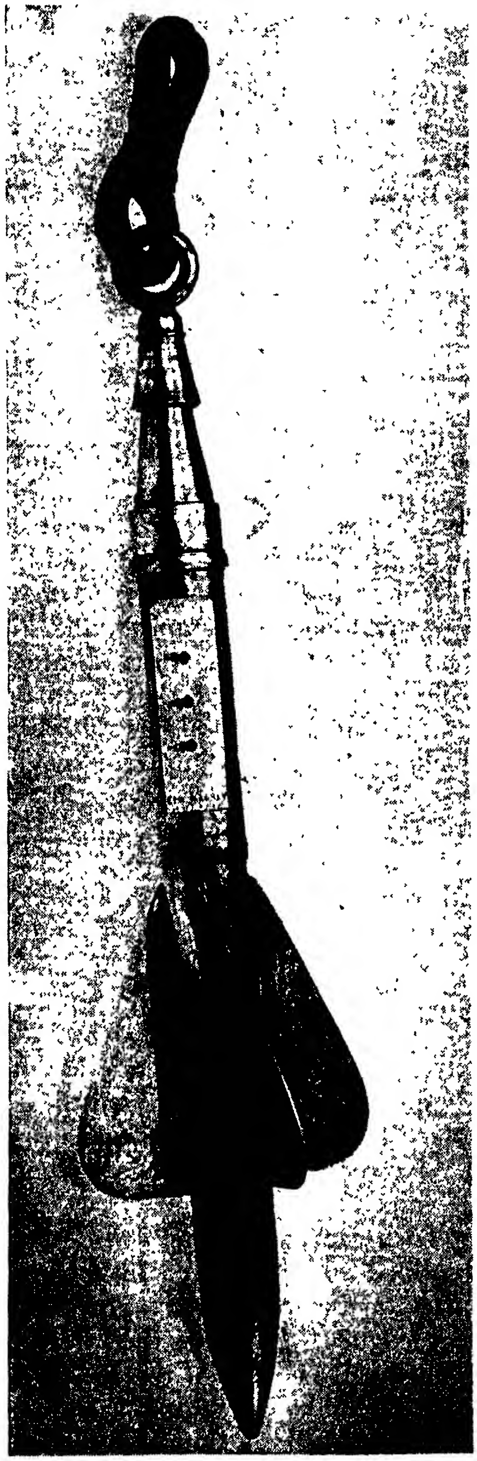


FIG. 7. A harpoon log, Massey's patent device for determining the speed of a ship. The dial is on the rotating part.

It was secured to the line, which might vary in length but which was always marked at known intervals by knots tied in the line. After being well soaked and stretched, it was wound around the reel. When a seaman threw the log over the side, he threw it to windward and paid out the line until a certain point was reached, marked with a red rag. At this stage he called to a companion to "Turn the glass."

This glass was a sand glass. For the purpose of reading the log special sand glasses were designed to run for short durations of time—20, 28, or 30 seconds—according to the space between the knots in the line. They were affected by damp and changes of temperature. Running sand was not ideal as a time measurer, but it had to serve because the mariner had no other.

The distances between the knots in the line had a direct relationship to the duration of the running sand in the glass. For example, a line knotted at every 47 feet 3 inches has the same relationship to a nautical mile of 6080 feet as a 28 seconds glass has to the 3600 seconds in an hour. When the seaman operating the log had paid out five knots of line during the running of the glass, the vessel was said to have travelled five units, that is, it had covered five times the distance of 47 feet 3 inches in 28 seconds, or five nautical miles during the hour.

The speed at which ships travel is still reckoned in "knots" (never knots per hour), a term which combines distance with time, a derivation due to the use of the knotted line in the old logs.

Several types of log are now in use. The Taffrail log (sometimes called the patent or cherub log) consists of a propeller shaped rotor towed at the end of about 150 fathoms of line. The rotor is so adjusted that it makes one complete revolution in a predetermined distance, and the number of revolutions is transmitted through the line to a recorder on the taffrail, which converts the revolutions into distance covered. The great defect of this type of log is its liability to inaccurate rotation when fouled by seaweed or other floating objects. The Nicholson log has a tube which projects from the bottom of the ship into the water. It calculates the speed of the vessel by pressure, which causes the water in the tube to rise proportionately. The Forbes log also uses a projecting tube, but has added a propeller shaped rotor at its projecting end. Both these tube logs have electrical connections by which speed and milage may be read on the bridge.

The Rodmeter used in the United States Navy is an instrument which protrudes through the bottom of the ship's hull into relatively still water. It has a pitot tube with orifice in the leading edge to transmit dynamic pressure, and holes at the bottom and side to transmit static pressure. Both the dynamic water pressure caused by the forward movement of the ship and the static water pressure are conveyed through hydraulic systems to a speed indicator, where a motor-driven centrifugal pump develops a pressure proportional to the dynamic

pressure in the Rodmeter. This can be read on a dial indicator where it has been converted through a series of gears into knots.

The log book used to record a ship's progress from day to day came into early use and, as measurement became more and more accurate, not only were the records more carefully and accurately kept but more uniform detail was entered.

Lunar Reckoning

For somewhat obvious reasons the navigators of the Mediterranean Sea had calculated their longitude only in an eastward direction, using the most westerly known point of land, Ferro in the Canaries, as their starting point. Later geographers adopted St. Michael in the Azores as their zero line because they were under the impression that the meridian of no magnetic variation passed through it. This latter method was in use at the time when mariners pinned their hopes of determining longitude by relating it to variations in the compass needle. Some early American geographers constructed their maps with Philadelphia as their prime meridian with this purpose in view.

Foremost among those striving for better results in the determination of longitude were the English, and they concentrated for many years upon an old idea of using the moon for this purpose. In order to establish the most accurate tables relating to the moon's position, they established the Observatory at Greenwich in 1765, and in their calculations fixed the meridian passing through that place as the prime meridian for all longitudinal measurements.

The principle of determining longitude by the moon is based upon measuring the angular distance between the moon and certain fixed stars as seen at a fixed point on shore (that is, Greenwich). Tables were constructed to give these angles for every day of the year. Local measurements of the angle were made by the mariner who, by comparison with the Greenwich tables, then computed his longitude. It was a slow. complicated, and inexact method but it was so generally adopted that it was hard to dislodge.

The Chronometer

The disasters to which we have alluded, and many others only of lesser gravity, befalling the British navy spurred the Admiralty to offer substantial rewards for a reliable method of determining longitude. The solution of the problem lay in the determination of the exact time, and since time is reckoned by the motion of the earth in relation to celestial bodies, the methods of using the moon were not fundamentally unsound. The mistake lay in the difficulty of predicting the position of the moon from the observations then obtainable. The fact that errors crept into their calculations through aberration and mutation was not unknown to the astronomers and, while they may have been

vaguely conscious of the superior advantages offered by relative comparisions with the altitude of the sun, there was no way by which a check between local time and time at a fixed point could be made.

Local time could always be determined when the sun was not obscured, but the means of having the time at a fixed point accompany the navigator was still unsatisfactory. The practical difficulty of the clocks and watches of the time resided in their incapacity to keep accurate record of the hour because of changes of temperature and the motions of the ship. For some inexplicable reason the discovery of the spiral balance spring by Huyghens in 1675 did not instantly appeal to the time-piece makers. Not until 1735 was a clock submitted to the "Commissioners for the Discovery of Longitude at Sea" as a potential solution to the problem of determining longitude.

The inventor, John Harrison, based his invention on the principle of the gridiron pendulum compensated by two metals of unequal contraction to overcome changes of temperature. Unfortunately for Harrison, the clock was given for test to a confirmed believer in the lunar method of reckoning longitude, and he had to wait for many years before receiving the award which had been promised. By this time he had greatly improved his time-piece by adapting it to a watch through the employment of a "grasshopper" escapement.

Although Harrison is entitled to the credit of being the first to provide a way of carrying fixed time on shipboard for comparison with local time ascertained by solar observation, the modern chronometer is not the direct descendant of his time-piece. Much remained to be done by French and British horologists (one of the latter, Arnold, gave it the name "chronometer"), before a completely satisfactory instrument was produced, suitable for placing in the hands of the mariner and adjusted in price to suit his purse.

The modern chronometer is a marvel of accuracy, its regularity varying only a few seconds in the course of a month.

We know that time is determined by the rotation of the earth, or as navigators say, by the "apparent" revolution of the sun about the earth. Since this apparent revolution of the sun is accomplished in 24 hours, we know that the sun appears to travel over 15 degrees of the earth's surface in one hour (one twenty-fourth of 360 degrees), or one degree in four minutes. Thus, when the sun is directly over the meridian passing through Philadelphia (not necessarily directly overhead) it is twelve o'clock noon, but as the sun will not reach the meridian of Chicago until an hour later (it is approximatley 15 degrees away) it is only eleven o'clock there, and nine o'clock in San Francisco.

With this unvarying rotation of the earth through fifteen degrees every hour, it becomes possible to fix one's location east or west of a given point at which the hour is known. All navigating charts fix this point along the meridian which passes through Greenwich, England,

which is marked zero on the charts. The navigator sets his chronometer by Greenwich Civil Time (or Greenwich Mean Astronomical Time, often called Universal Time, where the hours begin at midnight), so that he always has a constant reference to the zero point on his chart. By measuring the altitude of the sun at any time he can, by reference to his chronometer, find how many hours and minutes he is ahead or behind Greenwich time. By allowing four degrees for every four minutes of difference, he can determine how many degrees he is east or west of the zero line on his chart.

Formerly, three chronometers were carried so that when an irregularity occurred on one it was revealed by the others. Today, when time can be checked by means of radio signals from authorized stations, one or two serve the purpose.

With the chronometer in his possession, the mariner was now fully equipped to ensure accurate navigation. North, south, east or west his course could be laid and followed with an assurance such as never before had been known. For a full thirty centuries we have followed the mariner's progress in navigation: from Solomon's admission of ignorance concerning the navigator's skill up to the time of the modern shipmaster's complete knowledge. One thing only has remained unchanged during this long period of time, and that is the Sea itself. Whether the controlling hand had to steer the sea raider's long ship or the grimy little coaster was driven by a Diesel engine, the sea has remained unchanged in its behavior and still challenges the wit of man. No instrument has ever been invented that can replace the unceasing vigilance with which the mariner must observe its vagaries and caprices.

HIGH-SPEED DIRECT-INKING RECORDING SYSTEM *

BY

MARTIN A. POMERANTZ[1] AND ROBERT C. PFEIFFER[1]

ABSTRACT

A direct-inking recording system which is capable of resolving impulses separated in time by 0.01 sec., and which can operate continuously for long periods of time, has been utilized in telemetering cosmic-ray signals from balloon-borne equipment. A tape-reader mechanism permits rapid analysis of the records. The device has applications in various types of physical experiments, as well as in other fields.

I. INTRODUCTION

The well-known limitations of photographic recording techniques become particularly manifest in applications involving long periods of continuous operation combined with the requirement of a relatively short resolving time. It therefore becomes necessary to resort to devices embodying a direct-inking pen, tape puncher, or other printing arrangement for producing marks on a paper tape. Instruments of this type (**1**, **2**)[2] have been utilized, for example, in the recording of radio signals transmitted from equipment carried aloft by free balloons. The speed with which previous systems could be operated, however, has been limited by the mechanical response time of the pen assembly; consequently only rather low, or scaled-down counting rates could be handled.

A system incorporating a commercially available high-speed magnetic oscillograph has proved extremely satisfactory for continuous recording of signals received from cosmic-ray radiosondes. Comparatively inexpensive, light in weight, and easily transported, the arrangement to be described has been in use for two years in an extensive program of cosmic-ray investigations. It has provided trouble-free performance both in the laboratory and under difficult field conditions.[3] Furthermore, it has operated without interruption during flights remaining aloft as long as 15 hr.

Although designed primarily for telemetering information relating to cosmic-ray intensity measurements (**3**), this arrangement could easily serve various other purposes, as will become apparent from the description of its characteristics. For example, it might be utilized

* Assisted by the Joint Program of the Office of Naval Research and the Atomic Energy Commission.

[1] Bartol Research Foundation of The Franklin Institute, Swarthmore, Pa.

[2] The boldface numbers in parentheses refer to the references appended to the paper.

[3] During the National Geographic Society-AAF Eclipse Expedition to Bocaiuva, Brazil in May, 1947.

advantageously in conjunction with ionization chambers, proportional counters, high speed counting circuits, or counting-rate meters. In fields other than physics, such as biology or astronomy, the recorder could be readily adapted to kymographic, chronographic, or other similar applications.

II. RECORDING APPARATUS

The present device incorporates the Brush BL201 magnetic oscillograph [4] with certain modifications. The magnetic penmotor (4) reproduces electrical signals in the frequency range from d.c. to 120 cps. Multichannel oscillographs could equally well be modified for this

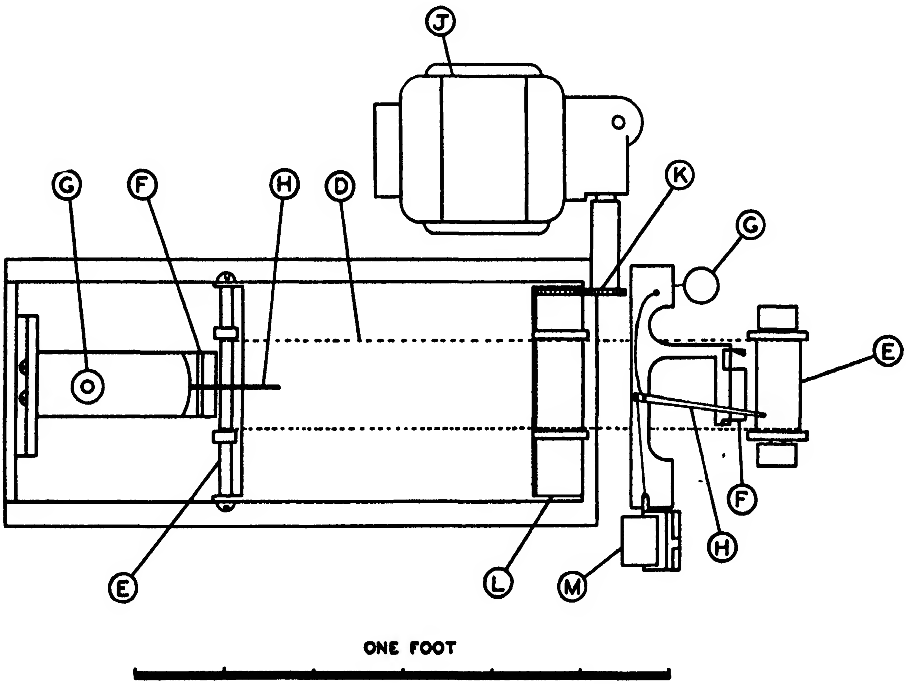


FIG. 1. Diagrammatic sketch of the high-speed direct-inking recording arrangement as viewed from above.

application. In its original manufactured form, however, the instrument is not designed for extended operation at high speeds, and in fact the chart paper capacity is sufficient for runs of a maximum duration of 6 min. This is remedied by the arrangement illustrated in Figs. 1 and 2. Although the paper-drive mechanism and gear-train for selection of speed have been removed for the present purposes, it is convenient to retain the other components.

The paper tape is fed from the large reel *C* which can accommodate 2500 ft. of paper. The outer disc is demountable, being held in position

[4] Brush Development Company, Cleveland, Ohio.

by a knurled threaded retainer. The rolls of paper are supplied with wooden center cores into which a brass bushing is forced. The latter is provided with a pin which fits into a groove along the axle of the reel assembly, thereby preventing slipping of the paper with respect

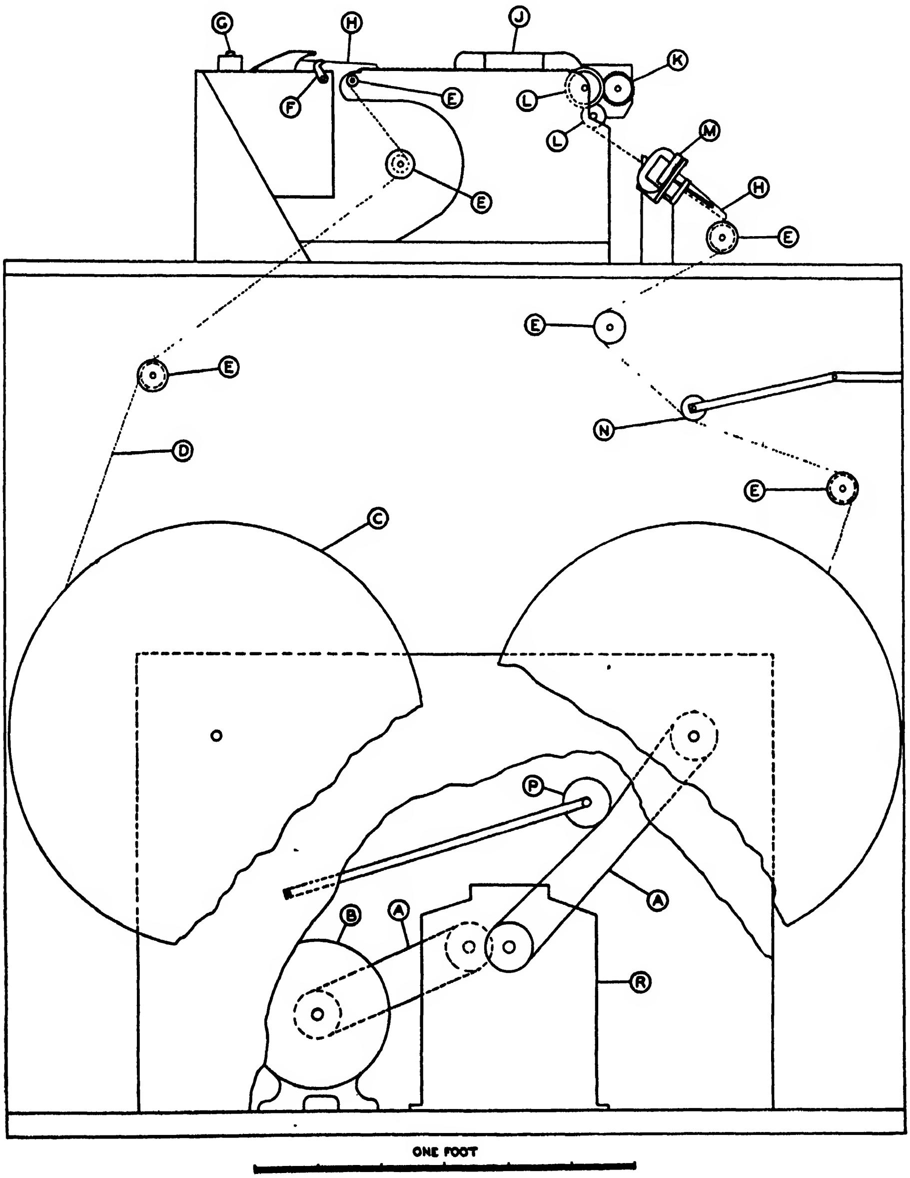


FIG. 2. Diagrammatic sketch of the high-speed direct-inking recording arrangement as viewed from the side.

to the reel. The paper tape *D* then passes over a roller *E* and on to the platten via two rods *E*. The paper drive is accomplished between the two pressure rolls *L*, the upper of which is powered by a speed-reducing

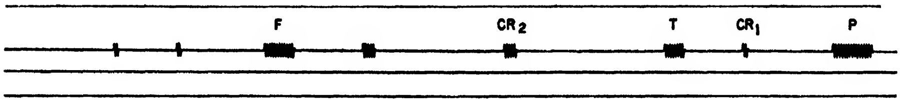


FIG. 3. Typical tape record obtained with the recorder during a free-balloon flight. mitted by an instrument incorporating an Olland-type of barograph. CR_1 and CR_2 represent identified by their duration, the amplitude being independent of the strength of the received 1/60 sec. The pulse labelled *TM* is applied once per minute.

motor [5] *J* through gear *K*. Additional rollers *E* then conduct the tape to the take-up reel which is similar to the feed reel.

A 0.1-h.p. motor coupled by a belt to a 22-1 speed reducer *R* drives the take-up reel. The coupling between the latter is effected by means of a leather belt *A*. An idling pulley *P* or any other arrangement, based upon the same principle, automatically compensates for stretching of the belt. The slippage of the belt over the pulley permits the take-up wheel to rotate with the correct angular velocity as the radius of the roll of paper changes. The idler *N* which is free to move vertically maintains constant tension in the tape.

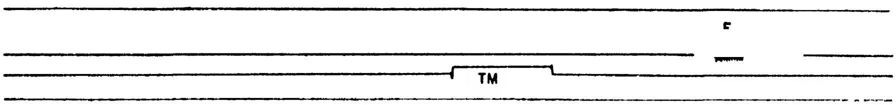
The elements of the magnetic penmotor include the inkwell *G*, pen *H*, and pen-lifter *F*. A duplicate set of these elements is mounted after the tape drive assembly for providing a time-marker trace. In this case the pen is attached to a relay *M* which is actuated periodically by a set of contacts on a small synchronous motor geared down to 1 r.p.m. Several independent time or other markers have been mounted in a subsequent modification of this arrangement. During flight operations, it is convenient to key coded notations on the tape without interrupting the recording.

When equipped with a Brush BL913 direct-coupled amplifier the pen will deflect one chart millimeter per millivolt input with a uniform frequency response up to 100 c.p.s. Many applications, such as that for which the present arrangement was developed, require neither uniform response nor high input sensitivity. In some cases no amplification is necessary, whereas in others input circuits having special characteristics (for example, non-linearity) might be advantageous.

III. TAPE READER

The paper speed in the present installation is approximately 9 cm. per second. At this rate it is necessary to replace a roll every 2½ hr. These factors are governed primarily by the nature of this specific application, and modifications in either the size of the reel or the chart speed would introduce no difficulties. Figure 3 shows a typical tape record obtained during a balloon flight incorporating a barograph based upon the Olland principle of radio meteorography (5). In this system it is necessary to measure the time interval represented by the tape

[5] Bodine Electric Company, Chicago, Ill. 1/70 h.p., 23 r.p.m. output.



F and *F* are reference signals, and *T* and *P* temperature and pressure respectively, as trans-
cosmic-ray events which triggered the two independent circuits. The various signals are
radio signal in this case. The separation in time between adjacent peaks in a given signal is

distances between temperature, pressure, and fiducial signals. Although a system (3) devised more recently obviates the necessity of making these measurements, the number or counts per minute produced by a particular type of cosmic-ray event must be read from the tape. As may be seen in Fig. 3, a different pulse length may be employed for identifying each of several types of event.

Analysis which would entail a tremendous amount of labor may be completed quite rapidly with the tape reader arrangement shown in Figs. 4 and 5. Here, the tape *F* is taken up by the reel *D*, driven by a

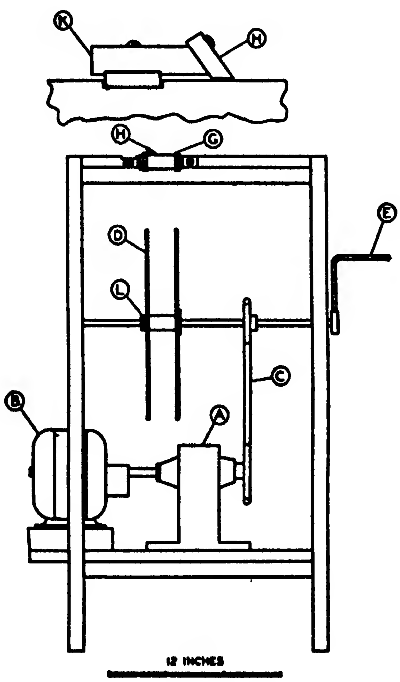


FIG. 4. Diagrammatic sketch of the high-speed tape-reading device.

d-c. motor *B* through a speed reducer *A*. The tape passes over a pulley *G* at either end of the table. The reels can also be turned manually by means of the handles *E*. The roll of tape is loaded by disconnecting the belt *C* and pulling the shaft forward. Construction of the reels is similar to that on the recording unit. To maintain tension in the tape,

friction is introduced into the feed reel through the arrangement *J* whereby a leather-tipped rod is pushed against *D* by means of a spring. The scale H is attached to brackets, the position of which are adjustable. These in turn are fastened to a rod of rectangular cross-section. The latter slides along a way so that the scale can be moved conveniently lengthwise.

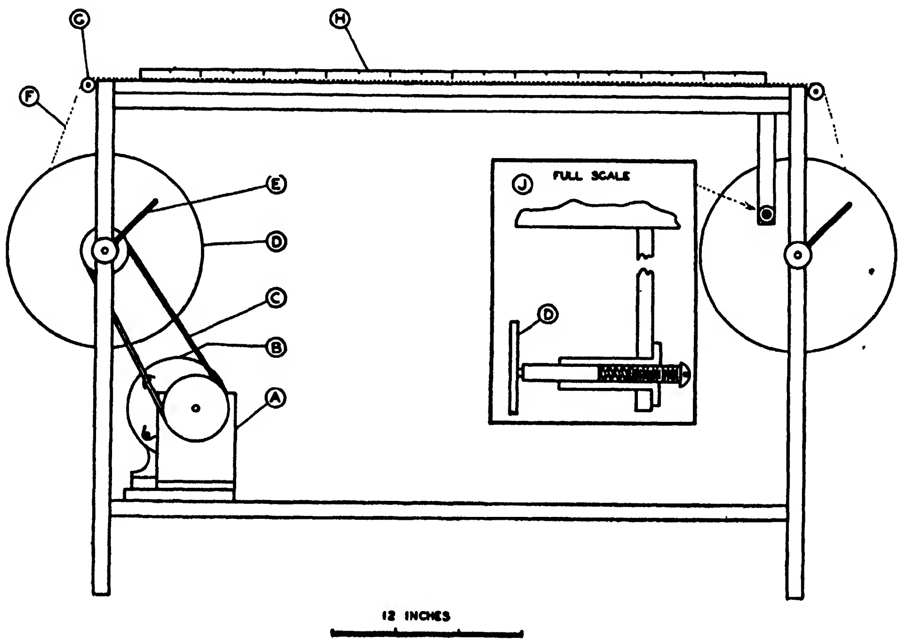


FIG. 5. Diagrammatic sketch of the high-speed tape reading device.

The motor is powered through a foot switch, and the speed controlled by means of a field rheostat mounted under the table. Other types of scale such as would be necessary for making amplitude measurements could easily replace the meter-stick.

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INACTIVATION OF MOLDS BY GERMICIDAL ULTRAVIOLET ENERGY

BY

MATTHEW LUCKIESH,[1] A. H. TAYLOR,[1] THOMAS KNOWLES[1]
AND E. T. LEPPELMEIER[1]

Spoilage of foodstuffs and manufactured products by molds results in great economic losses; therefore, any practical method of reducing these losses is of vital interest. Ultraviolet energy of certain wavelengths has been known to be effective for this purpose, and the advent of more efficient sources of such energy has stimulated interest in obtaining quantitative data regarding its effectiveness. Therefore, the results presented herewith pertaining to the effectiveness of ultraviolet energy in the spectral region of λ2537 are of considerable practical interest and importance.

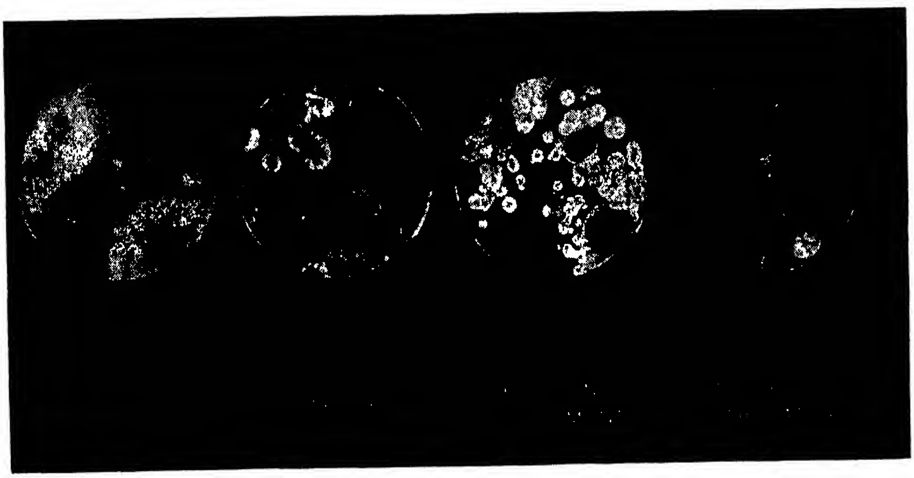

FIG. 1. Mold cultures demonstrating the prevalence of spores in air sampled at various locations.

Mold contamination probably results most generally from spores which settle out directly from the air, or which have recently been airborne. They have their origins on animal or vegetable matter, much of it outdoors, and may be brought indoors by air currents or on the clothing, shoes, etc. Prevalence of air-borne spores indoors may be shown by exposing open petri dishes with culture media for several hours, then incubating them for several days at room temperature. Figure 1 is reproduced from a photograph of four petri dishes exposed

[1] Lighting Research Laboratory, General Electric Company, Nela Park, Cleveland, Ohio.

in this way in various locations outdoors and indoors. The number of spores collected indoors is greatly increased during and after sweeping the floor, due to stirring up of contaminated dust.

Sterilization by heat is the most common method of preventing mold spoilage, but is not always applicable. It has long been known that ultraviolet energy can also be used as a germicidal agent (1),[2] but its use was not economically feasible before the advent of low-pressure mercury discharge lamps radiating nearly all their ultraviolet energy in the spectral region of λ2537. Fortunately, this is the spectral region of maximal germicidal effectiveness. Following their researches (2) on air-borne micro-organisms, the authors have been investigating the possibilities of mold control by germicidal energy. A condensation of some of the most important findings of our researches, involving several hundred separate tests, is presented in this paper.

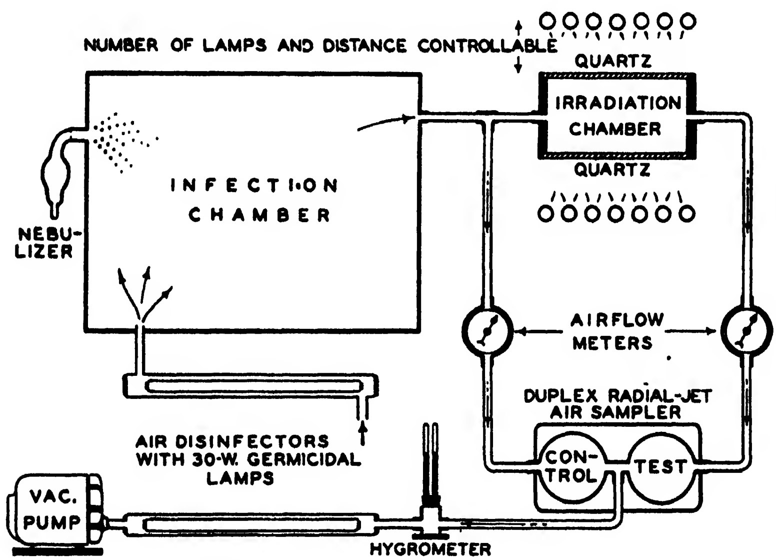

FIG. 2. A simplified diagram of apparatus used in studying the inactivation of air-borne bacteria and fungi by ultraviolet energy of λ2537.

In the period between the time mold spores become air-borne and the time when they produce spoilage, there are at least three periods during which they may be killed by germicidal ultraviolet energy, or at least inactivated so as to be incapable of reproduction. These periods are as follows:

1. When viable spores are air-borne and in motion in the room air. High intensities of suitable ultraviolet energy may be used to inactivate them before they settle onto foodstuffs, etc.

2. When viable spores have settled out of the air onto the floor or

[2] The boldface numbers in parentheses refer to the references appended to this paper.

objects which do not promote growth but from which they may later be stirred up and again become air-borne. Low intensities of ultraviolet energy reflected from walls and ceilings, when upper-air irradiation is used, may accomplish a high degree of inactivation over a period of many hours.

3. When viable spores have become attached to the surface of a medium or material upon which they can grow. Direct irradiation of the stationary spores may be effective in preventing growth, provided too long a period does not intervene between infection and irradiation.

This paper deals with results of our investigations of these three aspects and also with the range of resistivities of various species of molds which are common contaminants.

TABLE I

Dosages (Et) of germicidal energy required to inactivate 50% and 95% of various microorganisms irradiated in air and on the surface of culture medium.

E = microwatts of λ2537 energy per sq. cm.

t = exposure time in minutes

	Dosages Et to Inactivate			
	Air		Surface	
	50%	95%	50%	95%
Bacteria				
Mixed respiratory (from nebulized saliva)	0.4	3.2	—	—
Mixed respiratory (from sputtered saliva)	2.5*	23.0	—	—
Staphylococcus aureus	0.8	5.2	15.0	80.0
Escherichia coli	2.1	7.4	28.0	110.0
Alpha streptococcus	0.6	3.2	—	—
Beta streptococcus	3.0	11.5	—	—
Yeast				
Torula sphaerica	7.2	50	70	170
Mold Spores				
Mucor mucedo	340	1250	200	390
Penicillium chrysogenum	170	1150	120	320
Scopulariopsis brevicaulis	270	1450	210	490
Cladosporium herbarum	180	1350	120	410
Aspergillus amstelodami	320	1450	250	560
Rhizopus nigricans	250	580	—	—
Aspergillus niger	1800	9000*	1300	3000*

* Estimated value.

In photochemistry and photobiology, the so-called "reciprocity law" is often encountered. If a certain intensity E of radiant energy operating over a certain period of time t produces a given result, it is often found that the same result can be obtained if both of these factors are varied, so long as their product, or dosage, Et is unchanged. The application of this law to the inactivation of bacteria and mold spores

by exposure to germicidal energy has been verified experimentally by the authors for a range of 1000 to 1 in the two factors. The actual intensities of E were 0.5 to 500 microwatts per sq. cm. which cover the ranges encountered in practical installations of germicidal lamps for air disinfection.

In a previous paper (2) the authors discussed apparatus and techniques used in determining the lethal effect of various dosages of ultraviolet on different types of air-borne micro-organisms. A simplified diagram of improved apparatus for this work is shown in Fig. 2. A water suspension of the organism to be studied is nebulized into the infection chamber, from which it is drawn to the duplex radial-jet air sampler (3). By one path air is drawn directly to one unit of the

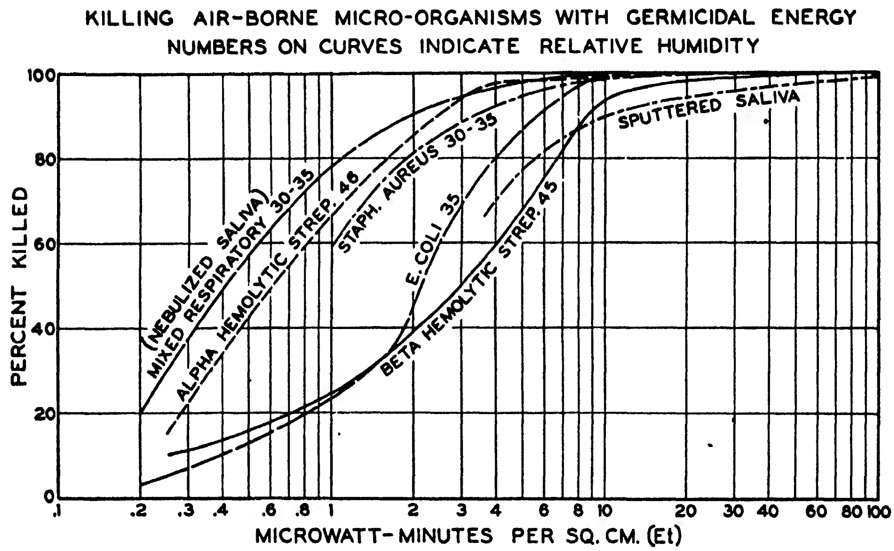

FIG. 3. Lethal effect of ultraviolet energy of λ2537 on air-borne micro-organisms as determined by use of apparatus shown in Fig. 2.

sampler, but air sampled by the other unit of the sampler passes through an irradiation chamber. The chamber has quartz windows through which the moving air is irradiated by energy from banks of 8-watt germicidal lamps outside the chamber. The number and distance of lamps and also the air velocity can be varied to obtain a wide range of known dosages. The actual available range of dosage Et was from 0.2 to 2400 microwatt minutes per sq. cm.

Various species of micro-organisms and mold spores were studied with this apparatus in the present and earlier forms. Some of the data obtained are shown in Table I and in Figs. 3 and 4. The lethal curves show a wide range of resistivity of various species of bacteria and fungi. The mixed respiratory organisms, obtained by nebulization of saliva taken from the mouths of human beings or sputtered directly from the

mouth, are very easily destroyed or inactivated while the mixed saprophytic organisms in dust, sampled in movie theaters during cleaning, are about several hundred times more resistant.

The diameter of droplet nuclei from nebulized saliva averages approximately one μ, which is 0.001 mm. or about 0.00004 in. The droplet size from sputtered saliva would probably vary from 7μ to 100μ and larger (4). These relatively large droplets would contain many times more micro-organisms than would be carried by droplet nuclei. The inactivation of micro-organisms by ultraviolet irradiation follows the exponential law (1). This phenomenon probably accounts in part

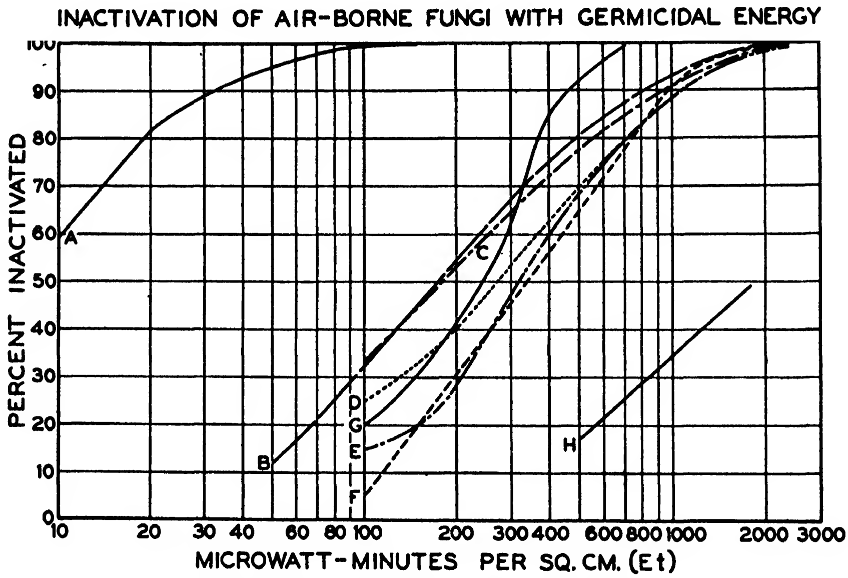

FIG. 4. Inactivation of air-borne fungi (produced by nebulization of water suspension of each) by ultraviolet energy of $\lambda 2537$.

A. *Torula sphaerica*, relative humidity = 65%
B. *Penicillium chrysogenum* RH = 41%
C. *Cladosporium herbarum* RH = 53%
D. *Scopulariopsis brevicaulis* RH = 79%
E. *Aspergillus amstelodami* RH = 67%
F. *Mucor mucedo* RH = 63%
G. *Rhizopus nigricans* RH = 62%
H. *Aspergillus niger* RH = 55%

for the greater apparent resistivity of the micro-organisms in sputtered saliva as compared with those from nebulized saliva as shown in Fig. 3. The authors have found, in schools and movie theaters, that an over-all reduction of 50 per cent of the air-borne micro-organisms can be obtained with practical installations of germicidal lamps. Since the respiratory organisms are much lower in resistance, it is logical to assume that they would be reduced by a much larger percentage, probably as great as 95 per cent.

The authors have found that *Escherichia coli* and the mixed organisms in saliva increase in resistance to germicidal energy with in-

creasing humidity. However, mixed air-borne organisms sampled in a poultry house appeared to be less resistant at high humidities. These results may indicate that air-borne bacteria are most resistant under humidity conditions most nearly approaching those of their natural habitats. The resistance of air-borne mold spores appears to be relatively unaffected by wide variations in atmospheric humidity.

In a previous paper (2) the authors showed that a very high concentration of air-borne mixed respiratory organisms (nebulized saliva) in a closed room could be rapidly reduced by upper-air irradiation by a single 30-watt germicidal lamp in a suitable reflector or fixture. Since the mold spores are much more resistant to ultraviolet energy than are the respiratory organisms, it was hardly to be expected that they could be inactivated at as high a rate as was found for the latter. Hence

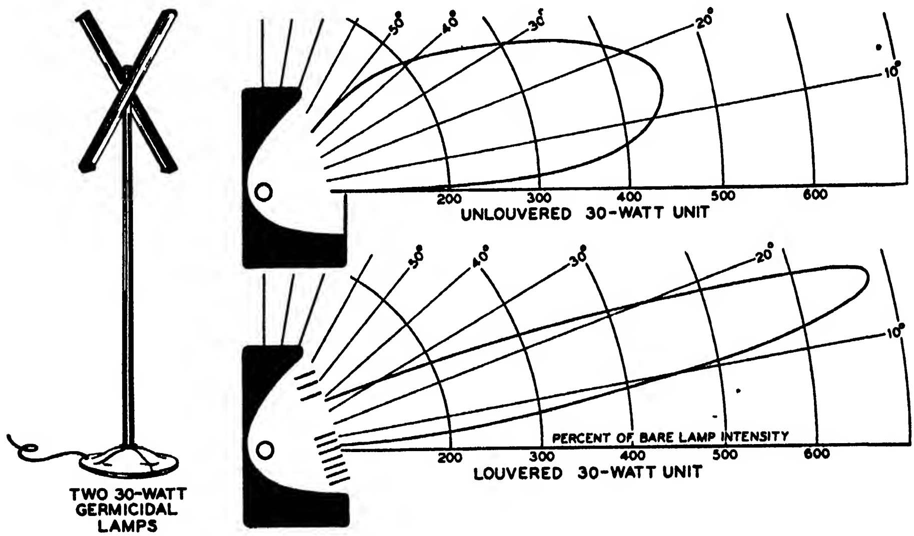

FIG. 5. Three types of germicidal lamp units used to study the inactivation of air-borne mold spores in an experimental room.

for mold-spore reduction it has been customary to recommend direct high-intensity irradiation by germicidal lamps during periods of non-occupancy of the rooms (5).

Intensive studies have been carried out by the authors to determine the lethal effectiveness of germicidal energy on air-borne mold spores in a closed room $17\frac{1}{2} \times 15\frac{1}{4}$ ft., with ceiling $11\frac{1}{2}$ ft. high. As a test organism *Penicillium chrysogenum* was chosen because it is a common contaminant mold of foodstuffs and appears to be fairly representative as regards resistance to ultraviolet energy, when the spores are air-borne. Three systems of irradiation were used, the types of units being as illustrated in Fig. 5. The two bare 30-watt germicidal lamps were used at the middle of the room mounted with their centers 4 ft. above the floor. In another installation the reflector fixtures were mounted so

that the germicidal lamp was about 7 ft. above the floor and irradiated the stratum of air above that level.

From an adjacent room rubber hoses attached to an air pump were carried to two DeVilbiss No. 40 nebulizers in the test room. A water suspension of the spores was nebulized for about an hour before air-sampling began. Usually the air in the test room was stirred by electric fans during the nebulization period, to make the concentration of spores as uniform as possible. The fans were then stopped. Before irradiation with ultraviolet energy, several samplings of the air were made by

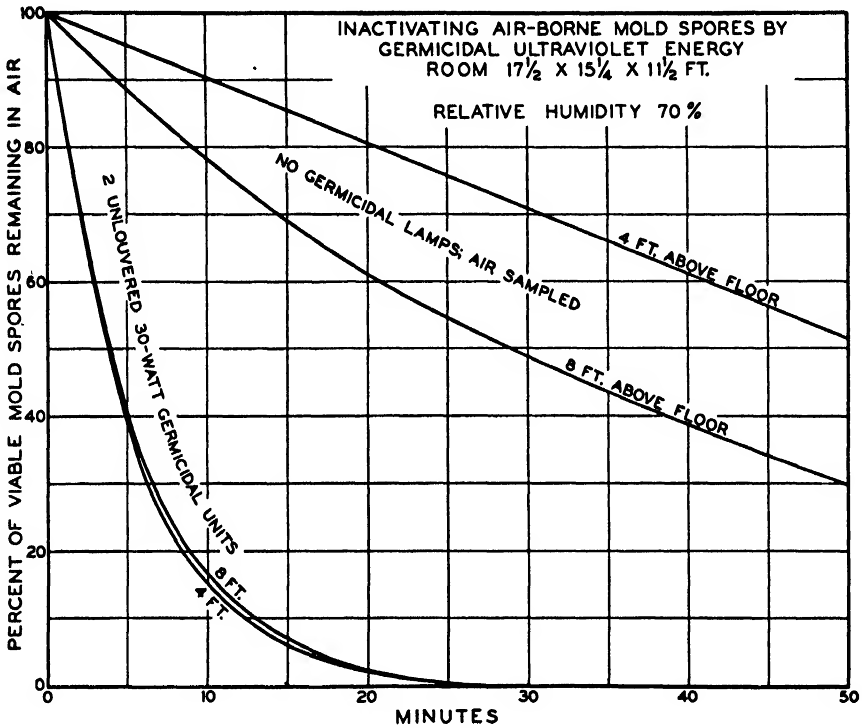

FIG. 6. The normal disappearance rate of *Penicillium chrysogenum* spores that were atomized into the air of a closed room is shown in comparison with the disappearance rate when the air was irradiated with ultraviolet energy.

means of rubber hoses whose intakes were located at the center of the room, 4 and 8 ft. above the floor. These hoses were attached to the two sampling units of a radial-jet air-sampler (3) located in the adjacent room. The germicidal lamp or lamps were then turned on and the air was sampled at various intervals to determine the rate of disappearance of the spores. The rate of disappearance was also determined *without* irradiation with ultraviolet energy from the germicidal lamps.

Many separate tests were made during winter months and typical results of several are illustrated in the charts of Figs. 6, 7, and 8. In

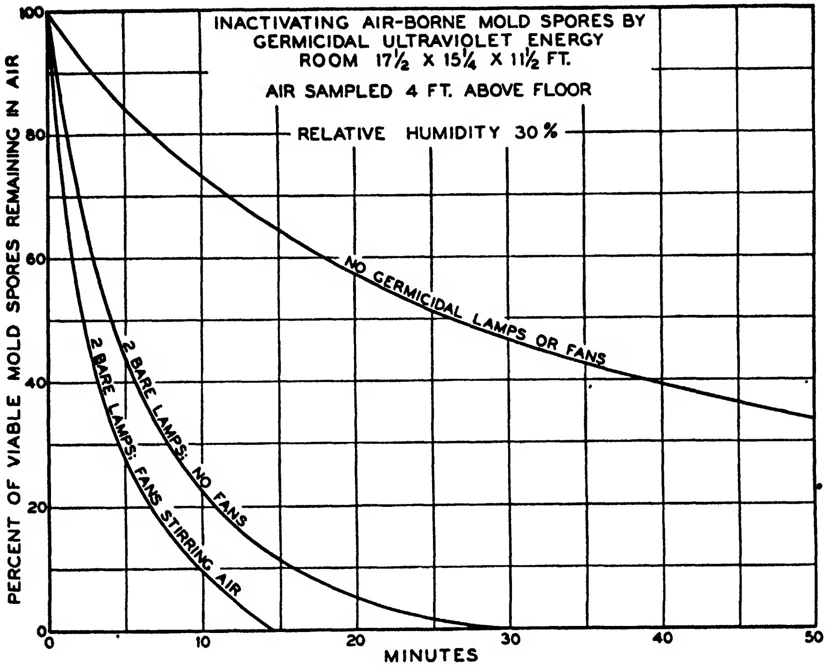

FIG. 7. Illustrating the effect of increased air circulation on the rate of air disinfection by irradiation with ultraviolet energy of λ2537.

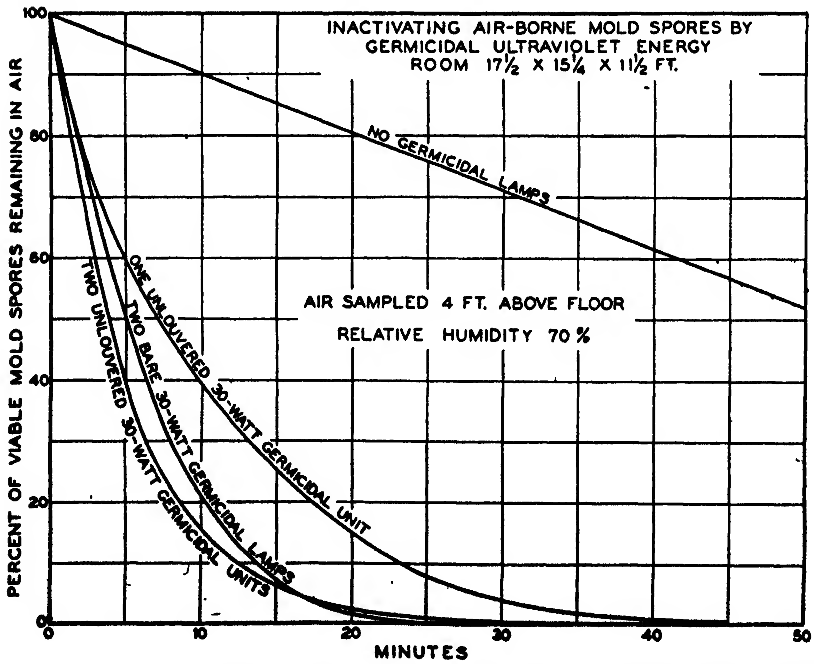

FIG. 8. Comparing the air disinfection rates achieved by three different methods of using the germicidal lamps. Almost identical results were obtained with two bare 30-watt lamps at the center of the room and with upper-air irradiation with two unlouvered units (see Fig. 5). The latter can be safely used during occupancy of the room.

most of the tests the air circulation was not forced but was that produced normally by the temperature gradients produced by the cold windows, a warm radiator, and the lamps used for illumination. In Fig. 6 are shown results of sampling at levels 4 and 8 ft. above the floor, when the relative humidity was 70. This high value of humidity was obtained by heating water on a hot plate which also probably aided air circulation. The fact that the disappearance curves at levels of 4 and 8 ft. are practically identical indicates appreciable air movement, since the intensity of germicidal energy was much greater at 8 feet than at 4 ft.

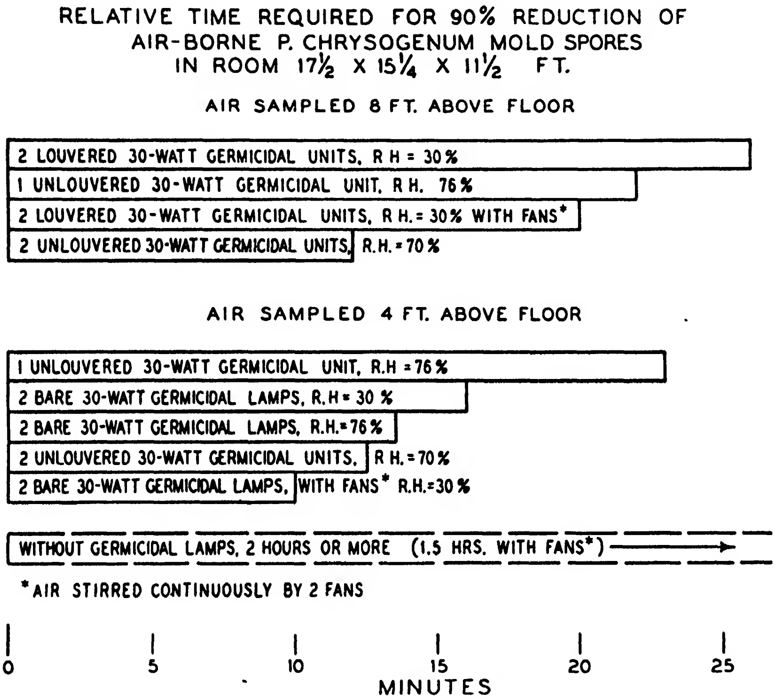

FIG. 9. A summary of disappearance rates of air-borne mold spores obtained by various methods of irradiation with ultraviolet energy of λ2537.

Figure 7 indicates an appreciable acceleration of the clean-up of air-borne spores when a fan was used to increase air circulation. In a closed room, such as this was, there is little control of air circulation unless fans are used. Consequently, the results shown without fans appear to be conservative, since there would usually be a greater degree of air circulation in an occupied room, especially with doors partially open and more or less activity on the part of the occupants.

In Fig. 8 the results obtained when using one or two unlouvered 30-watt germicidal units irradiating the upper air are compared with the results obtained when irradiating the air throughout the room with two bare lamps. The two unlouvered fixtures produced slightly better

results than the two bare lamps. This surprising outcome shows the feasibility of irradiating the upper air of an occupied room to reduce the possibility of contamination by mold spores. The authors have already proved that this type of air irradiation will greatly reduce air-borne respiratory organisms. Consequently, in areas where food processing is being done by a number of workers, the germicidal lamps can be used for simultaneous protection of personnel and products.

TABLE II

Time required for definite percentage reductions of air-borne *Penicillium chrysogenum* mold spores in a room $17\frac{1}{4} \times 15\frac{1}{2}$ ft. with $11\frac{1}{4}$ ft. ceiling. Data for natural reduction rate and accelerated reduction produced by germicidal lamps. When bare lamps were used, they were placed in center of room, lamp centers about 4 feet from floor. When louvered and unlouvered germicidal units were used, they were mounted 7 feet above the floor and irradiated the air above that level.

Relative Humidity, %	Germicidal Sources	Time for Indicated Reduction, min.			Reduction in 2 hr., %
		50%	90%	99%	
	AIR SAMPLED 8 FT. ABOVE FLOOR				
30	None	23	120	—	90
	2 louvered 30-watt units	4	26	60	
	None—with 2 fans*	17	90	—	97
	2 louvered 30-watt units with fans*	4	20	30	
70	None	30	—	—	
	1 unlouvered 30-watt unit	6	22	40	
	2 unlouvered 30-watt units	4	13	23	
	AIR SAMPLED 4 FT. ABOVE FLOOR				
30	None	26	over 2 hours		87
	2 bare 30-watt lamps	4	16	26	
	2 bare 30-watt lamps with fans*	2	10	14	
70	None	52	over 2 hours		85
	2 bare 30-watt lamps	5	13	21	
	2 unlouvered 30-watt units	4	12	25	
	1 unlouvered 30-watt unit	7	23	39	

* Fans stirred room air continuously during test period.

In Fig. 9 is presented a summary of results of many tests under various conditions. It shows that whereas two hours or more were required for disappearance of 90 per cent of the air-borne spores without germicidal lamps, this time could be reduced to 10 to 26 minutes with germicidal lamps. In Table II the data are summarized more completely.

It will be noted from Table II that the unlouvered fixtures (see Fig. 5) were appreciably more effective than those with louvers. The louvers are desirable when the fixtures are used in rooms with low

ceilings, so that the energy can be principally concentrated in a beam with axis not more than 10 to 15 degrees above the horizontal. The purposes of the louvers are to prevent excessive intensities of germicidal energy below the level of the fixture and to keep the energy reflected from the ceiling at a minimum, since sufficient dosages of this radiant energy will produce conjunctivitis and erythema (reddening of the skin). However, the louvers absorb energy and reduce the total output of germicidal energy by the fixture.

The high rate of inactivation of air-borne respiratory organisms (from nebulized saliva) due to irradiation with germicidal energy as compared with the inactivation of air-borne mold spores is clearly illustrated in Fig. 10 which shows the results of tests in the same room.

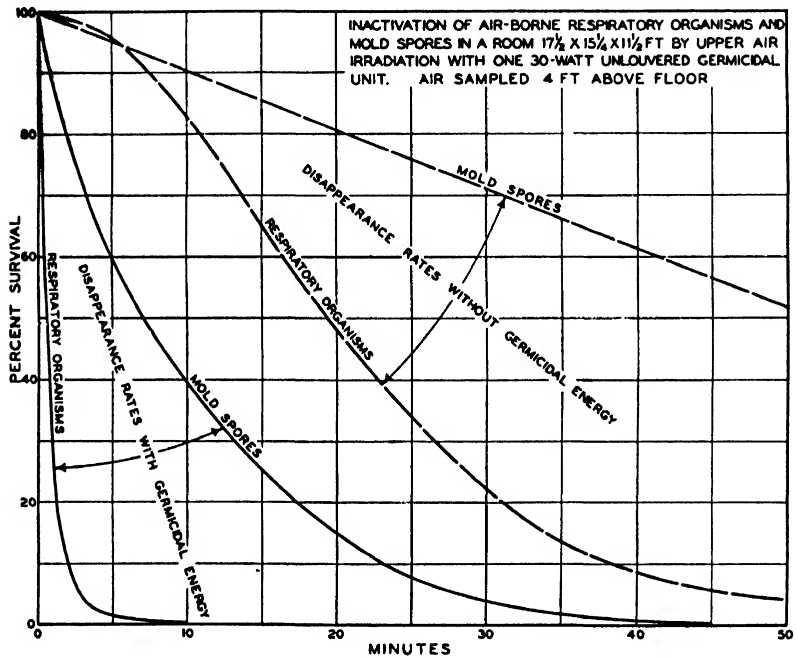

FIG. 10. Relative inactivation rates for respiratory organisms (from nebulized saliva) and mold spores by upper-air irradiation in a closed room without forced air circulation.

The room in which these tests were made has a waxed linoleum floor. Petri dishes attached to the ceiling, walls and floor showed that spores settled on the floor but did not attach themselves to the walls and ceiling. Several attempts were made to put them back into the air after they had settled on the floor (germicidal lamps off) by the use of fans vigorously blowing air against the floor. No increase in air-borne spores could be detected after this procedure, although culture media poured on the floor showed large numbers to be present. Experimental evidence seems to indicate that they are held on the linoleum by an electrostatic charge.

When upper-air irradiation was employed there was a small amount of germicidal energy diffusely reflected from the walls and ceiling. At the floor-level the intensity was about 0.3 microwatt per sq. cm. In one test this energy, applied over a period of about 20 hr., inactivated over 95 per cent of the dry spores of *Penicillium chrysogenum* on the floor. This important finding indicates that spores which settle out before inactivation may later be inactivated by prolonged irradiation with low intensities of germicidal energy. Hence air-disinfection for product protection can best be achieved by continuous operation of the lamps.

After the spores have settled on a surface or medium which will promote growth, they may be inactivated by a high intensity of germicidal energy acting for a short time, or by a low intensity continuously applied. Both of these methods were studied in these investigations. The dosage required to inactivate a given percentage of the spores on a culture medium is approximately $\frac{1}{3}$ as great as that required for the same percentage inactivation of air-borne spores. This relationship was found for several species of mold spores as shown in Table I. Previously the authors found that certain pathogens were 5 to 10 times more resistant when suspended in a thin layer of water (6) than when air-borne.

TABLE III

Inactivation of *Penicillium chrysogenum* spores on culture media as affected by elapsed time between seeding and irradiating with germicidal energy. Dosage was 300 microwatt-minutes per sq. cm.

Hours Elapsed Between Seeding and Exposure	Per Cent of Spores Inactivated
0	88
0.1	91
0.2	88
0.4	90
0.8	88
1.5	86
3	88
6	96
15	98
24	84

It is reasonable to expect that a spore will be more resistant to germicidal energy in the dry, dormant state than it will be in the vegetative, growing state. Consequently, it appeared important to study the effect of a definite dosage when applied to seeded plates at varying time intervals after seeding. A series of petri dishes was seeded by air-borne *Penicillium chrysogenum* spores drawn from the infection-chamber shown in Fig. 2, utilizing the duplex radial-jet sampler. In order to assure highest accuracy in the appraisal, one-half of each plate was exposed and the other half was used as the control. The results of this test are presented in Table III.

The variations in per cent inactivation during the first three hours are within the range of experimental errors. At 6 and 15 hr. the percentage was much higher, indicating a higher vulnerability in the early stages of growth. After 24 hr. there was slightly visible growth, and apparently a significant increase in resistivity.

The effect of *continuous* irradiation in inhibiting growth of *Penicillium chrysogenum* spores on a culture medium has been studied by exposing seeded plates at varying distances from a bare 30-watt germicidal lamp, in order to obtain a range of intensities of germicidal energy. Each dish was covered with a clear quartz plate masked around the edges to leave a clear central space two inches square. Paper numerals were attached to the quartz in the clear space, indicating the distance in feet between the lamp and the dishes. Continuous exposure for three days produced the results illustrated in Fig. 11 which is reproduced from photographs

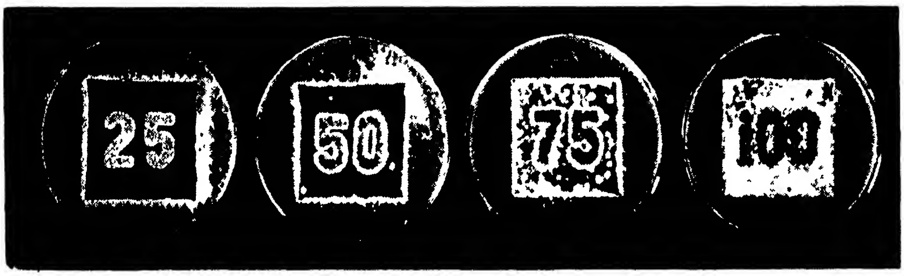

FIG. 11. Culture media in petri dishes were seeded with *Penicillium chrysogenum* spores, then exposed continuously for 72 hr. to a 30-watt germicidal lamp at distances of 25, 50, 75 and 100 ft. These results directly demonstrate the inhibition of mold growth by very low intensities of germicidal energy. (See text for details.)

of typical seeded plates. It is obvious that there is almost complete inhibition of growth at distances of 25 and 50 ft., and a definite retardation of growth even at 100 ft. from the bare 30-watt lamp. At 100 ft. the intenisty of germicidal energy was only 0.034 microwatt per sq. cm. These results indicate the value of low-intensity continuous irradiation such as may result from reflected energy in a meat cooler in which germicidal lamps are installed. The direct irradiation of air-borne spores in this case will inactivate many of them, and there is still the possibility of preventing spoilage by any spores that settle on the meats in locations where they receive low intensities of germicidal energy by indirect irradiation.

SUMMARY

Mold spores have their principal origins on decaying animal and vegetable matter, much of it outdoors. They are prevalent in outdoor and indoor air and cause great economic losses.

Spores can be killed or inactivated, that is, made incapable of reproduction, by irradiation with germicidal ultraviolet energy. The principal ways in which this may be accomplished are: (*a*) by high-

intensity irradiation of the air-borne spores; (*b*) by long continued low-intensity irradiation of the spores at rest on materials which do not support growth; and (*c*) by high-intensity short time or low-intensity long, continuous exposures of the spores on materials which promote growth.

FIG. 12. *Penicillium chrysogenum* mold spores were seeded over the entire area of the simulated ground around the miniature buildings and trees. The 2-watt germicidal lamp shown in the upper left-hand corner simulated a miniature sun. The germicidal energy inhibited the growth of the mold except in the protected areas or shadows cast by the miniature buildings, trees, etc.

The reciprocity law (Et = const.) has been proved to apply to both bacteria and spores over a range of intensity E of energy of $\lambda 2537$ from 0.5 to 500 microwatts per sq. cm. and obviously a corresponding range in time t.

The range of resistivities of mold spores and bacteria to germicidal energy of $\lambda 2537$ is commonly at least 1000 to 1.

The inactivation of air-borne mold spores in a closed room using several types of installations of germicidal lamps has been studied. Without germicidal energy there was a 90 per cent reduction in air-borne spores, principally due to settling out, in about two hours. With the germicidal installations the same results were obtained in 10 to 25 min., depending on the system used. Most of the spores which disappeared were probably inactivated and were no longer potential sources of contamination.

A suitable installation of germicidal lamps can provide a high degree of air disinfection for the protection of personnel and products, without exceeding intensities safe for the eyes and skin of occupants of the room.

Continuous irradiation of products subject to mold contamination, even with intensities less than 0.1 microwatt per sq. cm., affords a high degree of protection from the molds used in these investigations.

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Mobile Cashier Unit (*American Gas Association Monthly*, Vol. 31, No. 9).—A mobile cashier unit at which consumers in outlying communities can pay their utility bills conveniently is the latest addition to the vehicular fleet of Consolidated Edison Co. of New York, Inc. The new unit is being used exclusively in New York City's borough of Queens.

The mobile cashier, which began its service on July 1, 1949, stops once a month at specified locations in 21 different Queens communities. Nineteen of these are full-day stops, from 9:30 a.m. to 4:00 p.m.; the other two are half-day stops.

Locations were selected specifically to include those communities that are farthest away from any district offices, and from which transportation to district offices is least satisfactory. Residents in these communities can save time and expense by paying their bills regularly at the mobile cashier.

This new service was brought to the public's attention through circulars distributed with consumers' bills, local newspaper advertisements and newspaper releases.

The mobile cashier is operated by commercial relations department personnel and is staffed at all times by a minimum of two cashiers. Continuous year-round service will be provided except for possible limitations during the winter in the event that heavy snows hinder curb-side parking.

The mobile cashier truck has a standard Chevrolet forward control chassis equipped with a Grumman aluminum alloy Kurb Side body. Interior office space, located to the rear of the driver's seat, provides an area 128 inches long, 78 inches wide and 70 inches high.

Exterior of the truck is attractively colored red and white with grey paneling. It has two cashier-type windows on each side so that it can operate on either side of the street. When it pulls to the curb at any of the designated locations, the two cashiers on duty take their places at the windows facing the sidewalk and are ready for business.

A wide aluminum panel, faced with a promotional poster, obscures the windows on each side while the unit is in transit. When it is on location, the windows facing the sidewalk are exposed by raising this panel, which then provides an awning effect over the windows. Dates on which the unit will return to a location for the next two months are included in the receipt stamp imprint which leaves a picture outline of the truck on each bill stub when payment is made.

Interior facilities have been designed to meet all contingencies. Counter-desks extend the full length of each side wall, with seating space at each window, and include the necessary storage space and cash drawers to meet the requirements of each cashier at his pay window. Three movable stools can be used wherever needed.

Ventilation is supplied by two fans and there is a compact heating unit ready for use during cold weather. The roof and both sides are fully insulated and the smooth plate floor is covered with linoleum. Four dome lights give the necessary interior lighting.

Standard cashier's working equipment is provided, including a safe, two coin change machines, two adding machines and a portable typewriter.

THE BASIS FOR THE OPTIMUM AIDED-TRACKING TIME CONSTANT

BY

E. A. MECHLER,[1] J. B. RUSSELL [2] AND M. G. PRESTON [3]

ABSTRACT

This report explains why the optimum aided-tracking time constant for continuous signaling is between 0.2 and 0.8 seconds. The general formulae for aided tracking are developed and it is proven that with intermittent signaling the optimum aided-tracking time constant should equal the interval between signals.

I. INTRODUCTION

It isn't much of a trick to track a slow-moving target with a shotgun or a telescope. Under such circumstances the position of the gun or telescope is adjusted continually by the hunter to match the position of the target. The operation is called direct tracking. When the speed of the target increases, the interval available to get on target becomes shorter and the rapid motion makes the target increasingly difficult to follow.

We can help the gunner move his gun by driving it with a motor and so arrange the mechanism that he may turn the motor speed control to adjust the velocity and position of the gun until it matches the velocity and position of the target. This is called velocity tracking. Once the gunner obtains a match, he need not move the speed control unless the velocity of the target changes. And when changes are smooth and gradual, he can follow them by small movements of the speed control.

Sometimes it is difficult to get on the target with velocity tracking. To overcome this difficulty, a combination of direct and velocity tracking is used. The operator moves the gun until it points at the target. Through gearing, this direct gun motion also turns the motor speed control and so adjusts the velocity of the gun that it matches the target speed. This is called aided tracking. Theoretically the gun follows a constant velocity target without further adjustment. But in practice, more than one adjustment is required as a rule before the gun and target velocities and positions match continuously.

The procedure is simple and the tracking is stable. With aided tracking, the gunner can get on target quickly and easily, and, once on target, no adjustments or at the most only small adjustments to the

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gun position are necessary to follow usual target motions. To achieve aided tracking, a ball and disk integrator can be used to combine the position and velocity adjustments into a single control.

The ratio between gun displacement per unit control displacement and gun velocity per unit control displacement must be properly chosen. This ratio, called the aided-tracking time constant, is expressed in seconds. It gives the time required for the motor to rotate the guns through an arc which is equal to the arc of the initial displacement. This report develops the theoretical basis for choosing the proper aided-tracking time constant.

During the war a number of laboratories, interested in the study of aided tracking, reported that the most favorable aided-tracking time constant was in the region between 0.2 seconds and 0.8 seconds. This fact was reported by The Franklin Institute in investigations using aided tracking in the control of aircraft turrets, and by other United States and English investigators. Throughout all of these investigations in which different mechanical controls were involved and in which different target and gun velocities were used it was a striking fact that the optimum aided-tracking time constant was not less than 0.2 seconds and not more than 0.8 seconds. The investigators agreed also that within the range 0.2 to 0.8 seconds the choice of constant made little difference. Many control mechanisms were developed using the median aided-tracking time constant of 0.5 seconds or thereabout.

Despite the unanimity in opinion as to the range of optimum aided-tracking constant, for continuous signals and its applicability to a wide range of tracking situations, little progress was made in discovering why the optimum constant was of the order of 0.5 seconds.

Recently the Franklin Institute Laboratories for Research and Development undertook to design an aided-tracking system for intermittent signals. The achievement of this project is an interesting example of setting out to investigate one thing and succeeding in discovering something substantially more important. The authors were undertaking to determine the aided-tracking time constant for intermittent signals, the limits over which stable tracking could be achieved with any one constant, and the best constant for a particular time interval between signals. There was no intention, particularly at the beginning of the study, of ascertaining why the aided-tracking time constant is of the order of 0.5 seconds for continuous signals.

Nevertheless, this investigation shows why this is so, and the explanation is sufficiently basic and essential that it can be applied widely to all types of aided tracking using continuous and discontinuous signals.

II. THE PROBLEM

The problem of tracking a target appearing at regular intervals on a radar screen imposes some new factors on the conventional aided-tracking problem solved during the war for aircraft and coastal defense

gunnery. Most significant is the fact that the target appears only once during every antenna rotation. Its spot on the PPI scope remains for a few seconds due to persistence of the cathode ray tube screen and then fades out.

The presentation as shown on a north stabilized PPI scope is depicted in Fig. 1. Here the bearing of the pip on the screen is determined by the azimuth position of the target with respect to the aircraft which contains the radar set. A brightened sector containing cross hairs can be moved about on the PPI screen. The angular position of this sector is determined by the angular position of the azimuth shaft. In turn an aided-tracking device is used to turn and position the azimuth shaft. Range is obtained in a similar way.

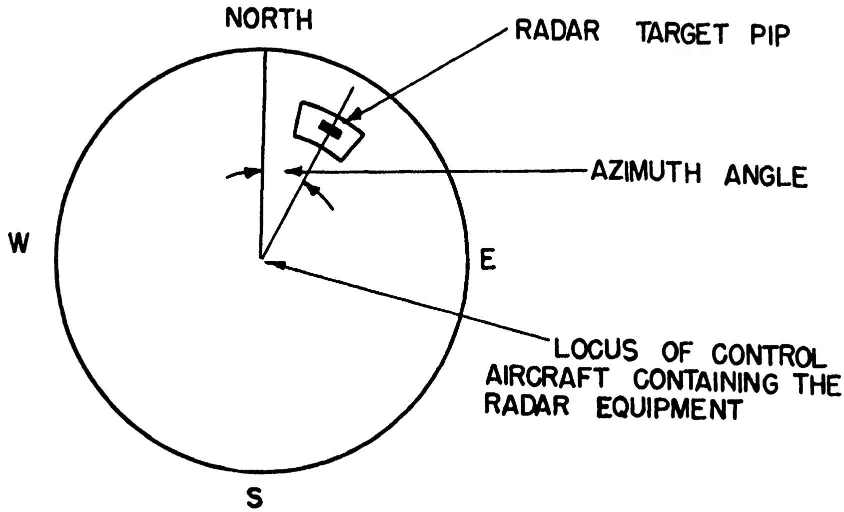

FIG. 1.

As a first approximation it may be assumed that the time intervals during which the target position is not observed are equal. One way to smooth these discrete target position data is to use aided tracking. The technique of tracking is to put the reticle quickly on the radar pip when it appears and then to make no further adjustment until the radar pip appears again.

The problem is to determine the optimum aided-tracking time constant which should be used in tracking this intermittent signal.

By definition, for a given rotation of the tracking knob,

$$\text{The aided-tracking time constant} = \frac{\text{Change in position of the reticle}}{\text{Rate-of-change in position of the reticle}}$$

If the cross hairs are centered on the radar pip at the start of a constant interval of τ seconds and the tracking rate is correct, the cross hairs will be centered on the radar pip at the start of the next τ second interval. If the rate is not correct the cross hairs will lead or lag the

pip by an amount $A°$. Assuming constant target and cross-hair rates, the error in the cross-hair rate is A/τ degrees per second. Thus, to track perfectly, a deflection of the tracking knob must change the position of the cross hairs by $A°$ and the rate at which the cross hairs move, by A/τ degrees per second.

To do this the aided-tracking time constant must $= \dfrac{A°}{(A/\tau)°/\text{sec.}} = \tau$ seconds. Consequently, when tracking a constant-rate target which appears at fixed time intervals, the aided-tracking time constant should equal the time interval in seconds. Theoretically this produces perfect tracking, and, when using this time constant, the cross hairs need to be positioned on the radar pip at only two successive intervals. Then the cross hairs will remain on the target until the rate at which the target moves under the reticle changes. This can happen when either the target or the observation position changes speed or direction.

III. MATHEMATICAL ANALYSIS

Aided-tracking control is a combination of two simple forms of control, namely, direct control and rate control. In direct control the reticle displacement is directly proportional to the control handle displacement, a relation expressed by

$$Y = AX \tag{1}$$

where Y = reticle displacement
X = control handle displacement, and
A = direct control constant.

A similar relation exists between the rates of displacement, or velocities, of the reticle and control handle:

$$\dot{Y} = A\dot{X}$$

where $\dot{Y}$ and $\dot{X}$ are the time rates of change of Y and X, respectively. In rate control the rate of change of reticle displacement is proportional to the control handle displacement, a relation expressed by

$$\dot{Y} = BX$$

where B = rate control constant.

In aided-tracking control the two effects are superposed and a displacement of the control handle produces both a displacement of the reticle and a change in the rate of reticle displacement, resulting in a rate of change in reticle displacement given by

$$\begin{aligned} \dot{Y} &= A\dot{X} + BX \\ &= A\left(\dot{X} + \frac{B}{A}X\right) \\ &= A(\dot{X} + aX) \end{aligned}$$

where $A/B = 1/a$ = aided-tracking time constant.

In what follows A can be taken as unity without loss of generality since it represents the gear ratio between the control handle and the rest of the system. Therefore, the choice of aided-tracking time constant will be studied using the relation

$$\dot{Y} = \dot{X} + aX.$$

Let the interval between target observations be τ and the actual target motion be $f(t)$. Then at the end of n such intervals the target position will be $f(n\tau)$. At this instant of time, $n\tau$, the reticle is placed on the target image thus making (see Fig. 2)

$$Y_{(n\tau+)} = f(n\tau+). \tag{2}$$

The $+$ is used to show that the relation holds for an instant of time slightly greater than $n\tau$ or at the beginning of the $(n+1)^{st}$ interval just after the reticle has been placed on the target image.

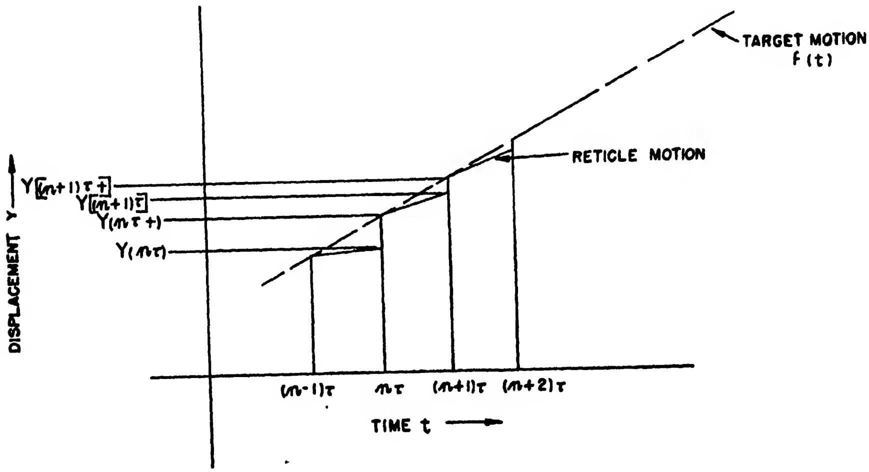

FIG. 2.

The value of Y at the end of this interval is

$$Y_{[(n+1)\tau]} = Y_{(n\tau+)} + aX_{(n\tau+)} = f(n\tau+) + \tau aX_{(n\tau+)} \tag{3}$$

where $X_{(n\tau+)}$ = control handle position after adjustment has been made at $n\tau$.

For ease in manipulation the operator $E(\tau)$ will be used where $E(\tau)$ shifts the time position of a function as shown below:

$$E(\tau)f(t_1) = f(t_1 + \tau),$$

$$E(\tau)Y_{(n\tau)} = Y_{(n\tau+\tau)} = Y_{[(n+1)\tau]}. \tag{4}$$

Using this in Eq. 3 gives

$$E(\tau)Y_{(n\tau)} = f(n\tau+) + \tau aX_{(n\tau+)} \approx f(n\tau) + \tau aX_{(n+1)\tau} \tag{5}$$

since X is the same at both ends of the $(n+1)^{st}$ interval and $f(n\tau) \approx f(n\tau+)$ because the value of the function changes only a very small amount during the adjustment of the control handle.

This may be written as

$$E(\tau)Y_{(n\tau)} \approx f(n\tau) + \tau a E(\tau) X_{(n\tau)}. \tag{5a}$$

It should be noted that $Y_{(n\tau)} \neq Y_{(n\tau+)}$ and $X_{(n\tau)} \neq X_{(n\tau+)}$ since X and Y are both suddenly changed at $n\tau$.

The change in X which must be made at each observation is equal to the error which exists at that time. Thus, at the end of the n^{th} interval of time or at $t = n\tau$

$$\Delta X_{(n\tau)} = X_{(n+1)\tau} - X_{(n\tau)} = f(n\tau) - Y_{(n\tau)}$$

and thus

$$E(\tau)X_{(n\tau)} - X_{(n\tau)} = f(n\tau) - Y_{(n\tau)}. \tag{6}$$

This gives

$$X_{(n\tau)} = \frac{1}{E(\tau) - 1}[f(n\tau) - Y_{(n\tau)}]. \tag{7}$$

Substituting (7) in (5a) gives

$$E(\tau)Y_{(n\tau)} = f(n\tau) + \tau a \frac{E(\tau)}{E(\tau) - 1}[f(n\tau) - Y_{(n\tau)}] \tag{8}$$

which may be written as

$$[E^2(\tau) + (a\tau - 1)E(\tau)]Y_{(n\tau)} = [(a\tau + 1)E(\tau) - 1]f(n\tau) \tag{9}$$

or

$$Y_{([n+2]\tau)} + (a\tau - 1)Y_{([n+1]\tau)} = (a\tau + 1)f(n+1)\tau - f(n\tau). \tag{9a}$$

This is a difference equation relating the values of the output Y at the beginning and end of each interval to the observed values of $f(t)$.

Equation 9 has a particular and a complementary solution. The particular solution is related to the form of the target function $f(t)$. The complementary solution is a transient and its form is independent of $f(t)$, but its magnitude depends on the conditions at the start of tracking.

To find the complementary solution $Y_{c(n\tau)}$, let

$$Y_{c(n\tau)} = Ce^{bn\tau}. \tag{10}$$

Substituting in the complementary equation

$$[E^2(\tau) + (a\tau - 1)E(\tau)]Y_{(n\tau)} = 0 = [E(\tau) + (a\tau - 1)]Y_{(n\tau)} \tag{11}$$

gives

$$[E(\tau) + (a\tau - 1)]Ce^{bn\tau} = 0,$$

$$Ce^{b(n+1)\tau} + (a\tau - 1)Ce^{bn\tau} = 0, \tag{12}$$

and

$$e^{b} + (a\tau - 1) = 0$$

which gives

$$e^{b\tau} = (1 - a\tau).$$

Thus from Eq. 9 the complementary solution is

$$Y_{c(n\tau)} = C(1 - a\tau)^n. \tag{13}$$

This equation shows that for $0 < a < 1/\tau$ the complementary solution approaches zero as time increases and does not change sign. For $1/\tau < a < 2/\tau$ the complementary solution also approaches zero but the terms alternate in sign. For the case where $a = 1/\tau$ the complementary solution is always zero, and this is probably the best choice of a. For $a > 2/\tau$ the complementary solution diverges. Thus for practical operation $0 < a < 2/\tau$.

For the case in which the target is moving at a constant rate K, the target position at each instant of observation will be

$$f(n\tau) = K(n\tau) \tag{14}$$

and the complete solution for the tracking output is

$$Y_{(n\tau)} = Kn\tau + Y_0(1 - a\tau)^n \tag{15}$$

where Y_0 is the initial tracking error and $Y_{(n\tau)}$ is the value of Y at the end of the n^{th} time interval.

Equation 15 shows that, for a constant rate target, setting the aided-tracking time constant $1/a$ equal to the time interval τ will give perfect tracking after the initial adjustment. This agrees with the results obtained in Section II, by reasoning. The choice of $1/a$ is also probably the best for other target motions since it eliminates the transient component of the response.

IV. CONCLUSIONS

The previous sections show that, where tracking is carried out on the basis of intermittent signaling, the optimum aided-tracking time constant will be the interval between signals. If, for example, the interval between signals is 10 seconds, the optimum aided-tracking time constant should be approximately 10 seconds. If on the other hand the interval between signals is 5 seconds, the optimum aided-tracking time constant should be 5 seconds. Clearly these facts suggest very strongly that the appearance of an optimum aided-tracking time constant in the neighborhood of 0.5 seconds with continuous signaling of the kind used at The Franklin Institute during the war, at the Foxboro Manufacturing Company, and at other places, may result from the operator treating a continuous signal as if it were a discontinuous signal with a silent interval of approximately 0.5 seconds. This conclusion finds accessory support from the well-established fact that visual attention is always discontinuous, never continuous,[4] as well as from the

[4] For a consideration of this fact, see "Experimental Psychology," by R. S. Woodworth, published by Henry Holt, pages 696–704.

fact that reactions are subject to refractory phase, that is, once we have made a certain response it cannot be repeated until a short interval of time has elapsed. Operators of tracking mechanisms undoubtedly are attending with blank intervals of from 0.25 to 0.75 seconds, as a consequence of which, even with continuous signaling, they are in a situation analogous to discontinuous signals with time intervals of this magnitude.[5]

When the aided-tracking time constant exceeds the time interval between signals (assuming it to be constant) the tracking with a capable operator will be stable and the transient component of error will decrease slowly and asymptotically to zero. If the aiding ratio has a value between the duration of the interval and half the duration of the interval, tracking will be stable and the transient component of the error will exhibit a damped oscillation around zero. If the aided-tracking time constant has a value less than half the time interval, the tracking is unstable and the error will increase indefinitely. When the aided-tracking time constant equals the time interval, tracking is nearly perfect, and errors are due mainly to variations of the rate at which the target moves under the reticle.

[5] An extensive discussion of the fact that discontinuous corrections are habitually superimposed upon a continuous motion is contained in the following two papers:

K. J. W. CRAIK, "Theory of the Human Operator in Control Systems": "I. The Operator as an Engineering System," *British Journal of Psychology*, Vol. 138, pp. 56–62 (1947); "II. Man as an Element in a Control System," *Ibid.*, Vol. 138, pp. 142–148 (1948).

NOTES FROM THE NATIONAL BUREAU OF STANDARDS *

CALIBRATION OF PHOTOGRAPHIC LENS MARKINGS

In the course of an experimental study of errors in the speed markings of photographic lenses, scientists in the optical instruments laboratory of the National Bureau of Standards have devised a convenient graphic method for converting each of these markings for a given lens to the corresponding "effective *f*-number"—an *f* number corrected for light losses within the lens. In this way it is possible to calibrate a lens so that losses of light from absorption, reflection, and scattering within the lens are taken into account, and a more accurate control of the amount of light admitted to the exposed film is obtained.

The method now in use for speed marking of lenses is based entirely on the ratio of the equivalent focal length of the lens to the diameter of the aperture. This ratio—known as the *f*-number—gives no consideration to the great differences in the useful light transmitted by various lenses.

The Bureau has also developed a method of testing the marked diaphragm openings so that values that entirely compensate for differences in transmission can be obtained and applied to the scale of *f*-numbers on a photographic lens. In this system the markings, known as effective *f*-numbers or *t*-numbers, are obtained by means of a photoelectric cell and a relatively simple photometric procedure, in which the diaphragm of the lens to be calibrated is adjusted to transmit the same amount of light as a similarly placed opening of standard size. The standard opening corresponds to an ideal lens of a given aperture ratio, in which incident light is wholly transmitted. A complete calibration is obtained by the use of a series of openings of graduated size corresponding to various aperture ratio values.

ACTION OF BORON IN HARDENING STEEL

An investigation at the National Bureau of Standards of the hardenability of boron-treated alloys and steels has shown that the action of boron in increasing the hardenability is due entirely to a solid solution effect at heat-treating temperatures.

High-purity iron-carbon alloys were prepared for use in this investigation and several commercial and openhearth steels were used. Determinations of the hardenability of the alloys were made in terms of the "critical cooling rate," which is defined as the slowest rate at which the alloy or steel can be cooled and be completely hardened. The standard

* Communicated by the Director.

end-quench test was used for evaluating the same properties of the iron-carbon-manganese alloys and commercial steels. Specimens prepared from the alloys in the conditions as cast, as forged and homogenized, and as heat treated in various ways to precipitate a boron constituent were studied metallographically. Experiments were also made to determine whether boron was lost during the decarburization of commercial steels and for determining the rate of diffusion of boron in austenite of these steels.

The Bureau's results and those published previously strongly support the conclusion that the effectiveness of boron in enhancing the hardenability of certain steels is due entirely to its action while in solid solution in austenite. Only the portion of the boron that is in solution at the time of quenching contributes toward an increase in hardenability. The boron undissolved or in the form of compounds is either without effect, or possibly decreases hardenability by acting as transformation centers for austenite in the temperature range where pearlite is formed.

When boron enhances the hardenability of a steel, it decreases the rate of nucleation and not the rate of growth of ferrite and carbide. The boron atom, believed to be located interstitially in the gamma iron lattice, is effective in retarding either the rate of formation of nuclei or the rate of their growth to the critical size necessary for transformation to begin, or both.

CAPACITANCE METHOD OF MEASURING WEAR

A convenient nondestructive method for measurement of the abrasive wear of textiles has been developed at the National Bureau of Standards as part of a program sponsored by the Office of the Quartermaster General, Department of the Army.

The method measures a property of the sample—namely, its capacitance—which is very closely related to the amount of material remaining in the portion of fabric tested. Since the specimen is not destroyed nor altered in any way during the capacitance measurement, a complete set of data may be obtained, showing the rate of abrasion at all times during the life of a single specimen.

The new method requires only three capacitance measurements to determine the extent of wear in a specimen over any interval of time. The electrodes of a precision capacitor are first adjusted to be slightly farther apart than the thickness of the thickest specimen to be studied. The capacitance C_a of the intervening air space is then measured. The capacitance C_0 with the unworn specimen between the electrodes is next obtained, and finally the capacitance C_R of the abraded specimen. The measure of destruction or ruin, Q, is then given in percentage by the formula

$$Q = \frac{C_0 - C_R}{C_0 - C_a} \times 100\%.$$

MAGNETIC MOMENT OF THE PROTON

The magnetic moment of the hydrogen nucleus, or proton, has been accurately determined in absolute units in the atomic physics laboratory at the National Bureau of Standards. When combined with other data, this work gives a precise absolute value for the electron charge-to-mass ratio, e/m. The gyromagnetic ratio of the proton, from which the magnetic moment was obtained, has been accurately determined by measuring the magnetic field and radio frequency required for magnetic resonance absorption in a proton sample. This is the first precise determination of the proton moment in absolute unit; all previous measurements approaching this precision have been made in terms of the relative values of other physical constants.

The new measurement provides an accurate secondary standard for magnetic fields. In the past, laboratory measurements involving both magnetic and electric fields have been limited by the low accuracy of the magnetic measurements. Now the situation is reversed, and magnetic fields can be measured more accurately than electric fields. This advance will be especially useful in the design and development of scientific and industrial apparatus employing magnetic fields where it is important to know accurately the special distribution of the field.

The nuclear resonance techniques developed in the course of this work can be applied to good advantage wherever the strength of a magnetic field must be regulated. The problem of magnetic field regulation arises widely in the use of scientific apparatus, such as cyclotrons, mass spectrographs, and beta-ray spectrometers, and in industrial equipment, such as servomechanisms and electromagnets.

EVALUATION OF PORCELAIN-ENAMEL TEXTURE BY A PLASTIC-REPLICA TECHNIQUE

A plastic-replica technique by which the texture of porcelain enamel may be conveniently studied and objectively evaluated has been developed at the National Bureau of Standards. The method of evaluation is based on the measurement of the haze of an ethyl cellulose replica of the enamel surface. Such replicas may be readily examined by either transmitted or reflected light, or projected for examination of the enlarged image. Details seen only with difficulty in the original surface are readily visible in the replica, which is a faithful reproduction of the surface texture. Another advantage is that the replica forms a permanent record for subsequent reference. By making a series of replicas during a test or service period, an investigator may follow the progressive breakdown of an enamel surface.

The replica technique should find application wherever small-scale surface roughness is evaluated. It can be used to classify surfaces as mat, semimat, or glossy, or to evaluate the degree of roughness de-

veloped in application of the enamel. It should lend itself readily to field inspections of enameled articles. Replicas made in the field can be examined by the investigator at his convenience, or sent to a central laboratory for study. A series of replicas of the same area of a test item, made before installation and after successive periods of service, will permit a study of the progressive breakdown of the surface. These replicas can later be compared with those from specimens treated in the laboratory to determine whether the laboratory test produces the type of damage observed in the field.

SKY COMPASS

The National Bureau of Standards has developed a sky compass, which indicates the direction of flight from which, with other data, the position of the aircraft can be determined. The compass is based on the investigations made by the late Dr. A. H. Pfund of Johns Hopkins University and is an outgrowth of his twilight sextant. The Pfund compass operates on the principle that the light of the sky during the day is partially plane polarized, the polarization being a maximum at right angles to the incident beam from the sun. The plane of polarization at any point in the sky thus contains both the observer and the sun. Establishment of this plane also gives direction, since it points to (or away) from the sun.

The principal advantage of the sky compass over the sun compass (which measures the direction of the sun directly) is its use during twilight, and when the sun is several degrees below the horizon, as well as when the region of the sky containing the sun is overcast, so long as there is a clear patch of sky overhead. The sky compass is thus of particular value when the sun compass and the sextant are not usable, after sundown before evening stars appear, and after morning stars disappear but before sunrise.

Essentially, the sky compass consists of an analyzer for determining the plane of polarization of the light, an azimuth circle on which the sun's computed azimuth can be set, and a clock that drives a chosen reference line in synchronization with the sun's apparent motion, that is, one revolution in 24 hours. For convenience in observing a portion of the sky around the zenith through the analyzer, the analyzed beam is turned from vertical to horizontal by means of a mirror.

PERFORMANCE OF PORTABLE ELECTRICAL INSTRUMENTS IN MAGNETIC FIELDS

Most portable instruments depend upon an internal magnetic field for their operation. A stray external field may, however, modify this internal field and make the instrument read incorrectly, particularly

when the external field is not taken into account. To determine the extent of such fields and how best to minimize their effects on voltmeters, ammeters, and wattmeters, an investigation of a representative group of commercial portable instruments was undertaken at the National Bureau of Standards. A brief study was also made of the stray field about some of the more common kinds of laboratory apparatus.

Magnetic fields of known magnitude, direction, and phase angle were set up about the test instruments by means of a pair of Helmholtz coils and suitable auxiliary apparatus. Auxiliary instruments, well-shielded and of high quality, were used as comparison standards. Placed well away from the Helmholtz-coil field, these instruments were read with a microscope of low magnification.

An analysis of the test results revealed that: (1) of eleven moving-iron-type instruments tested, seven exhibited field influence values within their accuracy figures in a 1-gauss field; in a 2-gauss field, four met this qualification; whereas none did in a 5-gauss field. (2) Of the seven permanent-magnet moving-coil instruments tested, two—having unshielded low-coercivity magnets—were affected by amounts comparable to their accuracy figures when subjected to fields of less than 1 gauss; and the other five—with high-coercivity magnets, or low-coercivity magnets combined with good shielding—behaved in a similar manner and could withstand 5-gauss fields or stronger without showing field influences greater than their accuracy figures. (3) Of seven electrodynamic instruments investigated, one (unshielded) in a 1-gauss field showed a field influence of 0.9 per cent, nearly equal to its 1 per cent accuracy figure; at 2 gausses the remaining six showed field influences less than their accuracy figures; while two of the six still met this qualification in a 5-gauss field.

The instrument itself (for most types) may be used to check the presence of harmful stray fields if its supply is sufficiently steady.

The Bureau's investigations show in general that the chief factor regarding the effect of stray fields on instruments lies in failure to recognize such effects and in not taking steps to avoid them.

THE FRANKLIN INSTITUTE

MUSEUM

The recent world war brought into prominence the problem of remote control in regard to the guidance of missiles and pilotless airplanes. The laboratories of The Franklin Institute played a not inconspicuous part in the solution of those problems. Meanwhile in numerous other laboratories other applications were being found for remote control and, after the cessation of hostilities we were suddenly confronted with a threat of a new science, that of cybernetics, in which the human being was surplanted in the performance of routine, repeated duties.

It is not, however, generally known except to Museum visitors that an interesting demonstration of an unconventional method of remote contol has been given daily in the Museum for over ten years.

In 1933 the late Thomas H. Dougherty of Wayne, while a voluntary worker on the staff, began to discuss with a colleague, William P. West, the project of building a remotely controlled boat. As a result of their consultations and that spirit of friendly cooperation which marks the staff relations, Dougherty began to construct a five-foot model of the 240-foot pleasure yacht *North Star*, while, at his request, West designed the mechanism which would permit the model to be manoeuvered in a manner very much like its prototype, both as to the operations performed and to the relative speed of response. After two years of work both men had completed their respective tasks. When the whole was assembled and tried out on a lake near Dougherty's home the two men had the satisfaction of seeing an excellent model yacht, complete in all its details and weighing forty-seven pounds, perform to near perfection in response to signals transmitted by the operator on the bank of the lake. So admirable was the accomplishment that only very few refinements were necessary to attain smooth and accurate control.

The boat is driven by a six-volt battery-powered motor and is steered by means of a motor driven rudder. The operation of both motors is controlled by a radio system which transmits through a telephone dial the number of impulses for any desired operation, and a stepping switch in the receiver which causes the correct relay to set in motion the manoeuver called for by the operator on "land." The stepping switch is a device widely used in telephone work. One impulse will cause circuit No. 1 to close, two impulses No. 2, etc. The transmitter is the push-pull type making use of a single type 19 tube. The frequency of transmission is in the five-meter amateur band, and the carrier is on the air continuously except during the transmission of the actual impulses.

Unhappily, Tom Dougherty's untimely death curtailed his enjoyment of and further experiment with the boat which he had projected with so much enthusiasm. However, many thousands of young people have delighted in its operation since Mrs. Dougherty presented the boat to the Museum. A special tank was built in the Marine Transportation Section and the boat is demonstrated here daily. Its popularity among the young and old visitors is apparent from the intense faces of the audience and from the numerous questions which are asked after each demonstration.

The remotely controlled boat is the final stage of man's development of his methods of transportation over water, the other steps of which are illustrated in the excellent series of models illustrating the progress made since the daring Norsemen set the prows of their longships in the direction of the West. But it is more than a symbol of man's conquest of the ocean. It is an example of the beginnings of that new science with the fearsome name of cybernetics of which we are likely to hear so much in the future.

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JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in the JOURNAL during the next few months:

YU, Y. P.: Applications of Network Theorems in Transient Analysis.

WHITE, W. C.: Electronics—Past, Present and Future.

FREUDENTHAL, A. M.: Workhardening of Metals.

GRIFFITH, ROY T.: Minimotion Typewriter Keyboard.

JACOBSEN, LYDIK, R. L. EVALDSON AND R. S. AYRE: Response of an Elastically Non-Linear System to Transient Disturbances.

MOON, PARRY AND DOMINA EBERLE SPENCER: A Modern Approach to Dimensions.

FANO, ROBERT M.: Theoretical Limitations on the Broadband Matching of Arbitrary Impedances.

FANO, ROBERT M.: A Note on the Solution of Certain Approximation Problems in Network Synthesis.

SILBERSTEIN, LUDVIK: Developable and Developed Silver Halide Grains.

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THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

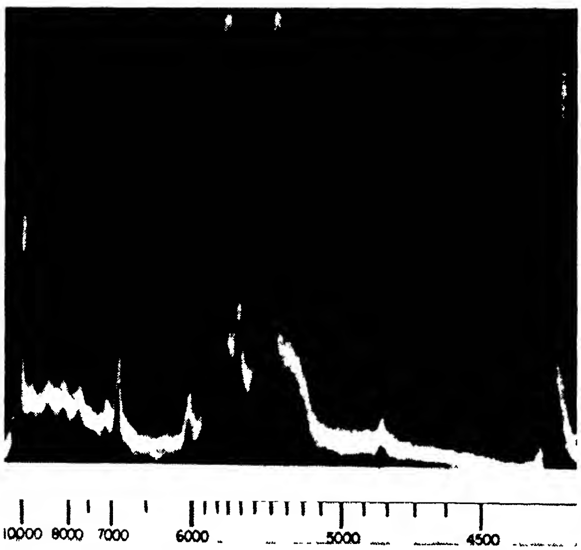
***Abstract of* The Image Orthicon in Spectroscopy.**[1]—R. E. BENN,[2] W. S. FOOTE,[2] AND C. T. CHASE.[2] Modern electronic techniques, applied to spectroscopy, not only enable one to

[1] Published in the *Journal of the Optical Society of America*, Vol. 39, No. 7, pp. 529–532, July, 1949. Based on work performed under contract with the Office of the Chief of Ordnance, Department of the Army.

[2] Research Engineers, and Assistant Director, respectively, Chemical Engineering & Physics Division, The Franklin Institute Laboratories for Research and Development, Philadelphia, Penna.

overcome certain limitations of the photographic plate or film; they also make possible the instantaneous observation or recording of spectral intensity as a function of both wave-length and time throughout a wide spectral region. One such application, in which the image orthicon replaces the photographic plate in the spectrograph, is discussed.

The image orthicon, RCA Type 2P23, is a recently designed television pick-up tube having high luminous sensitivity in the approximate range 3500Å to 10,000Å, with a peak in sensitivity near 4200Å. As a detector in spectroscopy the tube has a number of advantages of which three are outstanding: (1) For short exposure times it is much more sensitive to light of low intensity than any available photographic emulsion, since its signal-to-noise ratio is considerably greater than the ratio of density to background for the plate under these conditions; (2) It is sensitive not only in the visible region but also in the near-infra-red region; and (3) It can be made to respond to illumination, and provide a corresponding signal response, at any unpredetermined instant or succession of instants, and it exhibits, over fairly short times, a cumulative characteristic which for short exposures to weak illumination may be more linear than is the case for the photographic plate.



Engraving loaned through the courtesy of the Journal of the Optical Society of America.

Mercury spectrum as observed with recurrent sweeps on the orthicon spectrograph. (A scale of wave-lengths in angstrom units is attached.)

The output signal appears on an oscilloscope as a curve of intensity vs. wave-length in any range up to the limits, 3500Å and 10,000Å, imposed by tube sensitivity. If more than one signal is desired, a number of traces, separated in time by small predetermined amounts, may be photographed by means of a high-speed camera, or separated on the oscilloscope screen and photographed in a single exposure. This feature is of particular value in the study of rapidly varying or transient sources, especially where the illumination is so faint that a moving plate or film cannot be used in the spectrograph.

The application of the orthicon to spectroscopy was originally made in order to study a gaseous combustion process in which a few prominent spectral lines and bands were available. The time variation of the intensities of these lines and bands, unobtainable in many cases on a moving plate or film because of low intensities, is observable with ease and accuracy. Preliminary work of this nature has already been reported.[3] With this technique it should be

[3] AGNEW, FRANKLIN, BENN, AND BAZARIAN, *J. Opt. Soc. Am.*, Vol. 39, p. 409 (1949).

possible to follow the course of a chemical reaction where certain radicals are free and active only during the reaction, and are not available or detectable before or after the reaction.

Preliminary observations indicate that for short exposure times the orthicon is about fifty times more sensitive than the photographic plate, if the orthicon response is measured above noise and the plate response above background. The orthicon has the further advantage that the entire region between 3500Å and 10,000Å can be covered at the same time—no single photographic emulsion will provide this coverage. Errors due to the use of several plates of different types, especially if used at different times, or to low sensitivity in the overlap regions, are thus avoided.

NOTE: Reprints of above paper are available in limited quantities. Address: Administration Division, The Franklin Institute Laboratories for Research and Development, Benjamin Franklin Parkway at 20th Street, Philadelphia 3, Pa.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

The Changing Face of Research.—ELLICE MCDONALD, Director. In one lifetime, all arrangement of life has changed and it would be surprising if research, which is so much responsible for that change, should not have altered in its arrangement and orientation also. Perhaps research in itself is more responsible than anything for that change.

The pattern of life has altered. In one generation, standards have changed; there are no charts to guide the progress of present day affairs. All now must depend upon importunity: to do the best we can until we see how things break. Our fear is that, as a result of world affairs, less good things will result than the moderate good we had before and yet, as scientists, we can visualize a better arrangement than ever we had before, were we free to arrange for ourselves. Maybe this is our defect, but yet we are only slowly coming into our own: still confined and conflicted, made use of and depreciated by the ignorant politician or demagogue who has greater appeal and resource than the silent scientist, so long as people persist in classifying science as something apart from ordinary life.

Things have changed so in one lifetime that it is only in retrospect that we begin to appreciate how revolutionary the changes have been. It seems as though our advances in knowledge and technical capacity have gone so far ahead of our experience in government and control that there is a lag between them that will take a considerable number of years to catch up. Our technical advances in living, comfort, manufacture and general ease have been great, but there is a question whether the present deterioration of government inefficiency and honor has not depreciated all our effects, so that we are lagging rather than advancing in spite of all material progress. Certainly the cynicism of present day Statism (for that is what it is), seems a reaction and a rebuttal of all scientific efforts toward betterment. For why should we make such lifelong efforts toward the betterment and ease of mankind if a sign painter with a vogue for dramatic oratory was able to do more for destruction of the world than all the scientists have done to build it up? This man, incompetent, ignorant as he was, has changed the world and with it he has changed research. Or to put it in better words, the forces which allowed him to change the world had even before changed research.

These forces came of research. Such things would not have been possible forty years ago: all these are dependent upon research and discovery; good or bad as the results may be. Undoubtedly research in its passion for detail, precision and measurement has become a trifle

inhuman in the sense that it has not taken into account the low level of human appreciation. Scientists fulminate and adumbrate against present conditions to scientists, a limited group who are mostly inarticulate. The result does not touch the mass of the people who believe in headlines. Of course, one trouble with scientists is that they are trained to tell the truth and exact knowledge is a handicap to forcible statement. It is difficult for them to distort the truth, for their search is truth, and if they are in doubt, they modify their statements by so many qualifications that the final summary is denuded of its real value. They suffer from what a very interesting newspaper reporter once designated as "factism." They want to have such exact statement made that the main idea is obscured in detail: in other words, they are speaking to their fellows and, if we continue to speak to our fellow scientists, we cannot complain if our fellow scientists (and few of them) alone digest our statements. It is our own fault that we have not impinged more on the affairs of our own people to show them how management could be made useful. Minority pressure groups have usurped the place that the advice of scientists should have in affairs of government.

We have not even applied our principles to the most common incidents of life. The law of natural selection can only operate successfully with the law of natural elimination. We have done away with the latter with our intense preoccupation over lives that Nature will have none of by hospitals and welfare centers. This prudery and preoccupation of ours refuses to face an obvious fact and still less to put it into plain words. We give endless patience, trouble and thought to the breeding of our horses, cattle and dogs and none at all to the production of that infinitely more important man. The still small voice of science is drowned by the smooth speaking politician with his unlimited and irresponsible promises.

So research in spite of its great advances has lost out to the little people. Its defects were the faults of precision, exactitude and honesty and it must continue so if it is to be research, because the qualities of research are such that the last and most fatal ditch that any research man can fall into is to alter his results according to his need and away from the truth. This is so rare that in forty years of experience only one such has come under observation and he returned to "heil" the Hitler he then despised.

Research does not stir the soul of crowds like the preachers, Mohammed, Peter the Hermit, St. Paul, Luther and the other Pied Pipers. These subtle rhetoricians, seeking their own personal interests, endeavor to persuade by flattering base instincts and dwelling upon hardships and injustices. They then arouse in their hearers a faith which has always been the function of great leaders. It is not by students or philosophers, still less by scientific sceptics, that have been built the great movements

which have swayed the world. The researchers are too exact and too inarticulate.

One extraordinary thing is the poor quality of these leaders. Napoleon did not succeed immediately by his qualities and it was only an accident that he had such training as to develop him into a capable administrator. He had the capacity, as did Genghis Khan who was another great administrator and conquered most of the known world. However, the astonishing thing to scientists is the poor quality of human material that can sway the populace. Affirmation, pure and simple, kept free of all reasoning and all proof, seems to be one of the surest ways of making an idea enter the mind of the mass. The more concise the affirmation, the more destitute of every appearance of proof and demonstration the more weight it seems to carry. Thereafter, repetition and continued repetition induces a contagion and the uncertain basis of fact is lost and forgotten. The mass opinions are propagated by contagion, but never by reasoning, so that the scientist with his predilection toward exactitude and deduction has little influence upon world affairs, except as his discoveries can be useful to the politicians. When the scientist ceases to depend upon his results, then he loses his name. It is by word and not fact that crowds are moved.

The role of the research worker is not such as to wield immediate world influence, although his permanent influence on world affairs is greater than many of the ephemeral exploitations and explosions of the politicians which arouse in scientists a deep distaste and distrust.

However, after all that complaining, this is the world and we have to live in it. Some years ago, some of us tried to get a definition of sanity. In the group were two psychologists (whatever they are), one psychiatrist and a medical doctor. The best we could do was make a definition that sanity was "the degree of contact with reality." It takes one back to the two old Quakers—"All the world's queer but me and thee and I sometimes think thee a little queer."

In our time, however, the world has changed. It has changed in so many material ways that it is only by retrospect that we can appreciate the difference. The changes which have occurred can be listed under three heads:

1. Annihilation of distance
2. Scale of operations
3. Increased use of power.

These three, although very broadly stated, are at least, in part, the factors which have determined the enormous change in life in our time. Because this change has occurred within so short a time it is a little difficult to see in perspective. It has happened to us and the quick succession of events have followed in such sequence that they have overlapped and become a continuity, rather than being as the physicists say "particulate."

In the life history of lower organisms which can be studied in their various and succeeding generations on account of their shorter life span, succeeding generations will go along on an even keel of development and arrangement, so that each new organism maintains the character and resemblance of the previous parents. There then sometimes comes, from whatever unknown cause, a deviation from standard type and new generations of the organisms develop to a different type from their progenitors, so that the succeeding type of organisms is different from those which have gone before. This is called a "mutation period" where, in the course of evolution, the organisms, such as the drosophila or fruit fly for example, change their type.

In human affairs, this present time seems to be a mutation period for mankind. Such stirring events have happened in one man's lifetime of threescore years and ten that, if measured in events, such a man might consider himself to have lived perhaps 300 years rather than the allotted time of the Psalmist. The revolution of life cannot but have its effect upon the nature of man, although we may be too close to the picture to comprehend the vastness of the change.

What have these things to do with research? Has research altered as has life? Are these the product of research or does research advance result from the accumulated incidence of the discoveries of the last fifty years? As well ask, as the snow ball rolls and grows in size with accumulation, is it due to the momentum or to the adhesive qualities of the soft snow. Or is the hen before the egg which is before the hen which lays an egg. Incidentally, this last statement was debated by Aristotle.

Whatever the mechanism of the matter, there is no doubt that research has been profoundly altered, although some research workers do not yet see it, for, strange as it may seem, research men are sometimes the most conservative and reactionary of people although their sphere should be to do with new things.

Annihilation of distance and ease of communications have profoundly influenced research. Marconi has rendered research provincialism a migratory if not a meaningless thing. Physically, as a result of these discoveries, the continents are a thousand times closer together than were the scientists of a hundred years ago. The rapid dissemination of news makes all of us contemporary participants in the experiences of the whole world of science. A discovery is made and it is flashed around the world to be seized upon with eagerness by all those with research curiosity. "Let us check his conclusions and see if they can be verified." In almost no time, scientists are working, checking and developing the idea and applying it to their own problems.

After publication, the report arrives, perhaps in a day from Europe by transatlantic clipper, perhaps in a week by steamer. Details and methods are known and the whole world of science is instructed.

If there is a new development, even a new theory, men flock to the originator for instruction. In 1926, there were twenty American physicists in Göttingen, where Born and Jordon had announced a new development in quantum mechanics. A year later, they were at Zurich with Schroedinger; a couple of years later with Heisenberg at Leipsig; then Dirac at Cambridge was the attraction. The scientist, and quite right, will, as Horace says, take honey from any man's hive. A personal journey to hear Bohr in 1919 gave me a sentence which altered my entire view of biological research. Bohr then had only the first model of the atom and he said, "From now on the face of science will be changed. We can now study from the minute to the mass instead of from the mass to the minute." It struck home to me that at that I had been studying the processes of life from the mass to the minute, down the branches of the tree instead of from the roots.

Men's minds are quickened and their imagination stimulated so that thought is released from the shackles of authority and traditional dogma. The annihilation of distance, the ease of communications, with the size of operations, combined to accelerate the progress of research to an extraordinary degree. An example of this is the check that has resulted on the statement of Kögl in Copenhagen that, in cancer, the form of the amino acids was different from that in normal tissue. The unnatural or unusual form of glutamic acid, the $d(-)$ form, for example, was said to be more preponderant in cancer tissue than the usual or natural $l(+)$ form.

This statement, if verified, might be of enormous importance and might point the way to the cure of cancer where chemical differences from normal are slight. It would involve a new set of enzymes from the normal, different forms of metabolism, new chemical configurations and new properties. This study was a difficult one, for one reason because it involved animal cancer and human cancer experience, biology, chemistry and physics, branches of science which do not so often come together. For another reason, the chemical methods were difficult and fraught with pitfalls.

However, the statement was seized upon by researchers in England and the United States and, within a year, such doubt was cast upon the results that there was no longer hope of its accuracy. Our own laboratory contributed no little part to this work. On viewing this event in retrospect of my experience, 20 years ago this check would have taken five years' time and then might have not been completed nor indeed could have been done at all at that time.

Such a circumstance as this depends not alone upon quickness of communication but also upon size of operations. There are so many more people working in such larger coordinated groups that scientific research must advance by leaps and bounds. Coordination of the sciences, physics, chemistry and biology, makes it possible to gain a

comprehensive view of a problem where the isolated methods of the part would give no answer, but only come to the impasse.

Size of operations applies not only to research groups but to the scientific societies which actually exist for the exchange of information. The big aggregations of incorporated capital are one example of the increase in the size of operation and they, by the possibility of cooperation of large numbers, have made possible advances, based upon research, which have revolutionized human affairs. The dread spectre of war itself enlarged the hideousness of its countenance by the masses which it employed.

In science, large numbers of men exchange information, discuss methods and check results. An example is the American Chemical Society with 60,000 members, two large meetings a year attended by ten thousand chemists, and more than two hundred and ninety minor or local meetings a year. Another example is the American Association for the Advancement of Science with about 33,000 members. The programs of these meetings are now so large that they have almost become unwieldy, but a mere glance over the printed list of papers is stimulating to the imagination and gives inspiration to the mind.

Another example of the increase in the size of operations is the collection of scientific literature and the great number of scientific publications which are in existence at present, compared to the few of fifty years ago. In the Biochemical Research Foundation, for example, we have besides books and bound volumes a reprint collection of 37,600 items collected over 20 years. These are indexed and cross indexed as to author and subject with a separate index under "methods". A cross section of all papers pertinent to our type of research is always easily available. It is much easier to take ten or twenty reprints from the files instead of as many bulky bound volumes. The reprints can then be sorted out and such as are required will come easily to hand. If these reports were in books, the bound volumes would have to be strewn over a room with each place marked and would not convey the same impression to the mind as a number of thin pamphlets, easily placed on one's desk to be taken up, studied, and sorted with such ease. This collection which has taken constant care, time and money to create is the most valuable single item in our laboratory and in case of fire, there is an order that it should be the first thing saved, for it is not replaceable.

As to the third category of *increased use of power*, this has its prototype in science in the vast numbers of instruments of precision which have been devised to aid the scientific observation. The number of these would require pages to enumerate and every day brings another. These increase the power of man by giving him tools to extend the value of his bodily senses, just as power, produced by heat or electricity, extends man's muscular power.

The development of a new apparatus in Sweden means its use in a few days in the U. S. A. Not only in precision, but in the extension of our organs of sense by microscopes, spectrographs, photographic applications, etc., all bring methods of accuracy and record to make new advances possible. Indeed, the mere necessity of such apparatus, most of which requires specialists, makes larger aggregations and cooperative work necessary and makes the lone worker, the prima donna type, less effective. After all, the prima donna requires an opera company to support her genius.

A great deal of this advance has been due to development in physics. Until 1888, the physicists had really given up hope of increase in their knowledge of their science; all that was thought to be left was the elaboration of detail to the fifth decimal point. The knowledge of the visible frequencies was only a minute space in the whole spectral range. The discoveries by Hertz were the beginning, followed by Roentgen, Bohr, Hess, Millikan, Schroedinger, etc., and opened a new field in regard to matter and the structure of the atom.

Chemistry, physics, geology, astronomy and biology were conceived as parallel lines which might be infinitely prolonged without danger of meeting. Now all science is one. If chemical combination is the giving, taking and sharing of electrons, surely this makes it akin to physics. If life depends upon oxidation-reduction processes, these in turn must depend upon electron interchange. Biology, which includes the study of life and disease, on account of its many variables, has been the lagging phase, but with the advances of the recent past, it does not require much imagination to realize that in this branch are due future great discoveries. Its dawdling alone means that it must take up its slack.

These advances facilitate our work and are more than an omen, they are a cause of success. This is one of the pleasing surprises which often happen to active resolution, so that may things, difficult to design, by new tools prove easy of performance.

All this has been done without government subsidy and in the past Government agencies have had the criticism of taking others' work and exploiting it to bolster their claims for increased budgets. The real scientist looks with some trepidation to the plans for vast government subsidies and control of research. It is misleading to assume that expenditure is identical with efficiency. The commission form of operation is useless. It fixes neither responsibility nor authority and is deadly to initiative and enterprise which is the soul of research.

The argument for a planned and completely coordinated research program on a national scale is based on the rapid results obtained in the field of atomic fission, penicillin, other biotics, etc., by means of highly coordinated research efforts. However, an examination of the background of these problems shows that the rapid evolution was really in the field of *development* rather than in *research* and *discovery.*

The role of the exploiters was that of an *entrepreneur*, as Florey acted in the development of penicillin discovered by Fleming many years before.

However, in a field where no clearly defined line of approach can be said to exist it is doubtful whether a rigidly planned program would produce results more rapidly than the time-honored method of independent research activity, for discoveries are made by men, and if the free spirit of research is hampered, the vision is impaired. Discoveries are the unveiling of the unknown and so cannot be foretold in advance: if it were so, they would not be discoveries. A planned research tends to concentration in much detail in a few directions which are lucky if one succeeds in hitting the correct solution, an experiment and error type of research. Looked at in this light, the probability of hitting a correct solution is not so good.

The real scientist is a creative artist. No one must boss him or drive him. Certainly arrange that his life be free from monetary worries; but let us remember that his life interest and pleasure is research. Leave him to carry on undisturbed by the stormy blasts and distractions of a quarrelsome world licking its wounds after a terrific fight. Do not let him be regimented by the politicians, even those of his own class. Then, the upward curve of research and discovery can continue unhampered, which it will never do if the spirit and vision of research are not free for advance. These will come through the exceptional man and he is often quite unappreciated by those of bureaucratic tendencies. There is an old Scottish proverb, "Wha pays the piper, ca's the tune", and unless those who call the tune have a sympathy and understanding of the research instinct and the necessity for untrammelled development of its creative curiosity, the vast government subsidies will be worse than wasted.

The present is indeed an age of stupendous advance, but granting the utmost to the creative genius of our time, we should never have run our mile, but for the furlong achieved by the pioneers of science in the last three generations. The past is important, the future is exciting, but the present interests us because we live in it.

BOOK REVIEWS

STRENGTH OF MATERIALS, by J. P. Den Hartog. 323 pp., illustrations, 16 × 23 cm. New York, McGraw-Hill Book Co., 1949. Price, $4.00.

Many books have been written on strength of materials but it is seldom that one covers the field as well, and still makes such interesting reading, as Den Hartog's newest book. It is written as a text for use in a first course in the subject. Wisely, certain sections are marked for omission if the teacher believes them too difficult. There are over 200 pages of text followed by 87 pages of problems. The problems are better than those generally found in similar texts in that they require real thinking on the student's part. Certainly some of them are beyond his capacity while studying the subject for the first time, but with the assistance of a good teacher they will broaden his thinking and improve his methods of attacking a problem.

The first three chapters are entitled Tension, Torsion, and Bending and cover the most important cases of stresses in beams and shafts. These are followed by a chapter on Compound Stresses where Mohr's Circle is introduced together with the maximum-stress, maximum-strain, maximum-shear-stress, Mohr, and maximum distortion energy theories. The next three chapters adequately cover beams of different types with various loadings and thin- and thick-walled cylinders.

There is an excellent chapter on Experimental Elasticity covering Photoelasticity, Strain Gages, and Fatigue. The student should find the section on the use of Resistance Strain Gages interesting and extremely valuable information. The problem of determining the magnitude and direction of the principal stresses at a surface point is solved by the use of a rosette of three strain gages. This gives strains in three directions, 120 deg. apart, and Mohr's circle is applied to determine the principal stresses and their direction.

Although this book was written as an undergraduate text, it is highly recommended, because of its easily readable style, for the engineer who would like to brush-up on the fundamentals of the subject.

WM. M. MORSELL

ABSORPTION SPECTROPHOTOMETRY, by G. F. Lothian. 196 pp., tables, drawings and illustrations, 14 × 23 cm. London, Hilger and Watts Ltd., 1949. Price, 26 s.

This book is, as the subtitle states, a "Manual on the measurement of absorption spectra in the ultra-violet, visible and infra red regions of the spectrum by photographic, visual and photoelectric methods" and is of practical value especially to less experienced workers. It is divided into three sections dealing respectively with general principles, typical applications of spectrophotometry and the technique of measurement.

The first part contains a brief resume of the development of work in this field. For those not familiar with modern physical theory, some theory is included on the production of line and band spectra. The laws governing absorption, the definitions and nomenclature involved are discussed and well illustrated. The conditions for securing accuracy are clearly set forth and practical suggestions and information given concerning the choice of dispersing media, lenses, detectors, etc.

The three chapters comprising part 2 deal in more detail with the applications of spectrophotometry to the study of molecular structure, to chemical analysis and to biological and biochemical problems. Again the reader is introduced briefly to modern theory as a basis for a clear understanding of the relation between absorption and chemical structure. The absorption maxima of various chromophoric groups in the ultraviolet is given and the influence of other added groups illustrated. Absorption of vibration-rotation bands in the infra red is considered and illustrated by series of curves of related compounds. The usefulness of spectrophotometry in chemical and biochemical analyses is abundantly illustrated with references to much recent work.

Part 3 is devoted entirely to the description of available instruments and their merits as tools for the solution of particular problems together with practical instructions in the technique of their use and it can well be summarized by listing the chapter headings—visual spectrophotometry, photographic spectrophotometry, photoelectric spectrophotometry, spectrophotometers with thermal detectors, light sources, cells, standard substances, solvents and diffusing media.

Included also is a bibliography of more than 200 papers and an additional list of 18 books or papers not referred to in the text.

RACHEL FRANKLIN

STRUCTURE AND PROPERTIES OF ALLOYS, by R. M. Brick and Arthur Phillips. Second edition, 485 pp., illustrations, 15 × 23 cm. New York, McGraw-Hill Book Co., 1949. Price, $6.00.

The second edition of this volume of the well-known "Metallurgy and Metallurgical Engineering Series" embodies in an enlarged and improved form the principle underlying the first.

Revision and expansion of the former text, addition of two new chapters (on alloys of magnesium and on corrosion- and heat-resistant alloys), and inclusion of numerous new tabulated and plotted data and illustrative micrographs have resulted in a volume of 485 pages.

The method of giving all pertinent information on composition, treatment and specimen preparation and of including a discussion of the relevant features of the structure in the captions of the individual micrographs is certain to meet with the fullest approval of the users of this book.

The authors set themselves the double task of acquainting the reader with the principal classes of commercial alloys and with the industrial methods of their manufacture, and of providing him with theoretical concepts that would aid him in the interpretation of their structure and properties and of the physical changes induced by technological treatments.

This task is solved with great didactic skill by following the principle of presenting typical structures and properties of alloys in conjunction with the correlated typical equilibrium diagrams, and of arranging the material in the order of increasing complexity of equilibrium structure realized in the several classes of alloys.

At each stage, the presentation of a typical equilibrium diagram and the correlated structures and properties is followed by a discussion of the time-dependent internal processes, either spontaneous or induced by mechanical, chemical, or thermal treatment, by which these structures and properties are modified.

The authors succeed in giving much practical information on pure metals, copper-, lead,-aluminum-, and magnesium-base alloys, two-phase copper alloys, plain-carbon and alloy steels, cast irons, and sintered metal powders, which renders this volume valuable also as a reference book.

The theoretical concepts required for the discussion of the processes of crystallization, diffusion, precipitation, recrystallization, strain-hardening, quench-hardening, and stress relief are presented in a manner apt to kindle the reader's interest in the subject and to induce him to delve more deeply into it.

This reviewer's only regret is that the general excellence of this volume is somewhat marred by what he considers a quite unnecessary laxity of expression in the definitions of some theoretical concepts and in many descriptive parts of the text, all too commonly found in publications in the field of theoretical metallurgy.

OTTO R. SPIES

MATHEMATICS OF CIRCUIT ANALYSIS, by Ernst A. Guillemin. 590 pp., illustrations, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1949. Price, $7.50.

The text is designed specifically as a source book to supplement the usual undergraduate engineering mathematics curriculum. It is more appropriate as a textbook for a course in higher mathematics for engineers. No claim is made as to its rigor, rather it is advanced as a

book presenting an engineering viewpoint. The seven chapter headings are: Determinants, Matrices, Linear Transformations, Quadratic Forms, Vector Analysis, Functions of a Complex Variable, and Fourier Series and Integrals. These topics are adequately treated in surprising detail. Exercises are given at the end of each chapter.

The chapter on determinants covers material through Cramer's Rule, the rank of a determinant, and conditions for the existence of solutions of a system of simultaneous linear equations. The chapter treating matrices enables one to find the inverse of a matrix in several ways. It discusses partitioning of matrices and briefly introduces the idea of the latent roots of a matrix. This idea is carried further in the next chapter which commences with vector sets and takes the reader through the Cayley-Hamilton Theorem.

There is good continuity throughout the text. Each chapter lays the necessary groundwork for the material to be presented in the following one. The last three chapters are quite lengthy going into considerable detail. The sixth goes into the standard material in complex function theory including the principle of analytic continuation, Laurent expansion, contour integrals, Riemann surfaces and the linear fractional transformation; and ends with two articles treating stability criteria and physical impedance functions. In the last chapter on Fourier Series and Integrals the author has included a discussion on evalutaion of inverse transforms through complex integration, and approximate evaluation through use of the saddle point method.

The author is an engineer who has written two volumes on communication networks that are well known in their field. Though written by an engineer from his viewpoint, this text deals entirely with advanced mathematical topics.

DONALD B. HOUGHTON

ELECTRICAL TRANSMISSION OF POWER AND SIGNALS, by Edward W. Kimbark. 461 pp., illustrations, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1949. Price, $6.00.

"Electrical Transmission of Power and Signals" fills a long-felt need for a text which treats not alone the transmission of power, audio frequencies and radio frequencies, but which thoroughly deals with the transmission of all three varieties of electrical energy. As the author points out in his introduction, this concept in teaching is not new. However, it is believed that this is the first text to embody this principle. To the student, this book should be of great value, for even in this day of specialization in the electrical engineering field, the practicing engineer cannot totally disregard the transmission of types of electrical energy other than that with which he is primarily concerned. For the graduate engineer, the book is a good reference whether his main interest be power or communications.

Dr. Kimbark's book consists of sixteen chapters and two appendices. The text material opens with line parameters, goes on to steady state phenomena, then to transient phenomena, and finally to lumped parameter circuits. It contains individual chapters on impedance matching, filters, skin effect, and wave guides. At the end of each chapter is an excellent and very complete bibliography, in addition to which are a number of problems. Throughout the text the author presents numerous sample problems to assist in understanding the text material and to demonstrate practical applications of the theoretical material. The appendices contain much valuable information concerning the characteristics of conductors for open-wire power transmission lines and the characteristics of many types of transmission lines for both power and signals.

ROBERT S. GRUBMEYER

THERMODYNAMIC CHARTS FOR COMBUSTION PROCESSES, by H. C. Hottel, G. C. Williams and C. N. Satterfield. Part One—Text, 75 pp., 22 × 28 cm. Part Two—Charts and Tables. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd., 1949. Price, $2.60 and $2.40.

This book, presented in two separately bound sections, will undoubtedly prove to be a most valuable aid to engineers in fields of combustion and power generation. Part one consists of the text which introduces and describes the charts which are bound separately as part

two. As the authors point out several times in the text, the very rapid increase in the utilization of power cycles wherein the products of combustion of fuel and oxidant actually become the working fluid has not been accompanied by adequate data by means of which engineers might calculate or make a reasonable guess as to the performance expected. The compilation of such data is hindered by the difficulty of being able to accurately determine the thermodynamic properties of the endless number of complicated mixtures which might exist during the combustion of a given fuel and oxidant.

The first approach presented is the construction of a "modified-air chart" which is analogous to the Mollier diagram for steam. This chart is applicable in the low temperature regions where gas-law deviations may be important and involves the burning of moderately volatile hydrocarbons in air. The chart presented is for octene, and depicts the properties of both burned and unburned fuel-air mixtures, with the aid of supplementary plots. It can be used as a chart for pure air or as a mixture chart with allowances being made for moisture and combustion. Two illustrative examples involving the use of the chart are given for a Turbocompressor Power Plant and the Diesel Compression-Ignition Cycle. Fundamental data used in constructing the Modified Air Chart are presented.

At higher temperatures where chemical dissociation occurs, the Modified Air Chart is no longer applicable and a series of "Burned Mixture Charts" for several burned mixtures of octene and air are presented. Six of the charts give the thermodynamic properties of burned mixtures of $(CH_2)_x$ and air at higher temperatures (up to 5500° R) for ratios of fuel used to that required for complete combustion, F, of 0.8, 0.9, 1.0, 1.1, 1.2, and 1.5 while the seventh gives the properties of $(CH_{1.625})_x$ corresponding to 50 mole per cent xylene and 50 mole per cent octene (or 35.7 weight per cent benzene in $(CH_2)_x$), for F of 1.5 (50 per cent more fuel than is required for complete combustion.) Many illustrative examples involving the use of these charts are presented including; the throttled Otto Engine Cycle, the supercharged Otto Engine Cycle, Effect of Flame Travel (both at constant volume and with moving piston), Ram Jet, and the Turbocompressor Engine with Tail-Pipe Burner.

The last of four chapters comprising the text is a description of a general method of attack toward the analysis of any power cycle involving the combustion of a fuel and oxidant such that the four elements involved are essentially only carbon, oxygen, hydrogen and nitrogen. This approach is on the basis of the probability that by far the preponderance of combustion cycles in the near future will involve essentially these four elements.

Tables are presented giving equilibrium gas compositions at two pressures, 14.7 psia and 300 psia, for various mixtures involving H_2O, H_2, O, H, O_2, OH, NO, N_2, CO, CO_2, C, at 200 degree intervals over the temperature range 1600–3200° K. In addition, tables of generalized thermodynamic data involving enthalpy, entropy, internal energy and molecular weight are given as a function of chosen values of the atomic ratios, C/O, H/O, and N/O for the same two pressures and over the same temperature range.

The first chapter of the text contains a very clear discussion of the field to which these charts are applicable and a review of previous work in the field. A brief summary of fundamental data is also included for the purpose of quickly preparing the reader, who is assumed to already be familiar with thermodynamic concepts and methods, for the stress which is to be made on the chemical aspect of the problem. Basically, the complexity of the problem of preparing thermodynamic charts for combustion processes is precisely this difficulty in accounting for the effects of the release of chemical energy during the combustion process. This book represents probably the most practical approach to the problem which has appeared to date.

J. T. AGNEW AND W. E. SCOTT

BOOK NOTES

HIGHER ALGEBRA FOR THE UNDERGRADUATE, by Marie J. Weiss. 165 pp., 14 × 22 cm. New York, John Wiley & Sons, Inc., 1949. Price, $3.75.

Intended for students with a background of two years of college mathematics including calculus, this textbook introduces some of the simpler algebraic concepts. Such topics as

groups, rings, fields and matrices have been presented on an equal basis with the theory of equations. Adequate examples and exercises are given for the use of the student.

EXPERIMENTAL PHYSICS FOR COLLEGES, by Walter A. Schneider and Lloyd B. Ham. Revised edition, 442 pp., illustrations, 14 × 21 cm. New York, The Macmillan Co., 1949. Price, $3.80.

The authors have rewritten nearly half of the book in preparing this revised edition. Many new experiments have been added, while some old ones have been dropped. A rearrangement of the chapters has been made emphasizing the fundamental divisions of physics. The authors' primary aim of preparing a text on experimental physics for elementary students, which would be more than a set of laboratory instructions, has been maintained.

ANALYTIC GEOMETRY AND CALCULUS, by Frederic H. Miller. 658 pp., diagrams, 15 × 22 cm. New York, John Wiley & Sons, Inc., 1949. Price, $5.00.

Professor Miller in his most recent textbook has correlated the subjects of plane and solid analytic geometry, differential calculus and integral calculus as a single branch of mathematical analysis. He offers three specific advantages for this treatment, namely, that interest is more readily aroused, that the fundamentals of calculus may be introduced sufficiently early to be of use in other subjects, and that the reconsideration of a specific topic, when new concepts are introduced, serves as a review of basic principles.

Among the features which should be noted are the summaries at the end of each chapter; the appendix, with a list of symbols, formulas and tables; the 3025 exercises; and the treatment given such topics as maxima and minima, conic sections and the representation of loci in polar coordinates.

HYDROLOGY, by C. O. Wisler and E. F. Brater. 419 pp., illustrations, plates, 15 × 22 cm. New York, John Wiley & Sons, Inc., 1949. Price, $6.00.

With the ever recurrent appearance of items in the press concerning water shortages, lowered water tables and the like, it is obvious that a knowledge of hydrology would be useful to many persons. Water is needed not only for drinking purposes, for agriculture, for power, but increasingly in manufacturing processes, and for such new developments as air conditioning. These increased uses have necessitated tapping new sources of supply, re-use of water, and even limitations of some uses in certain areas where the essential needs could not otherwise be met.

Hydrology is a relatively new science which has seen a great increase in knowledge in the last two decades. The present book is intended primarily to serve as a textbook for college and university use, but secondarily in view of the limited material in book form, as a source of reference for anyone concerned with the problems it touches. In their presentation, the authors commence with a brief definition of hydrology and its importance, and follow this with a discussion of the hydrograph, a graphical representation of a stream's fluctuations in flow arranged in chronological order.

Subsequent chapters deal with the drainage basin, precipitation, water losses, infiltration, ground water, runoff, flood and stream-flow records. Each of these topics has been developed extensively with attention both to theoretical and practical considerations. The chapter on ground water by John G. Ferris points out that the supply is not inexhaustible and that it is necessary to achieve fuller use by sound and rational methods of a trained hydrogeologist and hydrogeological engineer.

There is much of interest in this book for the non-hydrologist and it offers a stimulating presentation of the vital problems involved to the student hydrologist.

Binac Demonstrated, New Electronic Brain.—The world's second all-electronic automatic computer was demonstrated in Philadelphia recently at the Eckert-Mauchly Computer Corporation by inventors Dr. John W. Mauchly and J. Presper Eckert, Jr. Named BINAC, it was built to order for Northrop Aircraft, Inc. of Hawthorne, California, first private concern to acquire an "electronic brain." BINAC calculates 12,000 times faster than a human being.

The only other computer like it is ENIAC, built by the same men in 1946 for the U. S. Army. BINAC is tiny in comparison with its 30-ton parent. The computing unit stands only 5 ft. high, 4 ft. long and 1 ft. wide and fits in a medium-sized office. It is the first "giant brain" not gigantic in size, and was designed especially for engineering problems.

"In designing today's advanced military aircraft and guided missiles we constantly encounter mathematical problems which cannot be solved in any reasonable period of time by humans working with pencil and paper or ordinary calculating machines," said John K. Northrop, President of Northrop Aircraft, Inc. "BINAC will be of extraordinary value in reducing preliminary design and test time on most research and development projects.

"In the past it has been practical to carry our analysis of a given design only to a certain point, after which costly and often destructive physical tests were necessary. With this new computer, calculations formerly impossible or impractical of solution can be completed rapidly with savings of hundreds of thousands or possibly even millions of dollars in time and money."

Actually twin computers, BINAC has duplicate arithmetic channels so it can check itself at every step, and two mercury tube "memories." Each twin has only 700 vacuum tubes. ENIAC has 18,000.

The new computer showed its mettle by solving "Poisson's Equation," a typical engineering problem. BINAC spent more than two hours of actual computation to obtain 26 solutions. For *each* solution the computer did 500,000 additions, 200,000 multiplications, and 300,000 transfers of control, all in the space of 5 minutes. A man with an adding machine would have needed years to complete the same job.

The audience also got a chance to test the computer. They were asked to select numbers at random. These were typed on a small keyboard having only eight keys. The BINAC used coded instructions from a magnetic tape to deal with these numbers. It calculated the square and cube roots of all the numbers in the fraction of a second.

For each square or cube root, hundreds of arithmetic operations have to be done by the computer. A human being with a scratch pad can find a square root correct to eight or nine digits in six minutes. With a desk calculator he can do it in two minutes. The BINAC does it in one-sixtieth of a second.

The BINAC operates in the binary system instead of the decimal one. In the binary system, it is required only to distinguish between zero and one. The computer has to convert automatically to binary numbers before beginning calculations and to convert answers back again to decimal form before printing

them out on an electric typewriter. Actually the computer receives and puts out figures in the octal system, from zero to seven, as this converts more easily to binary than the decimal system does. Decimal numbers may be easily expressed using pairs of octal numbers. The computer sorted them in increasing order from the smallest to the largest. This type of "collation sorting" has never before been done by any computer. It is done only for certain problems.

The audience was able to check results as they came out of the BINAC by consulting a familiar computer's handbook known as Barlow's Tables, showing answers to square and cube roots.

During a recent month the Eckert-Mauchly Computer Corporation ran a training school for computer operators in the plant, attended by Northrop Aircraft, Inc. engineers and mathematicians who will run the computer when it arrives in California. A typical computer staff consists of several operators and service engineers, one or two mathematicians to program problems, and typists to prepare magnetic instruction tapes.

The main reason for the BINAC'S smaller size is the mercury memory invented by Mr. Eckert. It replaces about 17,000 vacuum tubes and can store 15,000 binary digits. Electrical pulses representing numbers and instructions are sent through it at a rate of 4 million per second. They are held in the column by being rerouted through it for as long as necessary. The tremendous speed of the mercury memory makes it possible for the computer to handle a great volume of data.

Dr. Mauchly stated emphatically that the BINAC will not "think." It merely follows instructions given to it in the form of simple arithmetic operations.

"It often takes months and even years to bring a new industrial product from the research laboratory to the factory belt-line," he said. "This time can certainly be shortened through the use of computers such as the BINAC. The new era in industrial design forecast when ENIAC began operating, is now here."

A larger, more general-purpose type of computer named UNIVAC is also being constructed by the Eckert-Mauchly Computer Corporation for customers that include the U. S. Bureau of the Census, the Army Map Service, the Air Comptroller's Office, the Prudential Insurance Company of America, and other private customers.

ADDENDA OF USEFUL INFORMATION

Earlier "giant" calculators such as Harvard's Mark I and II, IBM's huge machine in New York, and the Bell Telephone Laboratories relay computers, are not all-electronic.

BINAC and ENIAC, both from Philadelphia, are the pioneers in this field. Other all-electronic digital computers began to be constructed soon after ENIAC was announced. They include Harvard's Mark III, MIT's Whirlwind, both intended for the Navy, RCA's and Princeton University's computer project, MANIAC, and the University of Pennsylvania's EDVAC. The latter was begun by Dr. Mauchly and Mr. Eckert in 1946 before they went into business for themselves.

Dr. Mauchly's interest in electronic computing was first aroused when he was at Ursinus College and noticed the over-whelming amount of data the weather bureaus have to contend with. He decided to hunt for a faster way of solving some of the complicated equations needed by the weatherman to predict weather accurately. His memorandum on creating an electronic computer came to the notice of Captain H. H. Goldstine of the Army's Ballistic Research Laboratory. Dr. Mauchly and Mr. Eckert were then with the Moore School of Electrical Engineering, part of the University of Pennsylvania. A contract was immediately arranged between the University and the U. S. Ordnance Department to develop the type of electronic computer Dr. Mauchly had described. Dr. Mauchly and Mr. Eckert were placed in charge of construction. The result was ENIAC, made public on February 16, 1946. Later that year the two men went into partnership and formed their own company.

There are also a group of analogue computers, some electronic. The first working analogue computer was constructed at MIT in 1930 by Dr. Vannevar Bush. Sometimes called the differential analyzer, this type of machine represents numbers in terms of some physical quantity such as the angle of rotation on a turning shaft. The results come out in graph form drawn by a pencil. The machine cannot achieve the accuracy or speed of a digital electronic computer that represents numbers by separate pulses and gets its answers in actual numbers on a typewriter.

Standardization of Oil Seals?—The following is taken from a letter written to Dr. Allen, Executive Vice President and Secretary of The Franklin Institute, from E. P. Stahl, oil seal specialist. It is published, with the consent of the writer, in an effort to determine what interest there is in the proposal to standardize oil seal sizes. Suggestions and letters of inquiry relative to the subject may be addressed to Mr. E. P. Stahl, 306 Pierson Ave., Newark, New York.

"I am a bearing technician and an oil seal specialist. I am a strong believer in standardization and simplification. I believe that this is one way American industry can reduce costs in the face of rising material and labor costs.

"Recently I have been advocating the standardization and simplification of unit oil seals for rotating shafts. Published sizes of unit oil seals have reached absurd proportions. The major oil seal manufacturers in this country now list and make from 1200 to 4000 oil seal sizes in 12 to 14 different types. I believe these size lists could be boiled down to 300 to 400 sizes and still have an adequate number of sizes to meet the requirements of industry. What is your opinion on this subject?

"It would be nice if the major oil seal manufacturers in this country would standardize and simplify their oil seal lists to conform to the British standards established and adopted in 1947."

Farms Need Landing Strips.—In areas where spraying or dusting by airplane has become a routine farm service, a farmer or group of farmers can well afford to provide a grassed landing strip that will lead the pilots to quote lower rates for their work. It is more economical to take a smaller and safer load for a nearby strip than to ferry a heavy load a long distance, says Arthur

Gieser of the U. S. Department of Agriculture. Gieser is the experienced chief pilot of the Division of Grasshopper Control in the Bureau of Entomology and Plant Quarantine. He has been flying for 19 years, and for 7 years has been specializing in insect control by means of aircraft. In suggesting that farmers plan for suitable landing strips in each neighborhood where plane service is needed frequently, he points out that the substitution of pasture for cultivated crops on a few level acres would be repaid by the lower rates for dusting or spraying for control of insects, plant diseases, or weeds.

When a pilot drives an airplane at a speed of 100 miles an hour and treats a strip 100 feet wide, he is covering about 20 acres a minute. Gieser uses this simple fact as a base for calculating loads and discharge rates. If the speed is 70 miles an hour and the treated strip is 60 feet wide, then 0.7 times 0.6 equals 0.42, and this fraction of 20 acres, gives 8.4 acres a minute as the area covered. This serves as a basis for determining how many pounds of dust or how many gallons of spray must be discharged each minute to distribute the quantity of insecticide required per acre for the particular job.

Weston Announces Simplified, Budget-Priced Exposure Meter.—Known as the Weston Cadet and designed especially for travelers, casual photographers and those who want a small, easy-to-use meter for everyday use, a precise new exposure meter has been announced by the Weston Electrical Instrument Corporation, Newark, N. J. It lists at $21.50.

Though small enough to fit into vest pocket or purse, the Cadet is equipped with the famous Weston instrument movement and Photronic Cell, and gives accurate shutter and diaphragm settings for all general amateur photography with either still or motion picture cameras, and for both black-and-white and color film.

The Weston Cadet can be used for measuring either reflected or incident light. A transluscent light collector, pivoted on the back of the meter, converts the Cadet from a reflected light to an incident light meter in an instant.

And the Weston Cadet is easy to use—simply take a light reading in the usual manner, set the light value over the emulsion speed by turning a convenient knurled button, and read the correct camera settings.

Atomic Energy Commission Starts Trial Program to Make Technological Information Available to American Industry.—A trial program for examining selected declassifiable technological information in the field of metallurgy with a view to determining its possible value to American industry has been inaugurated by the Atomic Energy Commission. To assist in developing the program, a temporary advisory committee of representatives of professional societies and the business press has been appointed.

The purpose of the program is to determine the possibility of providing to industry information about pumps, blowers, valves, techniques of handling metals, and other metallurgical data which will be of immediate value to industry. The trial program will determine whether such selected information can be gathered, screened, and declassified for release in a way which will not reveal the specific relevance of the information to the atomic energy program.

The setting up of the advisory committee is the first step. This group will recommend members for a small working party of engineers, representatives

of professional societies, and members of the business press in the field of metallurgy who will receive full security clearance. After the working group members are cleared, the Commission will make available for examination by the working party metallurgical information which may be declassifiable.

From the information made available to it, the working party will recommend to the Commission those portions which it believes should be considered for declassification because of their usefulness to American industry and technology. The Commission will exercise, of course, the final responsibility for declassifying any recommended material, taking into account the considerations against declassification as well as those favoring declassification.

Any information which the Commission may actually declassify would be made available for publication by established engineering and industrial journals in the field of metallurgy. If the trial program in metallurgy proves effective, the Commission will consider the application of similar procedures to other fields of technological and industrial information.

An Electrical Bridge De-icer (*Electrical Engineering*, Vol. 68, No. 8).—Electric current, flowing in cables buried below the surface of the concrete roadway, will keep Brooklyn Bridge free of snow and ice in future winters, if New York authorities adopt the proposal of Robin Beach, head of Robin Beach Engineers Associated, Brooklyn, N. Y. The renovation of the famed Manhattan-Brooklyn artery, so that it will accommodate six lanes of traffic, now is being planned. According to Mr. Beach, electric heat can remove snow and ice from the bridge as fast as it appears on the road surface. Therefore, he suggests that the installation of heating cable be a part of the program.

It is reported that the bridge's traffic lanes will be surfaced with steel-grid-reinforced concrete. Concrete, of course, is a relatively poor conductor of heat, but the steel surface grid would provide an efficient metallic conducting network for distributing the heat right at the roadway surface, where it is desired.

Lead-covered insulated resistance wire would be the heating cable used. It has been utilized successfully in the past for the purpose of clearing eaves and leaders of buildings, sidewalks, and other surfaces of ice and snow.

This heating cable would be embedded in the concrete roadway, and would be connected to operate on 220 or 440 volts. The higher voltage is thought to be more economical, however. The heating elements of the cable can carry as high a load as 25 amperes.

Power cables for energizing the resistance wire could be run along the sides of the bridge, from which connection would be made to the heating cable through appropriate weatherproof outlets. Protective fuse cut-outs similar to those found in conventional home or industrial circuits would be a part of the equipment.

The continuous warming of concrete surface with heating cable would require the application of electric power of the order of 20 to 40 watts per square foot of roadway. Using the maximum value of 40 watts per square foot, the six 10-ft. traffic lanes for the 6,000-ft. length of the bridge plus the terminal approaches would necessitate a power expenditure of 16,000 kw.

Assuming an electric rate of one cent per kilowatt hour, the cost of heating the entire bridge, including traffic lanes and approaches, would amount to

roughly $1.60 per hour. When the temperature is a little below the freezing point, 20 watts per square foot might be adequate, thus halving the cost.

In operation, the use of the electric heating would be required for a few days per year. The instantaneous removal of the snow as it falls would require only a fraction of the expense now incurred with manual labor.

Portable Atom Sorter (*Instrument Practice*, Vol. 3, No. 9).—Research representatives of the leading American oil companies saw recently for the first time a new portable "atom sorting" device, an accurate, high-speed gas analyser for oil refining and prospecting.

The analysing machine was demonstrated at the Westinghouse Research Laboratories by Dr. E. U. Condon, one of the leaders of the laboratory, and Dr. J. A. Hipple, research physicist who developed the device and named it a "portable mass spectrometer," or "atom sorter."

The "atom sorter" can sort out by weight the molecules and atoms which are the building blocks of all matter. It can quickly answer questions about intricate combinations of gases which are very difficult or impossible by ordinary chemical methods.

With a little further development by the oil laboratories, the "atom sorter" could probably improve the quality of refinery products by keeping a constant check on the separation and combination of gases. There are possibilities, too, that the instrument can discover new oil deposits by analysing soil gases.

Dr. Hipple, who designed the device, after building a room-sized mass spectrometer, explained that the new instrument has been made compact and placed on wheels so it can easily be rolled from one job to another. Other uses include the analysis of gases for the heat treating and hardening of steel and the tracing of carbon and other elements in animal bodies.

America's First Zoo (*DuPont Magazine*, Vol. 43, No. 4).—Keeping pace with an age of television and supersonic speeds, despite a history that goes back to a yesterday of horse-drawn trolleys and high-button shoes—that's the story of America's oldest zoo, the Philadelphia Zoological Garden.

Although 1949 marks the Diamond Jubilee of this famous institution, it has not been allowed to fall behind the times. There are more than 1500 specimens here today, in contrast with the 282 animals exhibited on the opening day, July 1, 1874. Besides boasting many modern buildings and some of the newest methods of exhibiting animals, it is one of two zoological parks in the country that has its own staff photographer. And, as far as we know, the Philadelphia Zoo is the only one employing a woman staff photographer.

When this institution first came into being, its guide books and cage signs were illustrated with old-fashioned drawings. But with the manufacture of modern photographic products, the Garden's publications were filled with pictures, and many of its cage signs were embellished with colored photos of the animals.

Isabelle dePeyster Hunt began her work as staff photographer in September 1942. Since then many fine animal shots have won her a respected place in her field. The problems she encounters in "shooting" the mammals, birds and reptiles are very unusual, for they require patience and understanding demanded by few other jobs. Monkeys, hippos and tigers are not easy sub-

jects; they won't sit still and pose for their portraits. However, Du Pont products—"Defender" photographic films and papers—help make her "shooting" and darkroom work easier.

When the modern Tropical Room in the Reptile House was designed, it presented a new problem. An entirely different type of sign was needed—one that could be easily illuminated from the rear, could be easily read, and would show in natural colors the reptiles in each cage. This would be helpful in distinguishing one species from another when several were in the same pen. Du Pont solved this problem for Miss Hunt with "Adlux" photographic film for transparencies. She printed her negatives on this material, then colored the pictures with photographic oil paint. The descriptive text was printed by the Zoo's hand press, and a new and distinctive sign style had been created.

In the Tropical Room the snakes, frogs, turtles and lizards are displayed in well-lighted cages, behind glass, with backgrounds simulating natural habitats. The "Adlux" signs, illuminated from the rear, are set above each exhibit. The public corridor is dimly lighted, and the result is one of the most spectacular displays in the Garden.

The Students' Spectrometer (*Instrument Practice*, Vol. 3, No. 10).—This new addition to the Watts' range of spectrometers has been designed specifically for educational use.

For this reason, a considerable amount of thought has been given during its development to both questions of economy and durability. The design is simple, incorporating the minimum refinements compatible with good training. Nevertheless, the instrument is accurate to within the limits necessary for the work it will be called upon to perform, and its construction is of a type successfully to withstand really hard wear. All optical and mechanical adjustments are makers' adjustments, not accessible to inexperienced users.

A single divided circle is employed to measure the rotation of both telescope and prism table. The circle is graduated to single degrees and opposite verniers give readings to 2 minutes.

The circle normally rotates with the telescope while the verniers, which are engraved on a single ring moving inside the circle annulus, rotate with the prism table. Either may be unclamped for independent rotation.

It has the conventional 3-screw levelling type of table, 2.5 in. in diameter. It is adjustable for height and the table top has independent rotation for adjustment purposes.

Ring focusing is provided on the telescope and collimator, each having an aperture of 0.7 in. and focal length of 4 in. A Gauss eyepiece is fitted to the telescope.

The collimator slit has unilateral adjustment, the moving jaw being operated, through an internal cam, by a knurled head surrounding the slit. This arrangement gives a comfortable hand-hold with good control over the movement. Spring pressure automatically returns the jaw once the slit is closed, to prevent accidental damage to the slit edges.

The manufacturer is Hilger & Watts, Ltd., London.

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ELECTRONICS—PAST, PRESENT AND FUTURE

BY
W. C. WHITE [1]

The science of electronics is usually thought of as being new, but this is hardly the case. This year, the JOURNAL is celebrating its 123rd year and, of course, this antedates all except the most elemental observations in this science. However, quite a little had been done as long as 50 years ago.

In 1898 the word "electron" had been in print for seven years. The British scientist, J. J. Thomson, had made the first evaluation of the charge on the electron and Thomas A. Edison had received a patent (fourteen years earlier) on the use of a hot-cathode, two-electrode tube for use as a voltage regulator in the Edison central station system that he was developing.

In another branch of what today is considered electronics, a German physicist had established pretty thoroughly the fundamentals of the mercury-arc rectifier as we know it today. In still other branches of the science, the use of alkali metals for sensitive phototubes had been discovered and the cathode-ray oscillograph had been disclosed with its deflection of beams by electric and magnetic fields.

Of course, this is the familiar pattern of most technical developments. The fundamental research and the disclosure resulting from it precede by several years the practical application. An attempt has been made to show this in the charts that accompany this article. For this purpose the field has been divided into six classifications and the fundamental discoveries basic to each part of the field are listed in connection with a time scale. Included also are the development and practical utilization of the basic ideas that were later evolved.

It is of interest to consider further this matter of the time that elapses between the conclusion of a piece of basic research and when the product is generally available.

[1] Electronics Engineer, General Electric Research Laboratory, Schenectady, N. Y.

RADIO HIGH-VACUUM TUBES

Fundamental Research	Approximate Year (Publication / Availability)	Development
Loss of electric charge from hot metals (du Fay)	1725	
Year given is for date of publication or announcement	1800	The selected research results at the left laid the foundation for the products listed below with approximate year commercially available.
	1850	
Hot electrodes lose + and − charges at different rates (Guthrie) Conduction in vacuum from a hot cathode (Edison Effect) Word "electron" coined for indivisible unit of electricity (Stoney) Evaluation of charge of electron (J. J. Thomson)	1900	
Basic work on thermionic emission of electron (Richardson) Secondary electron emission (Austin and Stark) Electron emission from oxide-coated cathodes (Wehnelt) The grid electrode for control (de Forest)		The Fleming valve wireless detector The de Forest audion
Ductile tungsten (Coolidge) Pure electron discharge in high vacuum (Langmuir and Arnold)	1910	High-vacuum and high-power diodes and triodes
Diffusion pump for high vacuum (Gaede) High-vacuum condensation pump (Langmuir) Use of getters in electron tubes Additional grid for high amplification (Schottky) Electron emission from thoriated tungsten (Langmuir)	1920	Broadcast triode transmitting and receiving tubes
A-c heater cathode (Freeman) Metal-to-glass seal for large diameter cylinders (Housekeeper)		Screen-grid transmitting tubes Rectifier tubes for a-c receivers A-c operated receiving tubes Screen-grid receiving tube Magnetron (multi-anode oscillator)
Screen-grid for radio-frequency amplification (Hull and Williams) Demonstration of wave nature of electrons (Davisson and Germer) Secondary emission suppressor grid (Holst and Tellegan) Early work on electron microscope (Ruska and Rudenberg) Strain free alloy seals (Scott and Hull) Steel-envelope technique	1930	Pentode

		Steel-envelope receiving tube
		Beam power tubes
		Klystron
	1940	
		Disk-seal triodes

USE OF ELECTRON OR ION BEAMS IN HIGH VACUUM

Fundamental Research	Approximate Year Publication Availability	Development
Fluorescence from ion bombardment (Plucker)	1859	
	1860	
	1870	
Beam of electrons in high vacuum and its deflection by magnetic and electric fields (Crookes)		
	1880	
	1890	
Use of ion beam for oscillography (Braun)		
	1900	
Electron emission from oxide-coated cathodes (Wehnelt)		
The grid electrode for control (de Forest)		
	1910	
Pure electron discharge in high vacuum (Langmuir and Arnold)		
		Coolidge high-vacuum X-ray tube
Wave motion theory of electrons (de Broglie)	1920	
	1930	
		General use of cathode-ray tube for oscillography
Early work on electron microscope (Ruska and Rudenberg)		
		Cathode-ray tube for television reception
		Iconoscope camera tube for television
		Cyclotron (custom built only)
		Beam power tubes
		The Orthicon
	1940	One-million-volt industrial X-ray equipments.
		Electron microscope
		20-million-volt induction accelerator
		100-million-volt induction accelerator
		Mass spectrometer
		Electron diffraction instrument
		Phasitron tube for FM transmitters
		Image Orthicon camera tube for television
		Cathode-ray projection tube for television

Tubes Sensitive to Light—Phototubes

Fundamental Research	Approximate Year (Publication / Availability)	Development
Photo-emission of electrons from alkali metals (Elster and Geitel)		
	1890	
	1900	
Secondary electron emission (Austin and Stack) Basic Theory of photoelectric effect (Einstein)		
		Phototubes available (custom built)
	1920	
		Sensitive material on an interior electrode Phototubes commonly available at low cost
	1930	
Secondary electron multipliers		
	1940	
		Multiplier phototube

Cold-Cathode Discharge Tubes

Fundamental Research	Approximate Year (Publication / Availability)	Development
Research on glow discharge in gases (Faraday)	1838	
Fluorescence from ion bombardment (Plucker)		
Argon discovered (Rayleigh and Ramsay) Neon, krypton, xenon discovered (Ramsay) Gas ionization by collision established (J. J. Thomson)	1900	
		Moore luminous tube lamps in use
Practical process to get the rare gases (Claude) Geiger particle counter devised (Rutherford and Geiger)		
	1910	Neon lamps introduced by Claude
	1920	
		Glow-discharge voltage-regulator tubes Grid-glow tube
	1930	
	1940	
		Geiger-Mueller counter tubes made in quantity

MERCURY POOL CATHODE TUBES

Fundamental Research	Approximate Year (Publication — Availability)	Development
First mercury arc (Way)	1860	
	1880	
Rectifying effect noted in mercury arc (Jamin)		
	1890	
Basic work on mercury arc in vacuum (Arons)		
Gas ionization by collision with electrons established (J. J. Thomson)	1900	
		Mercury-vapor lamp Mercury-vapor pool rectifier in glass
	1910	
		Pumped tank mercury-arc rectifiers
Grid control in mercury-pool tube (Langmuir)		
	1915	
High-vacuum condensation pump (Langmuir)		
	1920	
Control of arc current by phase of grid voltage (Toulon) Metal-to-glass seal for large diameter cylinders (Housekeeper)		
	1930	
Large strain free alloy seals (Scott and Hull) The ignitor for mercury arc cathodes (Slepian) Steel-envelope technique		
		Pumped ignitrons Sealed steel-envelope ignitrons

HOT-CATHODE, GLOW-DISCHARGE TUBES—TWO–THREE–FOUR–ELECTRODES, THYRATRONS

Fundamental Research (Publication)	Approximate Year	Development (Availability)
Loss of electric charge from hot metals (du Fay)	1725	
	1800	
Research on glow discharge in gases (Faraday)		
	1850	
Hot electrodes lose + and − charges at different rates (Guthrie) Conduction in vacuum from a hot cathode (Edison Effect)		
Gas ionization by collision with electrons established (J. J. Thomson) Basic work on thermionic emission of electrons (Richardson) Electron emission from oxide-coated cathodes (Wehnelt) The grid electrode for control (de Forest)	1900	
	1910	
Grid control in mercury-pool tube (Langmuir)		
Use of getters in electron tubes		Tungar, low-voltage rectifier
	1920	
A-c heater cathode (Freeman) Control of arc current by phase of grid voltage (Toulon) Metal-to-glass seal for large diameter cylinders (Housekeeper)		
Screen-grid tube for radio-frequency amplification (Hull and Willlams) Research on hot-cathode glow discharge (Hull)		Hot-cathode mercury-vapor rectifier The thyratron
Large strain free alloy seals (Scott and Hull) Steel-envelope technique	1930	Ultra-violet incandescent lamps Shield-grid thyratron Steel-envelope industrial tubes
		Fluorescent lamps
	1940	

In the case of radio communication, the British scientist, Maxwell, in 1867 enunciated the theory of the transmission of light, heat and radio waves in space. It was 21 years later, and after the death of Maxwell, that Hertz published his paper which confirmed experimentally the theory of Maxwell. After a further lapse of six years, and after the death of Hertz, the Italian engineer, Marconi, transmitted "wireless" messages in 1894. It was about another ten years before there was much actual everyday use of his developments in radio communication. In this example there was a delay of about 35 years between the presentation of a theory and the common use of products based thereon. This was due in part to the lack of organized research and development.

Electronics for radio communication was somewhat more fortunate in this respect. Only about ten years elapsed between the fundamental work of J. J. Thomson on the electron and the fairly common use of the de Forest audion. The really rapid strides in this science took place after 1911, when organized research took up the work. However, as the charts indicate, some phases of what we now consider electronics were slow in getting under way. These charts show very clearly that like many other fields of science many minds working in many lands have contributed to the methods and devices that are now in common use.

It takes a large volume today to cover even the fundamentals of electronics much less its practical applications. However, one simple concept that embraces pretty much the whole field is to consider that, when an electron tube of any sort is incorporated in an electrical circuit, it introduces a gap in that circuit, where the continuous metallic continuity of that circuit is broken, and in its place is substituted a gaseous or vacuum medium across which the electrical current is carried by charged particles. This gap may be only a few thousandths of an inch in length or it may be measured in feet (Fig. 1). The fundamental point, however, is that in this gap useful effects occur and it is possible to affect the flow of electrical current in a number of unique ways. It is these unique methods of current control that, in general, make the electron tube the valuable device that it is today.

Looked at broadly, there are four of these unique features:

1. High-vacuum tubes have ability to interrupt, re-establish or vary the current in this gap at tremendously high frequency, up to tens of billions of cycles per second. Therefore, in comparison with most other electrical devices, an electron tube designed for use at high frequency is independent of frequency through a very wide range.

2. Most electrical devices for the control of current effect that control in a step-by-step manner. An electron tube designed for the purpose does it with a degree of smoothness unrivaled by any other method. It is this characteristic that allows it to reproduce music and speech with such perfect quality.

3. The flow of electric current across the gap which is fundamental to all vacuum tubes can be controlled by means of the application of an extremely small amount of electrical power. In many cases, kilowatts of energy may be accurately controlled from a few watts of controlled power. This phenomenon, of course, is spoken of as amplification.

4. Many electron tubes are efficient rectifiers both for heavy currents and extremely high voltages.

FIG. 1. In every form of electron tube, there are at least two electrodes. The vacuum or gaseous space between these electrodes represents a gap in the circuit. The current control that can be effected in this gap of the physical phenomenon occurring therein is the basic effect which gives electron tubes their unique properties. This gap may be only a few thousandths of an inch in length as in the "lighthouse" type of microwave tube shown above or it may be many feet in length as in the case of a neon sign.

Although there are a number of electrical devices that incorporate one or two of these four unique properties, the electron tube in its more common forms combines several to a remarkable degree.

Electronics is a science of extremes. Electron tubes are in common use that operate at 25 cycles and during the war radar systems were evolved utilizing both transmitting and receiving tubes generating and

responding to about 25 billion cycles. Special tubes are available that act as electrometers for detecting currents of the order of 10^{-16} amperes and others are used for controlling the heavy currents to resistance welders involving the conduction of peak values through the tube of

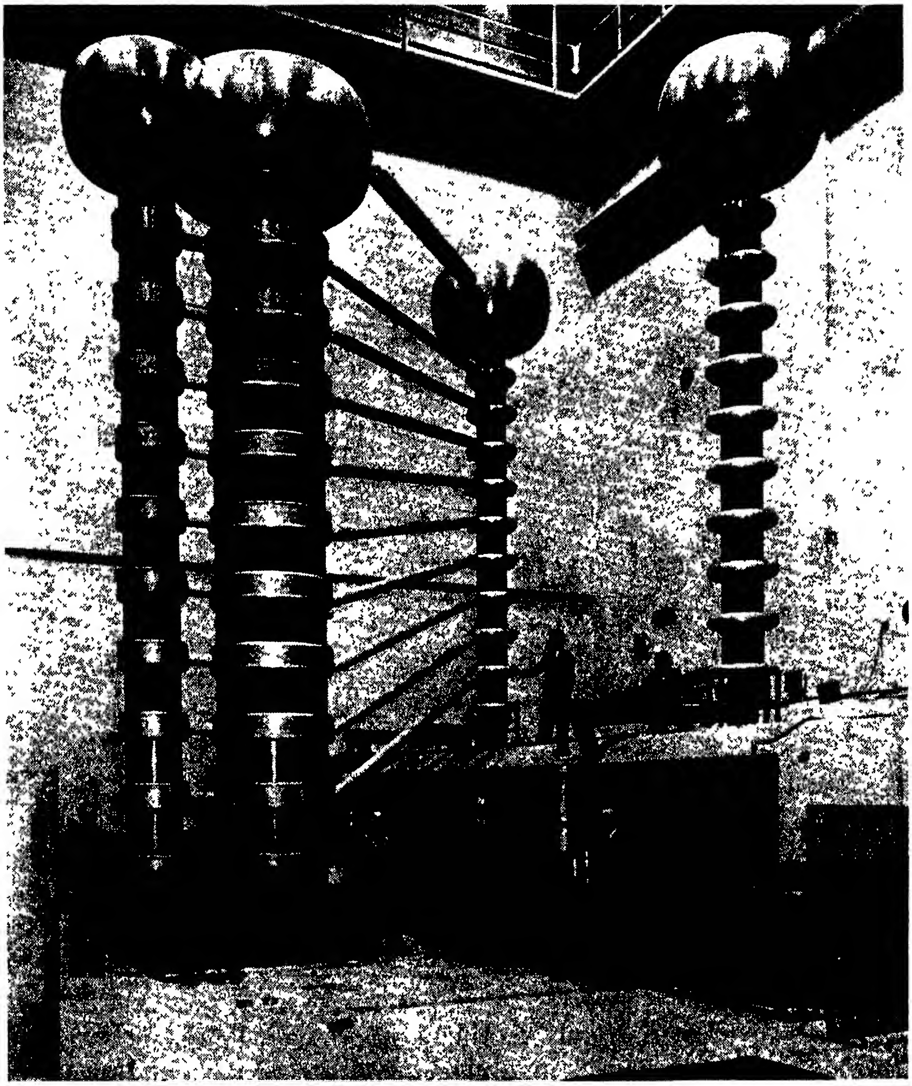

FIG. 2. Electronics is a science of extremes not only in electrical quantities, such as frequency, current and voltage, but also in size. Electron tubes range in size from the tiny ones in a vest-pocket hearing aid to the huge X-ray equipment shown above.

about 10,000 amperes. In communication, sensitive measurement tubes are available that can detect and amplify usefully a few millionths of a volt. At the other extreme, the so-called electron accelerators used for X-ray production and in the studies of nuclear physics operate in the millions of volts.

In physical size there is similar extreme range. Tubes have been built about the size of a kernel of grain. At the other extreme, some forms of tank rectifiers are large enough so that a man could move around inside (Fig. 2).

It must be admitted that one reason for the versatility and extremes of characteristics exhibited by the various kinds of electron tubes is in a measure accounted for by the breadth of activity which today is covered by the term "electronics." This term has broadened greatly in not too many years. Twenty-five years ago, it was applied almost exclusively to high-vacuum tubes used for radio. Today it has been

FIG. 3. Electronics plays an important part in complicated computing devices, such as this differential analyzer, that perform mathematical operations. It can do quickly and without error mathematical work of certain clerical types that otherwise would involve routine mental processes and pencil pushing.

extended to cover mercury-arc rectifiers, fluorescent lamps, neon signs, cyclotrons and other devices involving the electrical current through a vacuum or gas. There is even a tendency to apply the word to devices in which the conduction is through a solid if this conduction has certain unique properties.

Of late, it has become increasingly apparent that electronics is molding a new career for itself. Over the past many years, engineers and particularly electrical engineers have devoted much effort to the creation of labor-saving devices. These have found their places in the home as well as in industry. As a result, much of the hard physical

work formerly done has been eliminated and the human element is now merely the controlling factor. The extent to which this has been done is clearly indicated by the increase of horsepower available to each worker in this country today as compared with not too many years ago.

FIG. 4. The microphone, loudspeaker and phototube, each with its associated electronic equipment, simulate the human senses of hearing, speech and vision to a certain extent. Recently this idea has been applied to another one of the senses, that of smell. The device shown above, known as a leak detector, responds to vapors of halogen compounds when present in air to the extent of only a few parts per million. It is finding application as a leak detector which sniffs suspected welds or joints in tanks or piping systems. For a leak test, the system is filled with air under pressure plus the addition of the vapor of some halogen compound, such as freon.

As has been indicated, electronics will probably enter the picture in a somewhat different way. It will be used to simulate, replace or supplement the mental activities of a human being and the accomplishments traceable to his several senses, such as sight, sound and judgment of temperature. This new concept has now been carried even further. One of the most modern electronic devices is the computer by which complicated mathematical operations can be performed (Fig. 3). One of the basic needs in the design of such a computer is a method of storing

up impulses so that they may later be combined. This involves the phenomenon of memory, and here electronics is playing an increasingly important function.

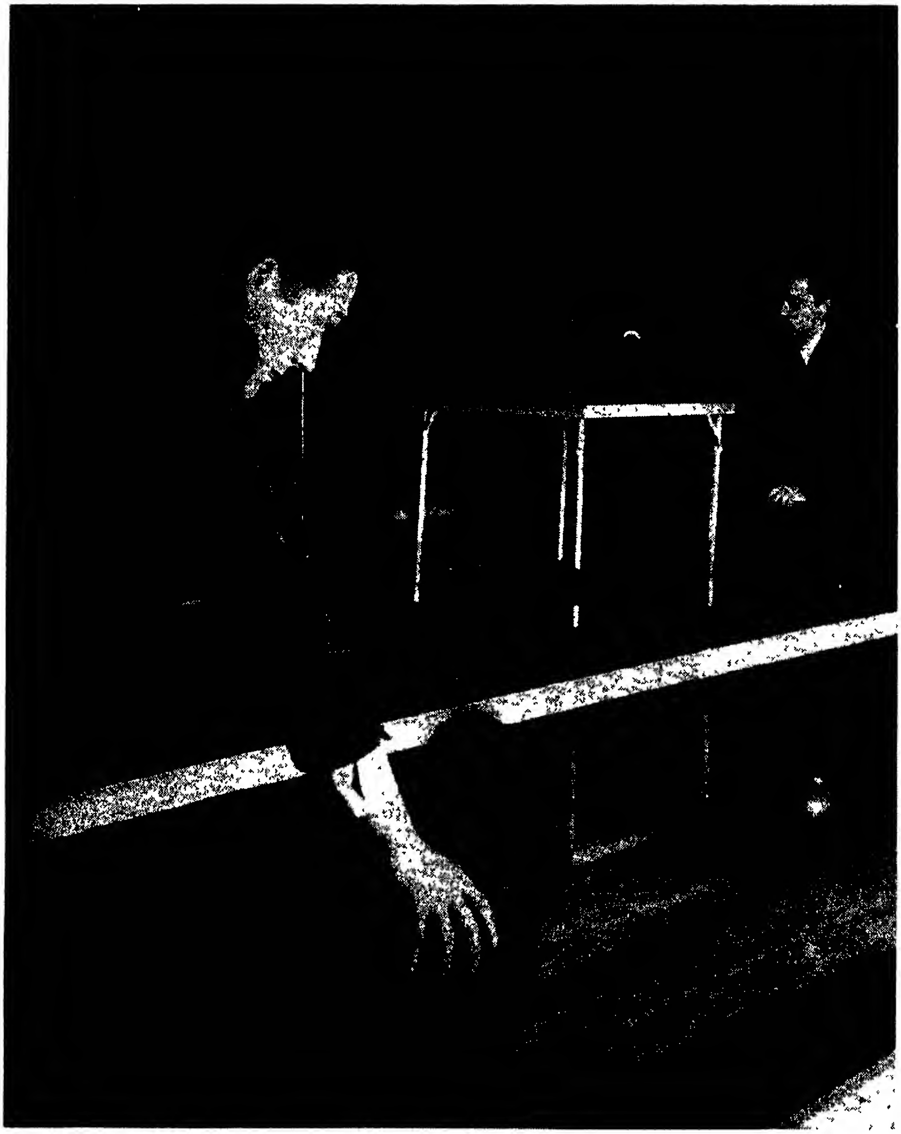

FIG. 5. Electronic devices of many sorts time industrial operations or measure time intervals so short in duration that they are utterly beyond human perception. The radar of World War II involved accurately generating and measuring time intervals of the order of a millionth of a second. In this picture the speed of a bowling ball is determined.

Electronic devices or apparatus in which electronics plays an important part are, as has been stated, used to simulate the common human senses (Fig. 4). Examples of this are the phototube which, like the eye, can respond to varying degrees of illumination; the microphone

with its amplifier that forms an ear, and the loudspeaker with its amplifier that simulates the voice. More recently, an electronic device has been announced with a sense of smell for certain classes of vapors and smokes.

These provide certain of the basic links by means of which the responses of a human being to different stimuli can be fed into an electrical circuit and made to do things that formerly required a muscular response. In other words, we are quite rapidly approaching the point where, by the aid of electronics, we can perform many functions that a human being performs that do not actually require reasoning or judgment. Of course, this process has been going on for years to a certain extent. A familiar example is the automatic home heating plant. Here a thermostat detects the drop of temperature in the room and starts the oil burner operating to raise the temperature to the proper value at which time it is automatically turned off. Formerly, one had first to feel chilly and then respond physically by putting more wood or coal on the fire.

Electronics is greatly extending the sensitivity and scope of even simple temperature-regulating systems. But more important, it is extending our several senses to a marked degree. Examples are modern lie detectors, electrocardiographs and a large number of devices that require timing of an operation in microseconds; many orders of magnitude beyond the human perception of time intervals (Fig. 5).

This idea of a science to simulate many non-muscular human activities is fairly new, but its ramifications and possibilities may well be staggering in their implications.

There is no reason to believe, however, that electronics will permanently be a dominating science. In popular appeal, nucleonics and atomic energy have already stolen from electronics its prominent position in the public's mind. Even technically other devices and methods from time to time invade fields that once were the strongholds for electronic methods. Examples of this are devices in the semi-conductor field, such as crystal diodes and transistors, which threaten to displace small high-vacuum detecting and amplifying tubes in radio broadcast receivers and repeaters for wire telephony. It is true that many physicists consider this new class of device electronic, although it is a far cry from the older and more restricted definition. Mr. D. G. Fink, Editor of the publication *Electronics*, presented a most interesting discussion on this point at the 1948 National Electronics Conference in Chicago, speaking on the subject "Decline and Fall of the Free Electron." He pointed out that all sciences as they pioneered and spread their influence experienced other factors that changed or narrowed their scope.

There is every reason to believe, however, that, when the JOURNAL celebrates it 150th year of publication, electronics will have extended to a wider field and will be functioning in ways in which we now little dream.

Low Voltage Electronic Flash Unit.—Better photographs at lower cost, is the promise of LUMAX Manufacturing Co. in announcing its new 2-way (a.c. and battery) low voltage electronic flash unit.

Electronic flash is growing in popularity because it provides more than 50,000 flashes without changing bulbs, and at practically no cost, once the equipment is acquired. But low voltage, the exclusive LUMAX development has made electronic flash a "must" in producing better pictures because its lengthened duration of flash makes a deeper penetration of the photographic film. While this has always been important in black-and-white, it is critical in color photography which requires ten times longer exposures.

When electronic flash made its bow in the early '40's it was 2500 to 4000 volts with a flash duration of 1/20000 to 1/10000 of a second. It was huge, heavy and costly—definitely not portable. Today's LUMAX units operate at voltages well below 500, flash duration is up to 1/125 of a second, size and weight are one half that of a portable typewriter, and the cost is down to $125 for the 100/watt/second 2-way portable, and as low as $60 for 1-way plug-in units, with self-payment budget plans. The $125 unit is equipped for an additional gun and for phototube wireless synchronizing of any number of units.

Fast, Precise Polarographic Analysis with New Type E Electro-Chemograph.—Designed for rapid, accurate polarographic analyses, a new Leeds & Northrup Type E Electro-Chemograph features one-piece console design incorporating a built-in Speedomax Recorder. Exceptionally flexible and stable, the instrument is convenient for both research and industrial process control laboratories.

Like the previous model of this L&N equipment, the Electro-Chemograph automatically plots diffusion current as a function of voltage. Now, however, the data-taking is done by a Speedomax Microampere Recorder, which provides greater speed of analysis and more complete detail of test results. A typical test run can now be made in about half the previous time.

A full range of variation in test technique is available through a set of controls, all centralized on one panel. Step-by-step instructions for operation, testing and calibration are included on the front of the instrument.

Choice of four degrees of damping enables the user to obtain a record that presents qualitative and quantitative data in the desired detail. Eleven current ranges cover full-scale readings of 1, 2, 3, 5, 7, 10, 15, 20, 30, 50, and 100 microamperes.

There are three polarizing voltage ranges: 0 to -2, -1 to -3, and $+1$ to -1 volts. To cover both anodic and cathodic polarization, any of these ranges can be reversed in polarity. Time to run through a polarizing range is ten minutes. Voltage can be adjusted to ± 4 mv by calibration against an integrally-mounted standard cell.

When it is desirable to obtain several different diffusion current curves on the same chart for intercomparison, the recorder zero can be readily shifted to separate the individual curves.

The equipment is shielded against electrical pick-up, and is not affected by vibration. Normally it is supplied for a-c. operation, but it can be supplied for use with a battery for the polarizer.

APPLICATIONS OF NETWORK THEOREMS IN TRANSIENT ANALYSIS

BY

Y. P. YU[1]

INTRODUCTION

Frequently, the transient solution of a given circuit may be somewhat simplified by the use of a suitable method of attack. Selected methods which have proved themselves to be most useful in simplifying the transient solutions of practical problems will be discussed in this paper. Attention must be called to the fact that each of these methods has one or more limitations which should always be strictly observed. Otherwise serious errors will result. A common limitation is that the values of all passive elements are assumed to be constant. In other words, we are restricted ourselves to constant resistances, inductances, and capacitances. Sometimes this is stated by saying that the passive elements are linear and bilateral.

In this paper, considerable use will be made of Laplace transformation.[2] It is desirable, therefore, to mention a few words about this method. The Laplace transformation of a function $F(t)$ is defined as:

$$L[F(t)] = \int_0^\infty \epsilon^{-st} F(t)dt = f(s);$$

then the inverse Laplace transformation of $f(s)$ is

$$L^{-1}[f(s)] = f(t).$$

For instance,

$$L[\sin wt] = \int_0^\infty \epsilon^{-st} \sin wt\, dt = \frac{w}{s^2 + w^2}$$

and

$$L^{-1}\left[\frac{w}{s^2 + w^2}\right] = \sin wt.$$

Furthermore, the Laplace transformation of the derivative of function $F(t)$ is

$$L[F'(t)] = sf(s) - F(0+)$$

where $F(0+)$ denotes the value of the function $F(t)$ at the instant

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[2] Detailed discussions of Laplace transformation may be found in many standard textbooks, for example: R. V. Churchill, "Modern Operational Mathematics in Engineering," New York, McGraw-Hill, 1944; Gardner and Barnes, "Transients in Linear Systems," John Wiley & Sons, Inc., New York, 1942.

immediately after $t = 0$. It is also important to know the Laplace transformation of the integration of function $F(t)$ which is

$$L\left[\int F(t)dt\right] = L[F^{-1}(t)] = \frac{f(s)}{s} + \frac{F^{-1}(0+)}{s},$$

where

$$F^{-1}(0+) = \int_{-\infty}^{0+} F(t)dt.$$

APPLICATION OF EQUIVALENT CURRENT GENERATOR THEOREM

Frequently, the node method of solution can be advantageously employed in analyzing transient phenomena of electric circuits. The node method which has been discussed in almost every text book concerning electric circuits is essentially a direct application of Kirchhoff's current law, which states that the algebraic sum of all currents directed toward a node or junction, is zero. Current generators are obviously

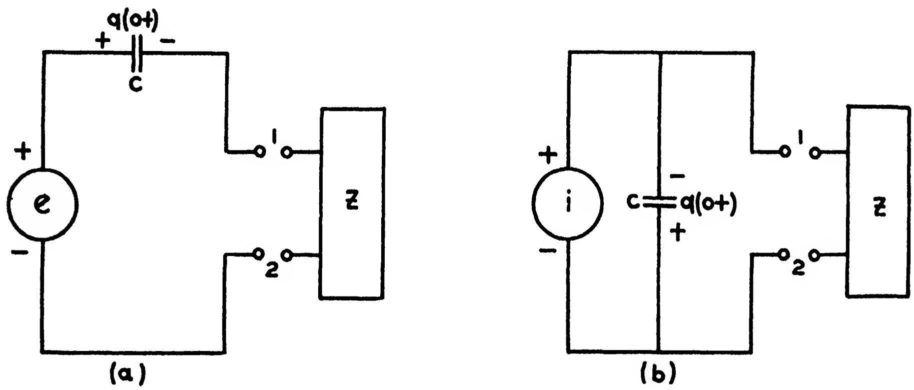

FIG. 1. Conversion of a voltage generator and its series capacitance.

convenient to handle whenever the node method is employed. A statement governing the conversion of a given network into an equivalent network containing a simple current generator shunted with an impedance may be generalized from Norton's theorem and is stated as follows:

A network of generators and passive elements having two terminals is equivalent in its external characteristics to a simple constant-current generator whose generated current is equal to the short-circuit current between the two terminals of the original network and whose shunting impedance has both its transient expression and initial conditions identical to the impedance "looking back" into the two terminals of the original network, measured with all voltage generators (exclusive of internal series impedance) short-circuited and all current generators (exclusive of internal shunt impedance) open circuited.

The above statement may be somewhat clarified by analyzing the following simple cases:

Case I

The conversion of the network to left of terminals 1–2 of Fig. 1(*a*), which contains a voltage generator in series with a condenser C having initial charge $q(0+)$, will be demonstrated in this case. The short circuit current i between terminals 1–2 of Fig. 1(*a*) may be determined from the following equation:

$$e = \frac{1}{C}\int_0^t i dt. \tag{1}$$

From Eq. 1, the Laplace transform of i* is found equal to

$$i(s) = Cse(s). \tag{2}$$

The current generator of the equivalent circuit shown in Fig. 1(*b*) will therefore have the Laplace transform of its current equal to $Cse(s)$.

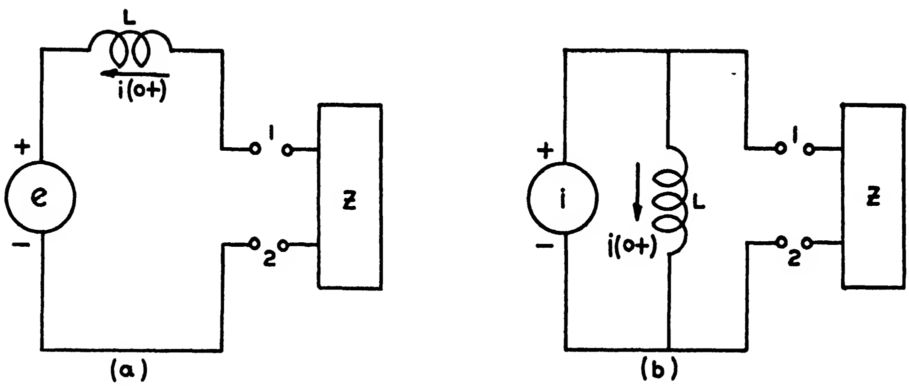

FIG. 2. Conversion of voltage generator and its series inductance.

In order to have the same direction of flow for the short-circuit currents in both networks, the top terminal of the current generator of Fig. 1(*b*) is assigned to be positive. The impedance looking back into terminals 1–2 is represented by condenser C with an initial charge of $q(0+)$. The polarity of the initial charge $q(0+)$ in Fig. 1(*b*) is determined from the original circuit. If the constant-voltage generator of Fig. 1(*a*) is short-circuited, a discharge current will flow out from terminal 2 to impedance Z. Similarly, if the constant-current generator of Fig. 2(*b*) is open-circuited, an equal discharge current should flow out from terminal 2 to impedance Z. Therefore the lower plate of condenser C in Fig. 1(*b*) is positive as far as the initial condition is concerned.

* It is noted that a constant is not the Laplace transform of any actual function. Instead it is the limit of the transform of a step function which is zero except when $t = 0$ at which the function is infinite. Such a function is called Dirac delta function. In case that a battery with potential E is employed for the voltage generator of Fig. 1(a), Eq. 2 becomes $i(s) = CS\left(\frac{E}{S}\right) = CE$. Here CE, a constant, is the limit of the transform of a step function i which is zero except when $t = 0$, the switch is being closed, at this instant the current i is infinite.

Case II

Let the network to the left of terminals 1–2 of Fig. 2(*a*) have a voltage generator and an inductance coil in series. The initial current of the inductance coil is $i(0+)$. Then the equivalent constant-current generator will have its current equal to

$$i = \frac{1}{L}\int_0^t e\,dt. \tag{3}$$

The Laplace transform of i is

$$i(s) = \frac{e(s)}{sL}. \tag{4}$$

The polarity of i, shown in Fig. 2(*b*), should be identical to that of the short-circuit current between terminals 1–2 of the original circuit. The shunting impedance is represented by L with an initial current $i(0+)$.

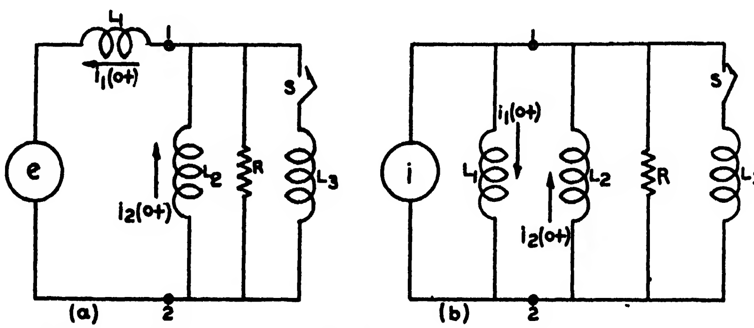

FIG. 3. Illustrating the conversion of a sinusoidal voltage generator into a current generator. $e = E_m \sin wt = 100 \sin 377t$, $L_1 = 0.1h$, $L_2 = 0.2h$, $L_3 = 0.3h$, $R = 100$ ohms.

The polarity of $i(0+)$ is determined in a manner similar to that of the initial charge $q(0+)$ of Fig. 1(*b*). Since the initial current $i(0+)$ of Fig. 2(*a*) flows inward from terminal 1 when its voltage generator is short-circuited, there should be an initial current of equal magnitude and polarity flowing from terminal 1 of Fig. 2(*b*) when its current generator is open-circuited. Thus $i(0+)$ is assigned to flow downward in Fig. 2(*b*).

To illustrate the application of this conversion method the following numerical examples are introduced:

Example 1

Consider a network of inductances and resistances that is energized by an alternating potential. At $t = 0$ a switching operation is performed that will suddenly alter the parameters of the circuit. Figure 3(*a*) is the schematic diagram of this circuit in which switch S is as-

sumed to be closed at $t = 0$. Find the voltage across L_3 at $t = 0.001$ second.

Solution:

The short-circuit current between points 1–2, by Eq. 4:

$$i(s) = \frac{e(s)}{sL_1} = \frac{E_m w}{sL_1(w^2 + s^2)}. \tag{5}$$

The impedance looking back into terminals 1–2 is represented by L, with initial current $i_1(0+)$. Both $i_1(0+)$ and $i_2(0+)$ can be determined from Fig. 3(a). Before $t = 0$ the voltage generator e is shunted by a steady-state impedance Z which is

$$Z = jwL_1 + \frac{jwL_2R}{R + jwL_2} = 36.3 + j85.5.$$

The current flowing through L_1 is e/Z, from which we can find $i(0+) = -0.993$ amp. by setting $t = 0$. The current flowing through L_2 is $\frac{e}{Z}\frac{R}{R + jwL_2}$. Setting $t = 0$, we can find $i_2(0+) = -0.835$ amp. The negative signs of these two initial currents indicate that they are flowing inward to the upper terminal of voltage generator e. Since $i_1(0+)$ of Fig. 3(a) flows away from terminal 1, $i_1(0+)$ of Fig. 3(b) should flow downward through L_1. By assuming v to be the voltage on L_3, current equilibrium with switch S closed may be expressed as

$$i = \frac{1}{L_1}\int v\,dt + \frac{1}{L_2}\int v\,dt + \frac{v}{R} + \frac{1}{L_3}\int v\,dt. \tag{6}$$

The Laplace transform of Eq. 6 is

$$i(s) = \frac{v(s)}{sL_1} + \frac{v^{-1}(0+)}{sL_1} + \frac{v(s)}{sL_2} + \frac{v^{-1}(0+)}{sL_2} + \frac{v(s)}{R} + \frac{v(s)}{sL_3} + \frac{v^{-1}(0+)}{sL_3}, \tag{7}$$

where the boundary values $v^{-1}(0+)/L_1$, $v^{-1}(0+)/L_2$ and $v^{-1}(0+)/L_3$ are initial currents in coils L_1, L_2, and L_3, respectively. Figure 3(b) indicates that: (1) the direction of positive current is assumed to be downward; (2) the initial current $v^{-1}(0+)/L_1$ flows downward; and (3) the initial current $v^{-1}(0+)/L_2$ flows upward. Therefore we have $v^{-1}(0+)/L_1 = +0.993$ and $v^{-1}(0+)/L_2 = -0.833$. Obviously $v^{-1}(0+)/L_3$ should be zero. Using the numerical values, the solution of Eq. 7 gives

$$v(s) = \frac{10^2[i(s) - 0.158]}{s + 1833}. \tag{8}$$

Substitute Eq. 5 for $i(s)$,

$$v(s) = \frac{10^5 w}{(s^2 + w^2)(s + 1833)} - \frac{15.8}{s + 1833}. \tag{9}$$

With the aid of equations developed by the use of Laplace transforms, the expression for v can be found as

$$v = 53.5[\sin(wt - 11.6^\circ) + \sin 11.6^\circ\, \epsilon^{-1833t}] - 15.8\epsilon^{-1833t}.$$

Finally, with $t = 0.001$, the voltage across L_3 is 8.47 volts.

Example 2

Assume that the current in the circuit of Fig. 4(a), prior to the closing of switch S, has reached its steady value. Switch S is closed

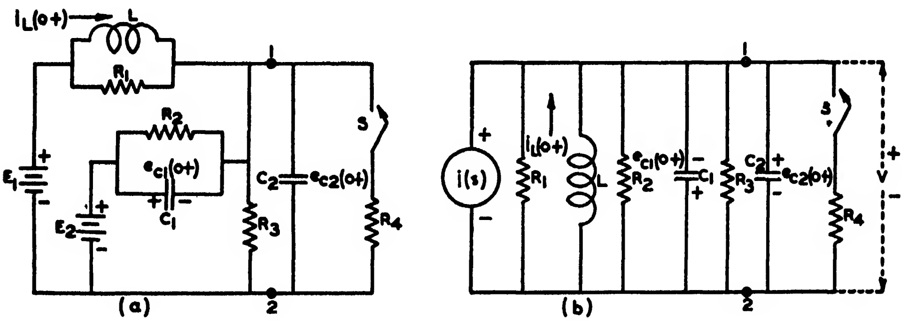

FIG. 4. The conversion of a complex circuit into a simple one-node circuit. $E_1 = 100$, $E_2 = 150$, $L = 1h$, $C_1 = C_2 = 10\,\mu f$, $R_1 = R_2 = R_3 = 1000$ ohms, $R_4 = 50$ ohms.

at $t = 0$. Find an expression for the voltage across R_4 after the switching action.

Solution:

By observing the circuit of Fig. 4(a), the short-circuit current i between terminals 1–2 is found to be the sum of the currents in L_1, R_1, R_2, and C_1, with R_3 shorted. By Eqs. 2 and 4, we can express the Laplace transform of i as

$$i(s) = \frac{E_1}{sR_1} + \frac{E_1}{s^2L_1} + \frac{E_1}{sR_2} + C_1E_2. \tag{10}$$

The impedance looking back into the network at terminals 1–2 with E_1 and E_2 short-circuited is a parallel combination of R_3, C_1, C_2, R_2, R_1, and L_1. The equivalent circuit is shown in Fig. 4(b). Initial currents and voltages are determined by analyzing the steady-state conditions of the original circuit with switch S open, that gives $i_L(0+) = 0.05$ amp., $e_{C1}(0+) = 50V$, and $e_{C2}(0+) = 100V$. The polarities of these values are indicated in Fig. 4(a). Since $i_L(0+)$ of Fig. 4(a) flows toward terminal 1, $i_L(0+)$ should flow upward in Fig. 4(b). Voltage $e_{C1}(0+)$ sets terminal 1 negative with respect to terminal 2 in Fig. 4(a); thus the upper plate of C_1 is initially negative in Fig. 4(b). The equation for

current equilibrium of Fig. 4(*b*) after the close of switch S is

$$L^{-1}i(s) = \frac{V}{R_1} + \frac{1}{L}\int v\,dt + \frac{V}{R_2} + C_1\frac{dV}{dt} + \frac{V}{R_3} + C_2\frac{dV}{dt} + \frac{V}{R_4}, \quad (11)$$

where V is the voltage across R_4. Transforming Eq. 11,

$$i(s) = \frac{v(s)}{R_1} + \frac{v(s)}{Ls} + \frac{v^{-1}(0+)}{Ls} + \frac{v(s)}{R_2} + C_1 sv(s) - C_1 e_{C1}(0+)$$
$$+ \frac{v(s)}{R_3} + C_2 sv(s) - C_2 e_{C2}(0+) + \frac{v(s)}{R_4}. \quad (12)$$

The initial current in coil L, namely $v^{-1}(0+)/L$, is equal to $-$ 0.05 amp. The negative sign is given because it is directed upward, in opposite with the assigned positive voltage direction. Conversely, $e_{C2}(0+)$ should be equal to $+$ 100 volts. Using numerical values, a simple expression for $v(s)$ can be found.

$$v(s) = \frac{10^2(s^2 + 150s + 50000)}{s(s + 45.5)(s + 1103)}. \quad (13)$$

With the aid of an equation developed by the use of Laplace transforms, the inverse transform of Eq. 13 becomes

$$v = 100 - 94.3\epsilon^{-45.5t} + 94.3\epsilon^{-1103t}.$$

It is interesting to note the simplicity contributed by employing equivalent current generators in analyzing the above examples.

APPLICATION OF THEVENIN'S THEOREM

Thevenin's theorem is one of the most useful tools in circuit analysis. Although the original statement of this theorem was written for use in steady-state direct-current circuits, its suitability extends to alternating current circuits and transient conditions. A generalized statement of Thevenin's theorem may read as follows:

A network of generators and passive elements having two terminals is equivalent in its external characteristics to a simple constant-voltage generator whose generated voltage is equal to the open-circuit voltage between the two terminals of the original network and whose series impedance has both its transient expression and initial conditions identical to the impedance "looking back" into the two terminals of the original network, measured with all voltage generators (exclusive of internal series impedances) short-circuited and all current generators (exclusive of internal shunt impedances) open-circuited.

The theorem of the preceding section is very similar in every respect to the present one with an exception that the former results in a current generator and an impedance in parallel while the latter results in a voltage generator with the same impedance in series. For this reason, further description of the conversion details seems unnecessary.

Example 3

The circuit shown in Fig. 5(*a*) contains a battery E and the two current generators i_1 and i_2. Before the closing of switch S, the circuit

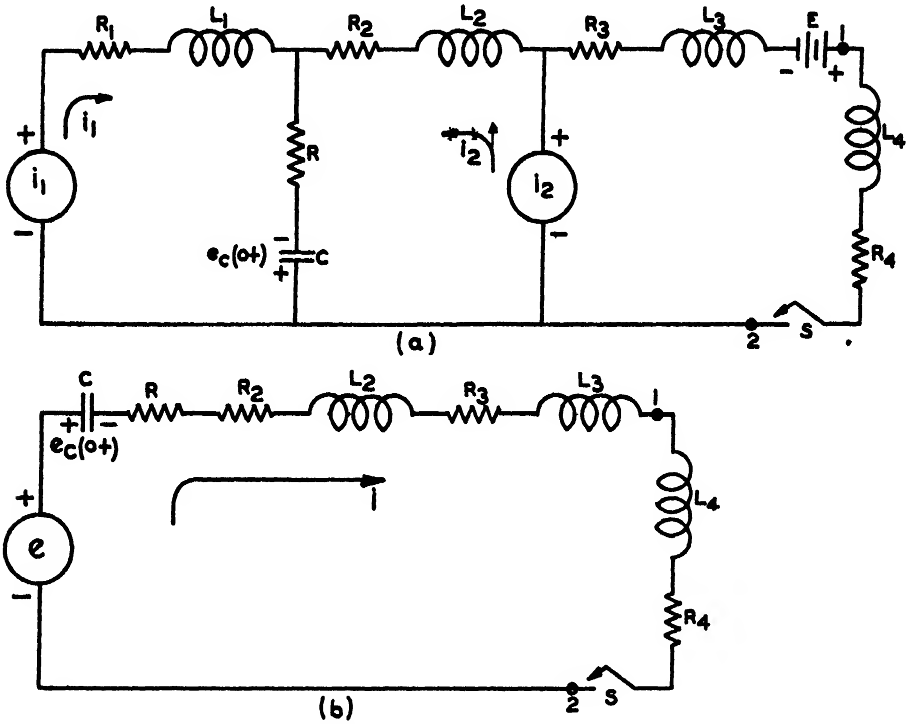

FIG. 5. Illustrating the application of Thevenin's theorem. $i_1 = 1 \sin wt$, $i_2 = 0.5 \sin wt$, $E = 100$, $R = R_1 = R_2 = R_3 = 100$ ohms, $R_4 = 200$ ohms, $L_1 = L_2 = L_3 = L_4 = 0.1h$, $C = 10\ \mu f$, $w = 377$.

is in its steady state condition. At $t = 0$, switch S is closed. Find the current in coil L_4 at $t = 0.001$ sec.

Solution:

To simplify calculations on this circuit, Thevenin's theorem may be used to replace the part of the circuit to the left of terminals 1–2. The open-circuit voltage e at these two terminals is the sum of battery voltage E, voltage drop across R_3L_3 which is zero, voltage drop across R_2L_2 due[3] to the flow of i_2, and voltage drop across RC due[4] to the flow

[3] Let e_{L2} be the voltage drop across L_2, we have $i_2 = \frac{1}{L_2}\int_0^t e_{L2}dt$. Applying Laplace transform to this equation gives $i_2(s) = e_{L2}(s)/sL_2$ or $e_{L2}(s) = L_2 s i_2(s)$. Also see Eq. 4.

[4] Let e_c be the voltage drop on c, then $e_c = \frac{1}{c}\int_0^t (i_1 + i_2)dt$. Transforming this equation, we have $e_c(s) = [i_1(s) + i_2(s)]/sc$. Also see Eq. 2.

of both i_1 and i_2. Therefore, we have

$$e(s) = \frac{E}{S} + R_2 i_2(s) + L_2 s i_2(s) + R[i_1(s) + i_2(s)] + \frac{1}{SC}[i_1(s) + i_2(s)]. \quad (14)$$

The impedance looking back into terminals 1–2 with all current generators open-circuited and all voltage generators short-circuited is a series combination of R_3, L_3, R_2, L_2, R, and C. The equivalent circuit is drawn in Fig. 5(b). Before the switching operation, the current in L_2 is i_2 which is 1 sin wt and the voltage across C is $(i_1 + i_2)/jwc$ which is 398 sin $(wt - 90°)$. Therefore, at $t = 0+$, $i_{L2}(0+) = 0$ and $e_{C2}(0+) = -398$ volts. With this information the polarities of initial values are given in both Figs. 5(a) and 5(b).

The transformed voltage equation for the circuit of Fig. 5(b) is

$$e(s) = \frac{i(s)}{sc} + \frac{i^{-1}(0+)}{sc} + Ri(s) + R_2 i(s) + L_2 s i(s) - L_2 i(0+)$$
$$+ R_3 i(s) + L_3 s i(s) - L_3 i(0+) + L_4 s i(s) - L_4 i(0+) + R_4 i(s). \quad (15)$$

The term $i^{-1}(0+)/C$ is the initial voltage on condenser C and equals $+ 398$ volts. The positive sign is used because i flows in the direction of increasing this initial potential. Except this term, all other initial values are zero. Employing numerical values, we can find:

$$i(s) = -870\left[\frac{s^2 - 361s - 54300}{(s^2 + w^2)(s + 233)(s + 1433)}\right]. \quad (16)$$

Using a Laplace transform equation, the inverse transform of Eq. 16 becomes

$$i = 0.842\epsilon^{-1433t} - 0.32\epsilon^{-233t} + 0.84 \sin (wt - 38.5°),$$

giving $i = 0.301$ amp. when $t = 0.001$ second.

In using both the equivalent current generator theorem and the Thevenin's theorem, attention must be paid to an important limitation which has been mentioned before and is reiterated here. An equivalent circuit obtained by using either of these methods cannot be employed to calculate either the steady-state or the transient voltage-current relations inside of the portion of the circuit already converted.

APPLICATION OF COMPENSATION THEOREM

In general, it is advantageous to analyze a series circuit with the loop method, and a parallel circuit with the node method. However, experience has shown that complexity would increase in using either of these two methods to attack a predominantly series circuit which contains one or more simple parallel branches, or to attack a predominantly parallel circuit which contains one or more simple series branches. This complexity that is due mainly to the lengthy and awkward mathematical expressions may be greatly reduced by using the compensation

theorem, which is generalized to suit applications in transient conditions as follows:

Any passive element or combination of passive elements—connected in series, parallel, or series-parallel—in a network may be replaced by: (1) a simple voltage generator of zero series impedance whose generated voltage is equal in every respect to the voltage developed across the replaced elements of the original network; or (2) a simple current generator of infinite shunting impedance whose generated current is equal in every respect to the current flowing through the replaced elements of the original network.

Obviously, it is advantageous to adopt the first equivalent for the loop method of solution, and the second equivalent for the node method of solution. The technical questions involved may be made a little clearer by the discussion of the following cases.

Case I

In the network of Fig. 6(a), a current i is assumed to flow through

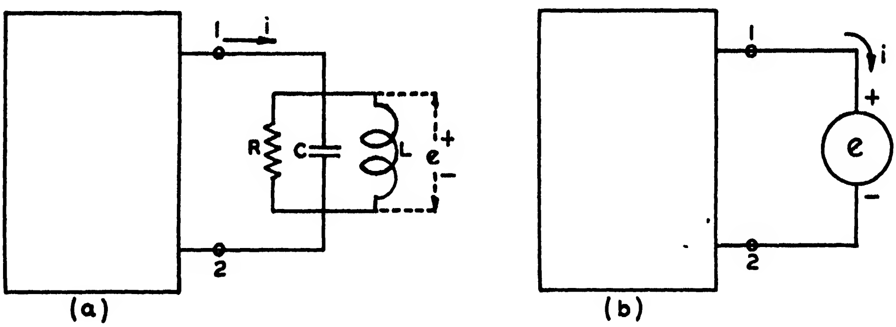

FIG. 6. The establishment of equivalent voltage generator by Compensation theorem.

the parallel combination of R, C, and L. Let it be required to replace this parallel combination with an equivalent voltage generator. If the unknown voltage developed across these parallel elements is represented by the symbol e, we have

$$i = \frac{e}{R} + C\frac{de}{dt} + \frac{1}{L}\int e\,dt. \tag{17}$$

Transforming Eq. 17

$$i(s) = \frac{e(s)}{R} + C[se(s) - e_C(0+)] + \frac{e(s)}{sL} + \frac{e_L^{-1}(0+)}{sL}, \tag{18}$$

where $e_C(0+)$ is the initial voltage across condenser C and $e_L^{-1}(0+)$ is the integral of the initial voltage across L. To evaluate these initial values, it is sometimes convenient to consider the term $Ce_C(0+)$ as the initial charge stored in C and the term $\frac{1}{L}e_L^{-1}(0+)$ as the initial current

flowing through L. Solution of Eq. 18 gives

$$e(s) = \frac{RLsi(s) + RCLse_C(0+) - Re_L^{-1}(0+)}{RCLs^2 + Ls + R} \tag{19}$$

as the Laplace transform of the generated voltage of the equivalent generator in Fig. 6(*b*). Finally, the top terminal of the equivalent generator is assigned to be positive, making the generated voltage equal in every respect to the voltage drop across the *RCL* combination of the original network.

The truth of this theorem is readily seen from the standpoint of Kirchhoff's voltage law. The voltage equations, summing up all potential differences in the network, will not be altered when the potential difference between terminals 1–2 of the equivalent network (Fig. 6(*b*)) is equal in every respect to that of the original network. Therefore, the use of the equivalent voltage generator makes no electrical difference anywhere in the entire network.

Case 2

In the network of Fig. 7(*a*), the voltage developed across the series combination of R, C, and L is assumed to be e. It is desired to replace

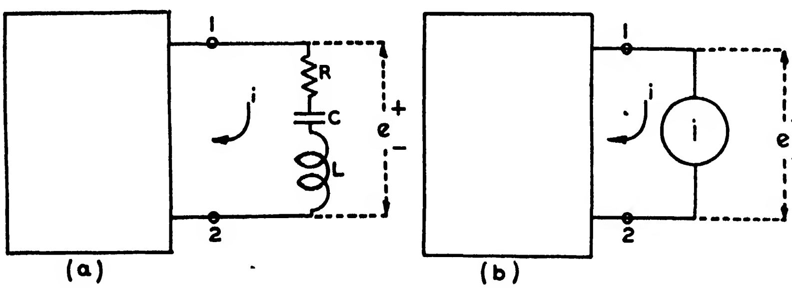

FIG. 7. The establishment of equivalent current generator by Compensation theorem.

this series combination by an equivalent current generator. Let i be the current flowing through these elements.

$$e = Ri + \frac{1}{C}\int i\,dt + L\frac{di}{dt}. \tag{20}$$

The Laplace transform of Eq. 20 is

$$e(s) = Ri(s) + \frac{i(s)}{sC} + \frac{i_C^{-1}(0+)}{sC} + L[si(s) - i_L(0+)], \tag{21}$$

where $i_L(0+)$ is the initial current through L and $i_C^{-1}(0+)$ is the integral of initial current through C. The term $i_C^{-1}(0+)$ may also be considered as the initial charge stored in condenser C. Solving Eq. 21

we have

$$i(s) = \frac{Cse(s) - i^{-1}(0+) + CLsi_L(0+)}{CLs^2 + RCs + 1} \tag{22}$$

as the Laplace transform of the generated current of the equivalent current generator in Fig. 7(*b*). The polarity is so assigned that the currents of both the original and the equivalent networks will be in the same direction.

The identity of these two figures may be proved by employing Kirchhoff's current law. The current equations, summing up all the branch currents in the network, will not be altered if the current flowing through terminals 1–2 of the equivalent network (Fig. 7(*b*)) is equal in every respect to that of the original network. Thus the replacement cannot affect the electrical phenomena anywhere in the entire network.

It is noted that this theorem applies even though the passive elements may neither be linear nor bilateral, because the replacement does not involve any variation in either potential or current anywhere in the network.

Example 4

In this example the use of equivalent current generator is considered. In Fig. 8(*a*), the current generator produces an alternating

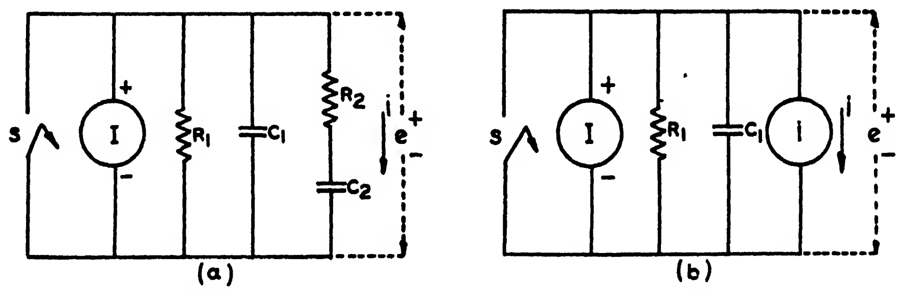

FIG. 8. Simplification by means of equivalent current generator. $I = 10 \cos 377t$, $R_1 = R_2 = 100$ ohms, $C_1 = C_2 = 100$ μf.

current $10 \cos 377t$ amps. Assume that both condensers have zero charge prior to the switching action.

Find the instantaneous voltage developed across condenser C_1 at 0.01 second after the opening of switch S.

Solution:

Employing Eq. 22, the generated current i of the equivalent current generator for R_2 and C_2 may be found. Substitution of $i^{-1}(0+) = 0$, $L = 0$, $C = C_2$, and $R = R_2$ into Eq. 22 gives

$$i(s) = \frac{C_2se(s)}{R_2C_2s + 1}, \tag{23}$$

where $e(s)$ is the Laplace transform of the voltage developed across the passive elements R_1 and C_1. The expression for current equilibrium of the circuit of Fig. 8(b) is

$$I(s) = \frac{e(s)}{R_1} + C_1[se(s) - e(0+)] + i(s), \tag{24}$$

where $e(0+)$ is the initial voltage across C_1 and equal to zero in the present case. Substituting Eq. 23 for $i(s)$, we have

$$e(s) = \frac{I(s)R_1(1 + R_2C_2s)}{(1 + R_1C_1s)(1 + R_2C_2s) + C_2R_1s}. \tag{25}$$

Using numerical values, the expression for $e(s)$ becomes

$$e(s) = \frac{10^5s(100 + s)}{(s^2 + 377^2)(s + 262)(s + 38)}. \tag{26}$$

The inverse transform of Eq. 26 is

$$e = 224 \sin (377t + 25.7°) - 89.3\epsilon^{-262t} - 7.32\epsilon^{-38t}. \tag{27}$$

Therefore the voltage across C_1 will be $-$ 208.5 volts at $t = 0.01$ second.

APPLICATION OF SUPERPOSITION THEOREM

To suit applications in transient conditions, the statement of superposition theorem may be generalized to read as follows:

In a network containing two or more generators, the instantaneous current flowing at any point is equal to the sum of (1) the instantaneous current at this point due solely to the redistribution of initial energy stored in the network with all generators replaced by their respective internal impedances and (2) the instantaneous currents flowing at the same point caused by each generator acting separately with all other generators replaced by their respective internal impedances and the initial stored energy disregarded.

This theorem is also true when potentials instead of currents are employed for consideration.

Obviously, this theorem is based upon the linear characteristics of the passive elements. When the values of all passive elements in a network are independent of the amount of current through themselves, superposing a second current by an additional generator will not affect the first current which has already been flowing in the network, and vice versa.

Example 5

Square pulses, that have been used extensively for testing the frequency response of electric circuits, provide good examples for the application of superposition theorem since a square pulse may be de-

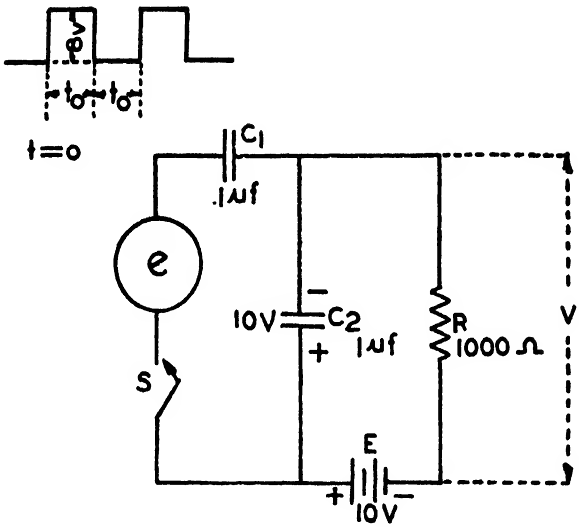

FIG. 9(*a*). Applying a square pulse *e* to a *R–C* circuit.

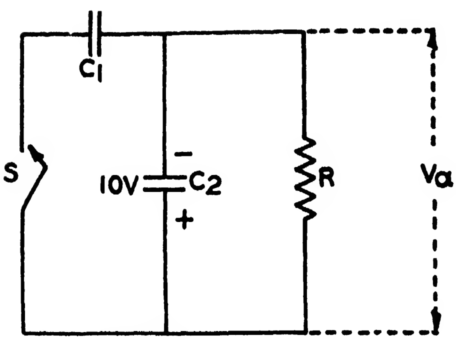

FIG. 9(*b*). Considering the response due to the initially stored energy.

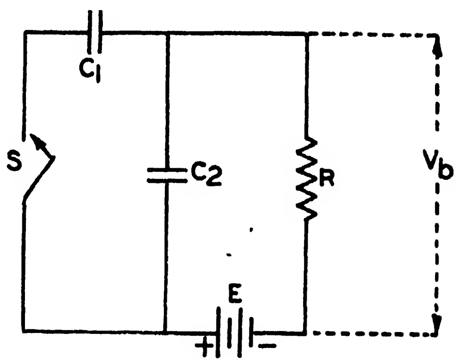

FIG. 9(*c*). Considering the response due to the battery *E*.

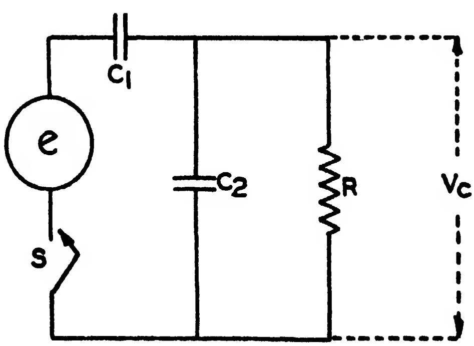

FIG. 9(*d*). Considering the response due to the square pulse *e*.

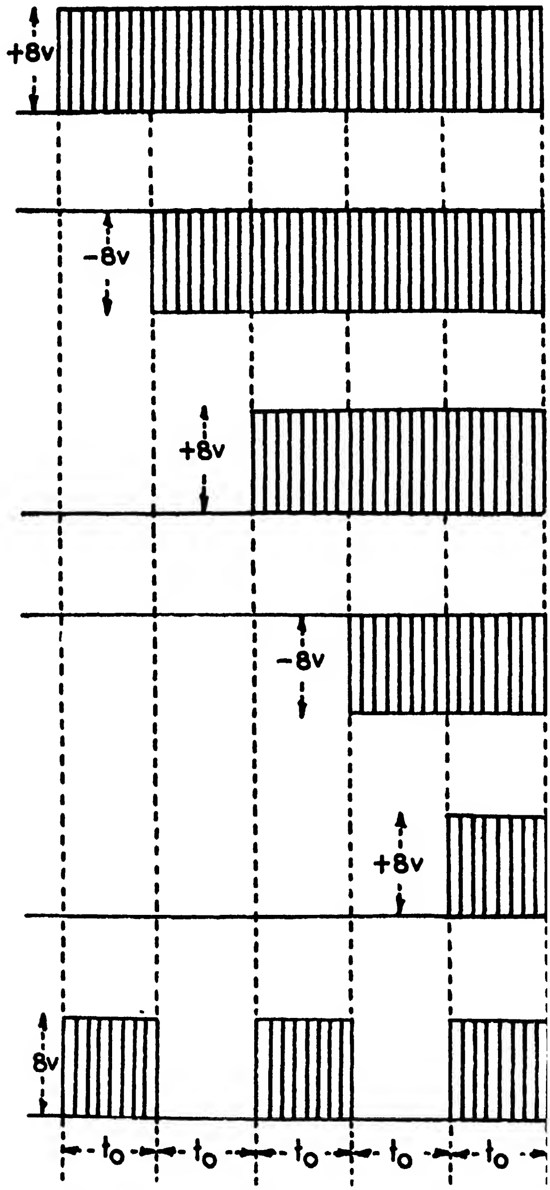

FIG. 9(e). Decomposition of the square pulse e.

composed into a number of components. In Fig. 9(a), the charge on C_2 has reached its steady-state value before $t = 0$, at which switch S is closed. Find an expression for the voltage developed across R after $t = 0$.

Solution:

Before $t = 0$, the voltage on C_1 is zero and on C_2 is 10 volts. After $t = 0$, the voltage developed across R due to the redistribution of the

energy stored in C_2, V_a, may be determined by using Fig. 9(b). This equivalent circuit is obtained from Fig. 9(a) with all voltage generators short-circuited since they are assumed to have zero internal impedances.

$$v_a(s) = \frac{1}{C_1 + C_2} \frac{-10}{s + \dfrac{1}{R(C_1 + C_2)}} \quad \text{or} \quad v_a = -5\epsilon^{-5\times 10^3 t}.$$

The response at R caused by the 10-volt battery can be found by using Fig. 9(c), which is obtained from Fig: 9(a) with the pulse generator short-circuited and the initial energy disregarded.

$$v_b = -10\epsilon^{-5\times 10^3 t}.$$

Finally, the pulse generator is taken into consideration by using Fig. 9(d). Decomposition of the square pulse gives the following components. The first starts at $t = 0$ with amplitude equal to $+8$ volts, the second starts at $t = t_0$ with amplitude equal to -8 volts, the third starts at $t = 2t_0$ with amplitude equal to $+8$ volts, etc., see Fig. 9(e). For the sake of calculation we may assume that the input square pulse is produced by a number of generators connected in series, and each of these generators produces one of the above mentioned components. Then the superposition theorem may be applied to analyze the circuit. The response at R_3 due to the various components of the input square pulse may be found by using the "operational impedance" method that will be discussed in the Appendix.

Due to 1st component

$$v_{c1}(s) = \frac{8}{s} \frac{\dfrac{1}{\dfrac{1}{R} + sC_2}}{\dfrac{1}{sC_1} + \dfrac{1}{\dfrac{1}{R} + sC_2}} \quad \text{or} \quad v_{c1} = 4\epsilon^{-5\times 10^3 t}.$$

Due to 2nd component

$$v_{c2} = -4\epsilon^{-5\times 10^3 (t - t_0)}.$$

Due to 3rd component

$$v_{c3} = 4\epsilon^{-5\times 10^3 (t - 2t_0)}.$$

Due to nth component

$$v_{cn} = (-1)^{n-1} 4\epsilon^{-5\times 10^3 [t - (n-1)t_0]}$$

where n is an integer and equal to $n = (t - t')/t_0$, where $0 < t' < t_0$.

Summation of the above equations gives:

$$v_c = v_{c1} + v_{c2} + v_{c3} + \cdots + v_{cn} = 4\epsilon^{-5\times10^3 t} \sum_{1}^{n} (-1)^{n-1} \epsilon^{5\times10(n-1)t_0}$$

$$= 4\epsilon^{-5\times10^3 t} \frac{1 - \epsilon^{5\times10^3 n t_0}}{1 + \epsilon^{5\times10^3 t_0}}, \quad n \text{ is even.}$$

$$= 4\epsilon^{-5\times10^3 t} \frac{1 + \epsilon^{-5\times10^3 n t_0}}{1 + \epsilon^{-5\times10^3 t_0}}. \quad n \text{ is odd}$$

Therefore the total voltage appearing across R is

$$v = v_a + v_b + v_c = \left(4\frac{1 - \epsilon^{5\times10^3 n t_0}}{1 + \epsilon^{5\times10^3}} - 15\right)\epsilon^{-5\times10^3 t} \quad n \text{ is even}$$

$$= \left(4\frac{1 + \epsilon^{5\times10^3 n t_0}}{1 + \epsilon^{5\times10^3}} - 15\right)\epsilon^{-5\times10^3 t}. \quad n \text{ is odd.}$$

APPENDIX

OPERATIONAL IMPEDANCE METHOD

It is known that two or more independent equations must be solved simultaneously in analyzing a series-parallel circuit. The number of independent equations usually equals the number of meshes in the circuit diagram. Thus in analyzing a circuit containing four or more meshes, four or more simultaneous equations are required to be solved.

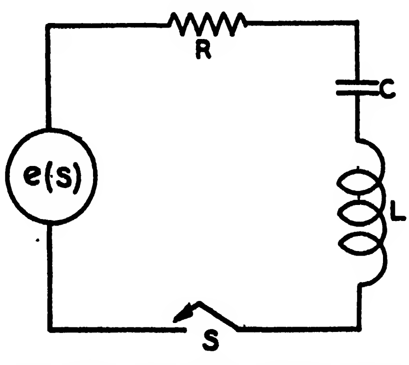

FIG. 10(*a*). Series *RCL* circuit used in connection with the "operational impedance" method.

This mathematical complexity may be eliminated by the use of the "operational impedance" method described here, which is essentially a variant of the "steady-state impedance" method used in elementary alternating-current theory. However, many new difficulties will arise if the circuit to be analyzed is initially active. For this reason, the use of the "operational impedance" method will be restricted here to those circuits which are initially at rest.

In the one-loop RCL circuit shown in Fig. 10(a) the switch is closed at $t = 0$. Initially, the circuit contains no stored energy. The unknown current $i(s)$ may be found as

$$i(s) = \frac{e(s)}{Z(s)}, \tag{28}$$

where $Z(s) = R + \frac{1}{sC} + sL$ is the total operational impedance of the circuit. The second case to be considered is the one-node circuit of Fig. 10(b); the opening of switch S at $t = 0$ allows a known current

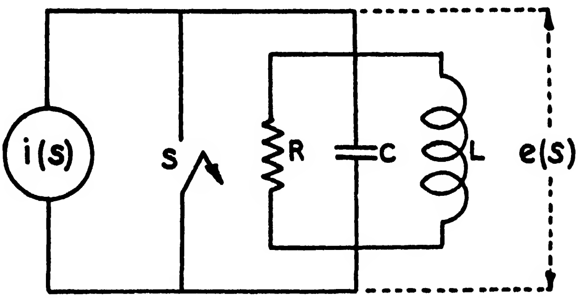

FIG. 10(b). Paıallel RCL circuit used in connection with the "operational impedance" method.

$i(s)$ to flow to the RCL parallel circuit. The initial voltage across C and the initial current through L are both zero. The unknown voltage may be determined as

$$e(s) = i(s)Y(s), \tag{29}$$

where $Y(s) = \frac{1}{R} + sC + \frac{1}{sL}$ is the total operational admittance of the circuit.

THE MINIMOTION TYPEWRITER KEYBOARD*

BY

ROY T. GRIFFITH [1]

SYNOPSIS

This paper first points out the defects of the typewriter keyboard now in general use, then describes the design of a keyboard that overcomes these defects and is claimed to afford the greatest attainable ease in writing average English. This is accomplished by determining the optimum key assignments to minimize hard motions (and maximize easy motions), whence the apt name "MINIMOTION."

The present paper concludes with a discussion of the outlook for general adoption of the MINIMOTION keyboard.

Appendices present statistical tabulations of the usage of single and adjacent letters in average English, convenient terminology for analyzing typewriter keyboards in respect to the motions required to write average English, and the comparative results of analyzing several keyboards. These appendices provide sufficient information that the basic statistics and the analysis methods can be applied to any keyboard.

1. INTRODUCTION

The typewriter ranks with the telegraph and the telephone as a medium of commercial communication; as a business machine it ranks with the cash register. Taking a broader view, nearly all the activities of the modern world are directed and recorded by typewriting. The numbers of typewriters in use run into the millions, but more impressive than large numbers is the thought that nearly everything we read: letters, newspapers, magazines, books—whatever the final form—was once typewritten.

The typewriter produces words one letter at a time, but in such a rapid succession that an observer's eye cannot follow the keyboard manipulation. Nevertheless, the operator must direct the motions of the fingers from key to key between each stroke. These motions are easy or hard, depending on the locations of the letters on the keyboard in relation to the adjacencies of the letters in the words being typewritten.

The motions required to operate so important a machine warrant the thorough study reported in this paper.

2. PREVALENCE OF THE QWERT YUIOP KEYBOARD

Nearly all the typewriters now in service have a keyboard bearing the letters of the alphabet arranged as shown in Fig. 1. Conveniently,

* Patent applied for.

[1] Deceased. Formerly, Assistant Engineer in Charge of Transmission, Western Area, Bell Telephone Company of Pennsylvania, Pittsburgh, Pa.

the top row of letters spells a distinctive and pronouncable name for this keyboard: QWERT YUIOP (kwert-you-ee-opp).

Amazingly, the QWERT YUIOP keyboard of Fig. 1 is almost identical to the keyboard that was devised about 1872 (**1**)[2] and used on the first commercial typewriter. The original keyboard seems to have survived the early years of typewriter development largely because the pioneer typewriter manufacturer continued to use it while attaining leadership of the new industry. At any rate, the early manufacturers who introduced other keyboards were relatively unsuccessful. By about 1900 the QWERT YUIOP keyboard had become the generally accepted standard, and the "touch system" of operating was steadily gaining acceptance. Since then practically all the typewriters have been made by a few large manufacturers who have never used any but the QWERT YUIOP keyboard. From time to time during these years various "improved" keyboards have been proposed, but most of these have been hardly noticed, and none has

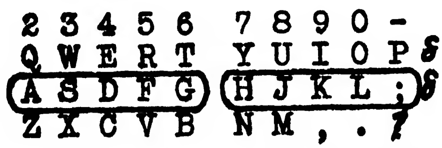

Fig. 1. The QWERT YUIOP Keyboard. The boxes enclose the ten keys that comprise the left and right halves of "Home Row." The two keys marked δ have little usage.

attained important acceptance. Accordingly, the peculiar QWERT YUIOP arrangement has become the very alphabet of the typewriter, and a machine having a different keyboard is quite a curiosity.

Generation after generation of operators has learned to use the QWERT YUIOP keyboard with good speed and accuracy by "touch" alone, with only occasional glances at the keyboard. The skill commonly acquired is such that a competent operator appears to manipulate the keyboard with almost effortless ease.

So the QWERT YUIOP keyboard is as old as the typewriter. It has resisted change for more than 70 years, and it seems to be easy to use. These explain the common presumption that this venerable keyboard must have inherent merit, however obscure. Nevertheless, the QWERT YUIOP keyboard has defects that begin to appear as soon as it is examined critically; finally, the most thorough-going analysis fails to reveal any adaptation to writing English easily.

This paper now proceeds to explain the defects of the QWERT YUIOP keyboard and describe how the MINIMOTION keyboard has been designed to avoid these defects.

3. DEFECTS OF THE QWERT YUIOP KEYBOARD

Five defects of the QWERT YUIOP keyboard are stated and explained below, each in contrast to the possible attainments of a well

[2] The boldface numbers in parentheses refer to the references appended to this paper.

designed keyboard. The statistical percentages apply when writing "average English" (see Part 6) with conventional fingering (see Part 7); the illustrative words are to be tried on a QWERT YUIOP keyboard.

First Defect

On the QWERT YUIOP keyboard 48 per cent of the *motions* to re-position the fingers laterally between consecutive *strokes* are *one-handed motions* rather than the easier *two-handed motions;* on a well designed keyboard one-handed motions can be reduced to 33 per cent, making 67 per cent of the motions two-handed.

One-handed motions are harder than two-handed motions because they afford little opportunity for overlap and anticipation—it is hard to beat a drum rapidly with one stick. A French student of typewriting motions found that one-handed motions consume, on the average, 73 per cent more time than two-handed motions (10). So *dfdfdfdf* or *jkjkjkjk*, done with adjacent fingers of one hand, are harder to execute rapidly than *dkdkdkdk*, which is done two-handed. The words *addressed*, *federated*, and *kiln*, written entirely with one-handed motions, contrast with *sicken*, which is done entirely two-handed.

Second Defect

The QWERT YUIOP keyboard requires *Reaches from Home Row* for 68 per cent of the strokes; on a well designed keyboard these reaches can be reduced to 29 per cent, putting 71 per cent of the strokes in Home Row. (This "Home Row," indicated on Fig. 1, contains the "reference keys" of the "touch system" of typewriting, as explained in part 7 and as diagrammed in Appendix 1.)

In the Home Row of the QWERT YUIOP keyboard only *A*, *S*, *H*, and *L* rank among the ten most common letters; moreover, the unimportant letters *J* and *K* are under the most agile fingers of the right hand.

The *easiest* motions of typewriting are *two-handed motions without reaches from Home Row*, like *dkdkdkdk*. On the QWERT YUIOP keyboard such motions are only 4 per cent and *half* seems to be the only common word that can be written entirely with them; on a well designed keyboard the two-handed motions without reaches can be raised to 34 per cent, and whole sentences can be written with them.

Third Defect

The *hardest* motions of typewriting are *one-handed motions with hurdles over Home Row* (or over the row next above). An extreme example of hurdles is the word *minimum;* other awkward words including several hurdles are *pumpkin* and *execrate.* On the QWERT

YUIOP keyboard 11 per cent of the motions are hurdles; on a well designed keyboard hurdles can be reduced to 1 per cent. (Use of the apt term *hurdle* in this connection seems to be due to Dvorak and his collaborators, (18).

Fourth Defect

The QWERT YUIOP keyboard *overloads the left hand* with 56 per cent of the total strokes; a well designed keyboard has the left hand carry somewhat less than half of the stroking load.

Fifth Defect

The QWERT YUIOP keyboard requires many *awkward fingerings* of one-handed motions yet *loads the fingers inequitably.*

Obviously all typewriter keyboards require repeat strokes on the *same key* to type double letters (as in the work *reef*). However a well designed keyboard should minimize consecutive strokes on different keys done by the *same finger* (as in *parted*). These same-finger fingerings are *S* and *A* because they permit no overlap or anticipation, hence are slow and awkward, like a pianist trying to do a "trill" with one finger. They are specially awkward if they involve a hurdle (as in *secede*). Consecutive strokes that must be done one-handed are somewhat easier if done by *adjacent fingers* or by *remote fingers*, which permit some overlap though much less than two-handed motion. (The word *treated* illustrates a variety of fingerings of one-handed motions.)

On the QWERT YUIOP keyboard about 7 per cent of all the motions are consecutive strokes on different keys done by the same finger, between many of these are hurdles, and several fingers are heavily overloaded; on a well designed keyboard the same-finger fingerings are reduced to about 4 per cent, the same-finger hurdles are almost entirely eliminated, and yet the individual fingers are loaded equitably.

4. ORIGIN OF THE QWERT YUIOP KEYBOARD

The foregoing exposition of the defects of the QWERT YUIOP keyboard calls for an explanation of the origin of the peculiar arrangement of the letters. The origin is obscure and the historians disagree. One writer who considers that the arrangement is not alphabetical says also that it is "nothing like the arrangement of the type in the printer's case (8); another says confidently that the keyboard "follows the idea of the printer's type case" (20). At least three writers (8,18,19) repeat a story that has become traditional: They say that the keyboard was designed to minimize the possibility of conflicting type-bars by keeping well separated those often used on consecutive strokes. This has been accepted without challenge for many years probably because it is plausible and not easy to refute.

The story about the conflicting type-bars is plausible in view of the early mechanical difficulties, but it does not stand up under statistical examination. First defining "close" type-bars as any two having not more than four others intervening on consecutive strokes, application of the statistics of average English reveals that the QWERT YUIOP keyboard actually uses more "close type-bars (26 per cent)

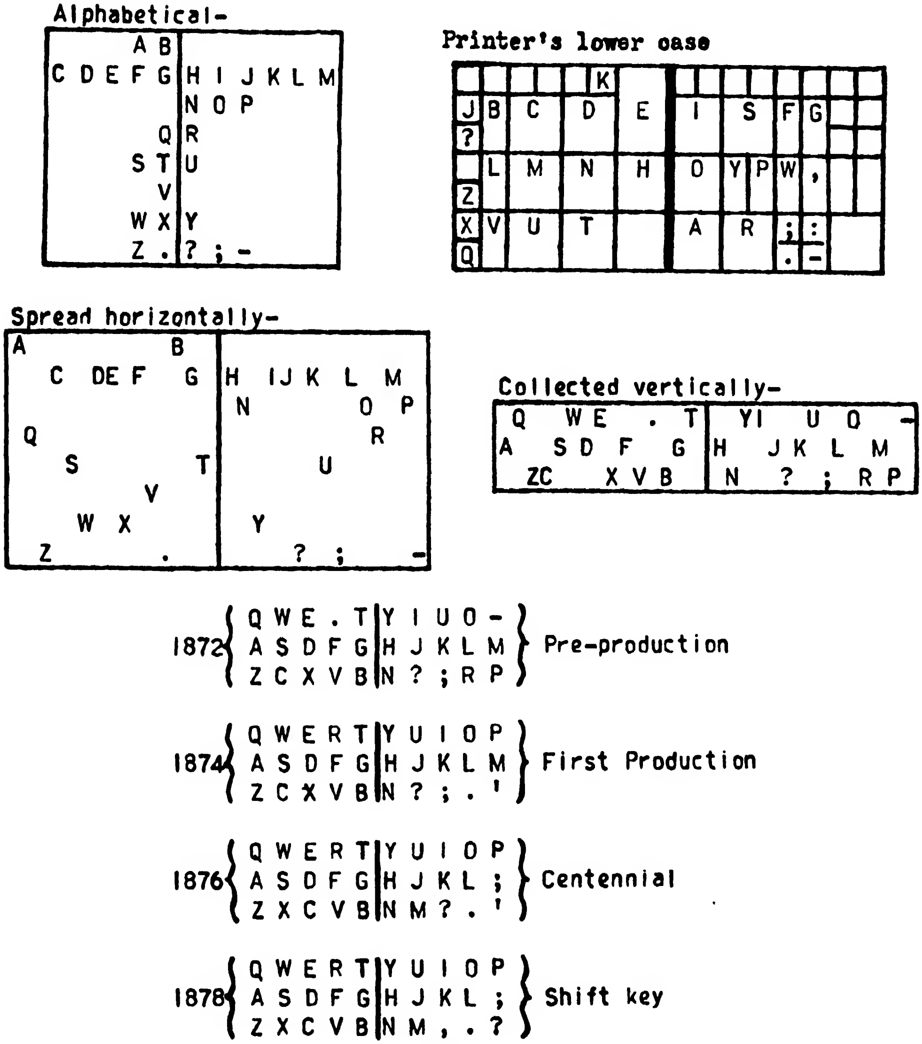

FIG. 2.

than the RANDOM[8] keyboard (22 per cent) and many more than the MINIMOTION keyboard (14 per cent) as described in Part 8. If the QWERT YUIOP keyboard really had been designed to minimize "close" type-bars, it would certainly have had fewer than the RANDOM keyboard; also it would probably have had fewer one-handed

[8] Described in Part 8.

motions, which would have made it an easier keyboard to use. But if the origin of the QWERT YUIOP keyboard was merely alphabetical, that would not adapt it to write average English easily. Nevertheless, the QWERT YUIOP keyboard does retain vestiges of an alphabetical origin. Figure 2 shows how the letters might have been laid out in eight alphabetical rows, then spread horizontally, finally collected into three rows to yield the 1872 keyboard (1). If this explanation is not convincing, at least none better has appeared. Figure 2 shows also the slightly modified keyboards of 1874, 1876, and 1878. The diagram of the printer's type case is presented to facilitate comparisons.

5. DESIGN OBJECTIVE OF THE MINIMOTION KEYBOARD

The general design objective of the MINIMOTION keyboard is to determine the optimum arrangement of the letters for writing average English with the greatest possible ease. This objective is desirable in view of the tremendous amount of human effort expended in typewriting. Attainment of this objective is difficult because the characteristics of average English are expressible only in statistical form. The most compact tabulation of these statistics adequate for use in typewriter keyboard design includes about 500 items. Then, as no keyboard arrangement can possibly conform to all the statistical indications, the optimum is necessarily a careful compromise among conflicting specific objectives. Nevertheless the MINIMOTION keyboard is claimed to attain the optimum compromise of compliance with the statistics of average English.

The five specific design objectives of the MINIMOTION keyboard are, each for each, the minimization of the five defects of the QWERT YUIOP keyboard described in Part 3:

1. First objective—To minimize one-handed motions.
2. Second objective—To minimize reaches from Home Row, and to maximize two-handed motions in Home Row.
3. Third objective—To minimize hurdles.
4. Fourth objective—To divide the stroking load equitably between the hands.
5. Fifth objective—To minimize awkward fingerings of one-handed motions, yet load the fingers equitably.

6. AVERAGE ENGLISH, STROKES, AND MOTIONS

As typewriters are used to write all kinds of material, the most appropriate basis for keyboard design and analysis is the performance when writing *average English*, comprised of the normal proportions of all kinds of material.

A truly representative sample of average English must contain at least 100,000 running words, so the sample is unwieldy until it is

reduced to statistical form. This is accomplished by first analyzing the sample for the statistical usage of words, then analyzing the words for the statistical usage of single letters and adjacent letters. These analyses must meet particular requirements to yield statistics suitable for application to typewriter keyboard design.

First, the statistics must consider not only *letters*, but also *punctuation marks* and *numerals*—indeed, the major punctuation marks have much higher usage than some letters. A convenient terminology defines *strokes* as the typewriter key operations that mark the paper, thus including punctuation marks and numerals, but excluding the horizontal *spaces* that are inserted between words by operating the letter space bar. Then, taking the number of strokes as the measure of the quantity of typewriting disregards the spaces—properly enough, because the spaces are inserted with negligible effort, and with no appreciable disturbance of the fingers, by either of two agile thumbs that are otherwise idle.

Second, the statistics must consider the usage of consecutive strokes in pairs, conveniently termed *adjacencies*, because these determine the *motions* required to re-position the fingers laterally between consecutive strokes. Directing the execution of these (lateral) motions is really the brain work of typewriting, while the (vertical) strokes involve merely physical effort. This is evident upon consideration that a motorized typewriter almost entirely eliminates the physical effort of strokes but leaves unchanged the mental effort of directing motions. Nevertheless this mental effort can be reduced by designing the keyboard in relation to the usage of adjacencies in average English. Further consideration makes clear that the motion to re-position the fingers between any pair of strokes is independent of the relative temporal order of the two strokes; also that the disregarding of spaces makes the final letter of each word adjacent to the initial letter of the next word (or to a punctuation mark), thus creating adjacencies between words that are treated just like the adjacencies within words.

In the lack of any statistical information satisfying these particular requirements, the author made a statistical study based on a carefully compounded sample containing 107,000 running words involving 510,000 strokes and motions. This study determines, for average English, with customary USA spelling, the usages *per 510,000 strokes* of 37 individual characters and 544 adjacencies of these characters. The final results of this statistical study are attached to the present paper as appendices, described as follows:

Appendix 8 presents the single and adjacent usages per 510,000 strokes for 37 typewriting characters (strokes). These usages are arranged in the form of a square table (37 × 37), with segregated vowels (vow) and consonants (Con) in alphabetical order, and the non-letter strokes (P + N) in order of usage. The horizontal and

vertical ruled lines follow sub-totals. The vowels are distinctively indicated by lower case letters, the consonants by capital letters, and the numerals by a subsequent sign # (the usages of numerals being assumed concentrated on 0#, 1#, and 2#). In this square table each item is entered twice and is the sum of the usages of the two possible temporal orders of the two component mutually adjacent strokes. For example, the largest entry in the body of the table, appearing in two places, is for the adjacency *TH* + *HT*, which is the sum of the usages of *TH* (as in *THe*) and *HT* (as in *rigHT*), including adjacencies occurring between words (as in *nexT HigH Tide*). Hence the bottom row and the column at the extreme right, both designated 2Σ, give double the total usage of the single strokes, and the grand total of the individual entries is 1,020,000. The usages of single strokes range from about 60,000 per 510,000 for *e* to 300 per 510,000 for the colon: usages of adjacencies range from about 16,000 per 510,000 for *TH* + *HT* to 10 per 510,000 for the smallest entries.

Appendix 6, derived from Appendix 8, presents in columnar form the usages of 37 single strokes, both per 510,000 strokes and per 1000 strokes, arranged in order of usage. The three columns to the right of the usages are Key Group numbers (see Part 7) for the stroke analysis of three typewriter keyboards.

Appendix 7, derived from Appendix 8, presents in columnar form the usages of 544 adjacencies, per 510,000 strokes (and per 1000 strokes for the large items), arranged in order of usage. The three columns to the right of the usages are motion codes (see Part 7) for the motion analysis of three typewriter keyboards. Experience shows that all close comparisons involved in typewriter keyboard design must be made in terms of the usages per 510,000 strokes and must include all the 544 items; however, the final results of such comparisons can usually be displayed best by translating the totals to usages per 1000 strokes.

7. KEY GROUPS, MOTION CODES, AND FINGERING

Appendix 1 explains a convenient terminology for the classification of strokes and motions. In general, the plan is to number the left and right halves of the four rows of typewriter keys as *Key Groups*. Motions as between any two Key Groups are then designated by two-digit *motion codes* formed from the numbers of the terminal Key Groups, with the smaller number always the first digit. This coding intentionally disregards as insignificant the temporal order of the terminal strokes, and is consistent with the designation of adjacencies in the form *TH* + *HT*. There are 903 possible different motions among 42 typewriter keys, but only 36 possible different motions among six Key Groups. Hence typewriter keyboards are much more easily analyzed and designed in terms of group-to-group motions than key-to-key

motions. Appendix 1 shows the 36 group-to-group motions by motion codes classified by use of hands and by relative difficulty. The lower left corner of Appendix 1 explains the code letters used to supplement the numerical motion codes to classify the key-to-key motions involved in fingering of one-handed motions.

The light curved lines on the keyboard diagram of Appendix 1 are intended to indicate the *conventional fingering* of a 42-key typewriter keyboard in the "touch system" of typewriting. These light curved lines emanate from the scalloped keys, which are the "reference keys." In "home position" the index fingers touch lightly (or hover over) the keys numbered 14 and 26. The index fingers are required to stroke the two keys in each Key Group nearest the left-right division line. The middle and ring fingers each stroke one key in each Key Group. The little fingers are required to stroke the outer keys and also operate the *CAP.* shift keys off the lower left and right corners of the keyboard. With practice the little fingers can operate these shift keys without getting much out of home position. The letter-space bar is operated by one of the thumbs; the other thumb has nothing to do.

8. THE RANDOM KEYBOARD

The RANDOM keyboard, diagramed and characterized in Appendix 3, was conceived by the author in 1942 as an idealized imaginary keyboard which is the composite of a large number of keyboards each having the letters assigned to its keys perfectly at random. While such a keyboard cannot be realized, its characteristics are completely predictable from probability considerations and are significant when compared to the characteristics of actual keyboards. In particular, the RANDOM keyboard reveals that the QWERT YUIOP keyboard has nearly the characteristics of a random arrangement; in respect to the other keyboards the RANDOM keyboard affords a criterion for appraising the accomplishments of deliberate design as distinguished from the expectations of mere chance.

The RANDOM keyboard has no numeral row of keys but assumes the total stroking load to be uniformly distributed over 30 keys, in three rows of ten keys each, and in six Key Groups of five keys each. Hence, such a keyboard will have one thirtieth of the total stroking load on each key, one sixth in each Key Group, one third in Home Row, and one half on each hand.

Wherever a particular stroke falls on the RANDOM keyboard, the next stroke is equally likely to fall on any one of the 30 keys. Hence, one half the motions will be two-handed; the two-handed motions without reaches from Home Row will be $(2/6)(1/6) = 1/18$ = about 6 per cent; and the one-handed motions with hurdles over Home Row will be $(4/6)(1/6) = 1/9$ = about 11 per cent.

More detailed analysis of the RANDOM keyboard yields the

complete distribution of motions by motion codes and the fingering *of one-handed motions shown on* Appendix 3. Finally an auxiliary study has determined *the usage of "close"* type-bars as reported in Part 4.

9. THE DVORAK-DEALEY KEYBOARD

The DVORAK–DEALEY "Simplified" Typewriter Keyboard (U.S. Patent 2,040,248 issued in 1936) was designed about 1932 by August Dvorak, a psychologist of the University of Washington who had specialized in time and motion studies of typewriting, and William L. Dealey, another educator (17,18). Their keyboard largely avoids the defects of the QWERT YUIOP keyboard, and the several thousand typewriters that have been equipped with DVORAK–DEALEY keyboards have demonstrated excellent performance (17,18). The DVORAK–DEALEY keyboard is diagramed and analyzed in Appendix 4 for ready comparison with the other keyboards, and the details of its strokes and motions when writing average English are recorded in one of the columns at the right of Appendices 6 and 7.

10. DESCRIPTION OF THE DESIGN OF THE MINIMOTION KEYBOARD

1. First Objective—To Minimize One-handed Motions

Referring to Appendix 5, the MINIMOTION keyboard diagram shows nine keys in the top row used for numerals. The candidates for the remaining 33 keys are as follows:

26 letters	26 keys
Comma, period, and hyphen, each double	3 keys
Semi-colon/colon, quotation marks, apostrophe, and question mark	4 keys
	33 keys

These 33 candidates are to be assigned 15 to the left side of the keyboard and 18 to the right side so as to minimize one-handed motions (or maximize two-handed motions). In general, this will be accomplished by assigning to opposite sides of the keyboard the characters that are commonly used adjacently.

The 33 candidates can be assigned as between the left and right sides of the keyboard in more than a billion different ways (15C33 or 18C33). Fortunately all but a few of these possible choices can be eliminated by careful study of the square table of Appendix 8 and classification of the individual characters into three groups which may be conveniently designated as follows:

Alphas—The 5 vowels, the consonant *Y*, and the ***punctuation marks***, which are all clearly more commonly adjacent to the major consonants than to one another. The alphas require 13 keys and they are assigned to the left side of the keyboard, primarily to obtain a desirable load for the left hand.

Betas—The 15 consonants *N S R L C M F W P B V X J Q Z*, which are all clearly more commonly adjacent to alphas than to one another. So the betas are assigned to the right side of the keyboard, opposite the alphas.

Gammas—The 5 consonants *T H D G K*, which do not have a clear preponderance of usage adjacent to alphas or betas.

An auxiliary study shows that the left-right assignments of the alphas and betas are correct for maximum two-handed motions no matter how the gammas are assigned. Although the assignments of alphas and betas leave two spare keys on the left and three on the right, arrangements can be made to accommodate any or all of the five gammas on either side of the keyboard. Hence the number of possible choices of gamma assignments is $2^5 = 32$. Each of these 32 choices has been studied in detail for the resulting two-handed motions per 510,000 strokes. The results for the ten best choices and the 32nd choice are shown in Table I.

TABLE I. *Two-handed motions after alpha and beta assignments for 32 different gamma assignments*

Rank of CHOICE	Gamma Assignments LEFT	RIGHT	Two-Handed Motions per 510,000	Two-Handed Motions per 1000
1st	G	THDK	340,452	668
2nd	KG	THD	340,274	667
3rd	H	TDGK	339,690	666
4th*	KH	TDG	339,502	666
5th		THDGK	339,396	666
6th	K	THDG	339,316	666
7th	GH	TDK	338,134	663
8th	KGH	TD	337,848	663
9th	T	HDGK	337,132	661
10th	GT	HDK	336,794	661
		(21 others not shown)		
32nd	KGDHT		293,848	576

* MINIMOTION choice.

The possible gamma assignments result in a rather small range of two-handed motions because the alpha and beta assignments have insured so many of the obtainable two-handed motions. Any of the 32 choices of gamma assignments yields more two-handed motions per 1000 strokes than the QWERT YUIOP keyboard (which has 523), and 20 of the choices yield more than the DVORAK–DEALEY keyboard (which has 653).

As indicated, the fourth choice is considered most satisfactory for the MINIMOTION keyboard. This choice yields 666 two-handed motions per 1000 strokes out of a possible 668 and has other advantages that appear in Parts 10.2 and 10.4.

The two-handed motions of the MINIMOTION keyboard could be increased to 722 per 1000 strokes by providing a second letter *T* on the

left side of the keyboard, but this large gain would be impractical to realize.

The two-handed motions of the several typewriter keyboards per 1000 strokes are:

QWERT YUIOP	523
RANDOM	500
DVORAK-DEALEY	653
MINIMOTION	666

2. Second Objective—To Minimize Reaches From Home Row, and To Maximize Two-handed Motions in Home Row

The MINIMOTION choice of assignments to minimize one-handed motions puts five of the ten most commonly used letters on each side of the keyboard. Hence reaches from Home Row are minimized by assigning *e a o i h* to Key Group #5 and *T N S R L* to Key Group #6. Including the use of the lower case of *L* for the numeral *one*, these assignments insure 709 strokes in Home Row per 1000 keyboard strokes, which is the maximum attainable on ten keys.

The ten letters that have the highest individual usage also have the highest usage of adjacencies as between two groups of five letters each. The MINIMOTION assignments to Key Groups #5 and #6 yield 335 two-handed motions in Home Row per 1000 keyboard strokes, which is the maximum practically attainable. These motions are those coded *56* and described as the *easiest motions* of typewriting. (The motions coded *56* could be increased to 341 per 1000 keyboard strokes by interchanging the assignments of the letters *T* and *H*, but this would decrease the two-handed motions for the whole keyboard from 666 to 661.)

The strokes and two-handed motions in Home Row per 1000 keyboard strokes for the several typewriter keyboards are as follows:

	Strokes	Motions
QWERT YUIOP	316	41
RANDOM	334	56
DVORAK-DEALEY	673	288
MINIMOTION	709	335

3. Third Objective—To Minimize Hurdles

The design to minimize one-handed motions (Part 10.1) has determined the assignments as between the left and right sides of the keyboard; further, the design to minimize reaches and maximize two-handed motions in Home Row (Part 10.2) has determined the assignments to Key Groups #5 and #6. The remaining choices of assignments above and below Home Row are now to be made so as to minimize hurdles, which are described as the *hardest motions* of typewriting. In general, this will be accomplished by assigning to the key groups above and below each half of Home Row characters that are not commonly used adjacently.

First, the low-usage letters *J Q* and *Z* are disposed of by assignment to the three available keys in Key Group #2 at the extreme right of the keyboard. Then there remain ten assignments to be made on the left as between Key Groups #3 and #7, and ten on the right as between Key Groups #4 and #8. The candidates for these twenty assignments are listed in Table II with their individual usages per 1000 strokes.

Evidently the principal problem is to minimize the hurdles coded 37 and 48; those coded 17 and 28 will be few because of the low usage of any characters in Key Groups #1 and #2, also a few strokes in #7 and #8; other hurdles, coded 15 and 26, will also be few and these have already been fixed by the assignments completed in Key Groups #1, #2, #5, and #6.

Considering now the right hand hurdles, the number of possible assignments of the ten letters as between Key Groups #4 and #8 is the combinations of ten items taken five at a time = 252. However, study

TABLE II

On the LEFT as between Key Groups #3 and #7, affecting Hurdles coded 37 and 17		On the RIGHT as between Key Groups #4 and #8, affecting Hurdles coded 48 and 28	
u	27	D	36
y	20	C	28
,	12	M	24
.	11	F	22
–	7	W	20
K	6	G	19
"	4	P	18
'	2	B	15
; :	2	V	10
?	1	X	1
	92		193

of Appendix 8 reveals that few hurdles will result from assigning *D* to Key Group #4, and *V* and *X* to Key Group #8, which reduces the number of possible choices of assignments to 35. Each of these 35 choices has been tested in detail for the resulting hurdles per 510,000 strokes. The results for the five best choices and the 35th choice are shown in Table III. As indicated, the 1st choice was used for the MINIMOTION keyboard.

The left hand hurdles were considered just as thoroughly, but only four choices needed to be tested in detail for hurdles per 510,000 strokes, the range of hurdles was from 2.2 to 3.1 per 1000 strokes, and the choice for the MINIMOTION keyboard tolerates 2.8 left hand hurdles per 1000 strokes to avoid key assignments that appeared otherwise objectionable. (– to #7, ' to u. c.)

In making the assignments to minimize hurdles the strokes in Key Groups #7 and #8 were minimized, as these are a little harder to

reach than Key Groups #3 and #4. Appendix 5 shows the final arrangement of these Key Groups.

The hurdles for both the DVORAK–DEALEY and MINIMOTION keyboards are shown by motion codes in Table IV.

TABLE III. *Right hand hurdles, coded 26, 28, 48*

Rank of Choice	Key Group #4 and #8 Assignments	Hurdles /510,000	Hurdles /1,000
1st*	D C F W G M P B V X	4,092	8.0
2nd	D F W G B C M P V X	4,192	8.2
3rd	D C F W B M G P V X	4,296	8.4
4th	D M F P B C W G V X	4,414	8.6
5th	D M F W P C G B V X	4,473	8.8
(29 others not shown)			
35th	D C W P B M F G V X	5,699	11.2

* MINIMOTION Choice.

TABLE IV. *Hurdles by motion codes*

	DVORAK–DEALEY		MINIMOTION	
Motion Code	Hurdles per 510,000	Hurdles per 1000	Hurdles per 510,000	Hurdles per 1000
15	1,776	3.5	321	0.6
17	70	0.1	0	0.0
37	804	1.6	1,107	2.2
Left	2,650	5.2	1,428	2.8
26	3,754	7.4	1,033	2.0
28	959	1.9	231	0.5
48	5,976	11.7	2,828	5.5
Right	10,689	21.0	4,092	8.0
Total	13,339	26.2	5,520	10.8
		=3%		=1%

The hurdles of the several keyboards per 1000 strokes are as follows:

QWERT YUIOP	113
RANDOM	111
DVORAK-DEALEY	26
MINIMOTION	11

The residual hurdles of the MINIMOTION keyboard mostly occur between words or involve a non-letter character. Some hurdles that occur within words, as in *aDVise, eXCept, lucKY,* and *suBJect,* are conspicuous but have little usage. Table V lists the ten most common hurdles, which account for nearly half the total.

TABLE V. *Minimotion keyboard, the most common hurdles*

Adjacency	Motion Code	Usage per 510,000
DM + MD	H48R	493
DB	H48R	398
," + ",	H37A	350
DP + PD	H48A	305
O#T + TO#	H26A	246
FM + MF	H48S	218
FB + BF	H48S	186
GM + MG	H48A	168
Y" + "Y	H37A	166
SO# + O#S	H26S	161
10 above		2691
Others		2829
Total		5520 = 11/1000 = 1 per cent.

4. *Fourth Objective—To Divide the Stroking Load Equitably between the Hands*

The MINIMOTION choice of assignments to minimize one-handed motions also results in an equitable division of the total stroking load between the hands. The heavier stroking load for the right hand (516 per 1000) seems in line with indications that more than 90 per cent of literate adults are right-handed (15). Some tests (6,12) have found about 10 per cent greater agility for the fingers of the right hand, which suggests that the ideal division of a load of 1000 strokes may be about 475 left and 525 right. At any rate the left-right stroking loads of several keyboards are:

	Left	Right
QWERT YUIOP	561	439
RANDOM	500	500
DVORAK-DEALEY	447	553
MINIMOTION	484	516

5. *Fifth Objective—To Minimize Awkward Fingering of One-handed Motions Yet Load the Fingers Equitably*

The assignments to minimize one-handed motions, reaches, and hurdles have been by Key Groups, with no regard to the assignments

of the individual keys within each Key Group. The particular assignments internal to the Key Groups determine the fingering of one-handed motions and the division of the stroking load among the fingers. These assignments will now be made to minimize awkward fingerings as well as can be without loading the fingers inequitably.

The several fingerings of one-handed motions are conveniently coded as follows:

Code K—The repeat strokes on the *same key* for double letters, which account for about 30 motions per 1000 strokes;

Code S—The consecutive strokes on different keys done by the *same finger;*

Codes A and R—The consecutive strokes on different keys done by *adjacent fingers* or by *remote fingers,* which account for the remaining one-handed fingerings.

TABLE VI. *Fingering of one-handed motions, per 1000 strokes*

All one-handed motions	QWERT YUIOP	RANDOM	DVORAK-DEALEY	MINI-MOTION
K = Same Key	30	30	30	30
S = Same Finger	73 = 7%	108 = 11%	39 = 4%	41 = 4%
A = Adjacent Fingers	204	160	139	142
R = Remote Fingers	170	202	139	121
Total one-handed	477	500	347	334
Total two-handed	523	500	653	666
Total Motions	1000	1000	1000	1000
Hurdles only				
HS = Same Finger	26	31	6	3
HA = Adjacent Fingers	54	35	6	4
HR = Remote Fingers	33	45	14	4
Total Hurdles	113	111	26	11

On the left side of the keyboard the number of permutations of assignments that are distinct in respect to fingering are 60 within *each* of the Key Groups #3, #5, and #7, thus making the possible distinct arrangements of the left side of the keyboard = 60^3 = 216,000; on the right side the same number of arrangements results from permutations within Key Groups #4, #6, and #8. However only 12 arrangements of the left side of the keyboard and 36 arrangements of the right side avoid finger loadings that appear inequitable. These 48 arrangements were tested in detail for same-finger fingerings per 510,000 strokes, with hurdles segregated for special consideration. The combination of left-right arrangements chosen for the MINIMOTION keyboard involves 41 same-finger fingerings per 1000 strokes (4 per cent), of which only 3 per 1000 are hurdles. These figures appear in the comparative summary which is Table VI. Analyses of one-handed fingerings per

1000 strokes are in the lower left corner of Appendices 2, 3, 4, and 5; details per 510,000 strokes are in the motion code columns of Appendix 7.

As suggested by Table VII, some of the residual same-finger fingerings of the MINIMOTION keyboard have sufficient usage to be annoying. However some such residuals are inevitable except by making other more serious sacrifices. Finally an auxiliary study indicates that a typewriter keyboard can hardly reduce same-finger fingerings below about 3 per cent no matter what finger loadings are tolerated.

TABLE VII. *Residual same-finger fingerings of the MINIMOTION keyboard*

Adjacency	Motion Code	Usage per 510,000		Usage per 1000
Ho + oH	55S	2,493		5
Yo + oY	35S	1,968		4
DS + SD	46S	1,352		3
BL + LB	68S	1,092		2
RC + CR	46S	979		2
NW + WN	46S	884		2
ue + eu	35S	826		2
,a + a,	35S	796		1
GT + TG	46S	697		1
TP + PT	68S	656		1
10 above.................. ...		11,743		23
Others........		8,948		18
Total		20,691	=	41/1000 = 4 per cent

A guide to equitable loading of individual fingers in recognition of their relative abilities is afforded by experimental findings of Hoke (6) and Riemer (12). Their finger abilities are shown in Table VIII,

TABLE VIII. *Finger abilities and finger loads, per 1000 strokes*

Finger designations	Total Left	L4	L3	L2	L1	R1	R2	R3	R4	Total Right
Hoke-Riemer abilities	475	104	112	126	133	140	139	128	118	525
QWERT YUIOP	561	85	80	182	214	196	88	127	28	439
RANDOM	500	100	100	100	200	200	100	100	100	500
DVORAK-DEALEY	447	83	86	130	148	171	138	138	106	553
MINIMOTION	484	84	92	149	159	192	127	107	90	516

averaged and adjusted to total 1000 for eight fingers, in comparison to the finger loadings of several typewriter keyboards. In relation to the Hoke-Riemer abilities the QWERT YUIOP keyboard overloads the inner fingers of the left hand; however the actual keyboards generally prefer to underload the outer fingers of each hand and let the stronger inner fingers carry the resulting overloads. Comparison of Appendices 4 and 5 shows that both the DVORAK-DEALEY and MINIMOTION keyboards assign such high-usage letters as *S* and *R* to the outer fingers of the right hand in order to attain desirable fingerings.

6. Letter Spacing and Line Spacing

Horizontal spaces between letters, to separate words, are inserted by operating the long space bar at the bottom of the keyboard. The space bar can be operated by either thumb, but assignment of the responsibility to one thumb or the other is desirable to avoid hesitancy. For the MINIMOTION keyboard there is a slight choice in favor of the *left thumb* because an auxiliary study (using statistical details not reproduced here) shows that more words begin and end on the right side of the keyboard than on the left, hence the left hand is less likely to be busy between words than the right hand. Incidentally, the auxiliary study showed that the DVORAK–DEALEY keyboard also should insert spaces with the left thumb, but the QWERT YUIOP keyboard should use the right thumb. These thumb assignments are indicated on the keyboard diagrams of Appendices 2, 4, and 5.

Vertical spaces between lines are inserted incidentally to returning the carriage, usually by a long lever projecting toward the operator from the left end of the carriage. Eventually typewriters will normally be fully motorized, and a suitable keyboard key will control line spacing and carriage return. Accordingly the MINIMOTION keyboard shortens the letter space bar for operation by the left thumb and provides a key convenient to the right thumb labeled *CAR.* and having a "double touch:" when depressed lightly it inserts a line space of pre-set width; when depressed firmly it also returns the carriage. These facilities are shown on the keyboard diagram of Appendix 5.

11. RESULTS OF KEYBOARD ANALYSES

Keyboard diagrams and the results of complete analyses of strokes and motions per 1000 strokes are given for four keyboards in Appendices 2 and 5. These Appendices also show some of the major indices rounded to the nearest whole per cent. Such *percentile ratings* are assembled in Table IX for ready comparison of keyboard performances.

TABLE IX. *Percentile ratings of four typewriter keyboards when writing average English*

	QWERT YUIOP	RANDOM	DVORAK-DEALEY	MINI-MOTION
Strokes by the Right Hand	44	50	55	52
Strokes in Home Row	32	33	67	71
Two-handed motions				
without reaches (Easiest) with reaches (Easy)	$\left.\begin{matrix}4\\48\end{matrix}\right\}52$	$\left.\begin{matrix}6\\44\end{matrix}\right\}50$	$\left.\begin{matrix}29\\36\end{matrix}\right\}65$	$\left.\begin{matrix}34\\33\end{matrix}\right\}67$
One-handed motions				
without hurdles (Hard) with hurdles (Hardest)	$\left.\begin{matrix}37\\11\end{matrix}\right\}48$	$\left.\begin{matrix}39\\11\end{matrix}\right\}50$	$\left.\begin{matrix}32\\3\end{matrix}\right\}35$	$\left.\begin{matrix}32\\1\end{matrix}\right\}33$
Same-finger fingerings	7	11	4	4

Comparison of these percentile ratings for the strokes and motions of the several keyboards supports the claim that the MINIMOTION keyboard affords the greatest facility for writing average English. This leads to the corollary claim that the MINIMOTION keyboard, being least tiring to the operator, will yield the highest attainable all-day efficiency.

Contrasting with the words used in Part 3 to illustrate the defects of the QWERT YUIOP keyboard, the following sentences have been devised to emphasize the merits of the MINIMOTION keyboard:

1. Done entirely by two-handed motions in Home Row—
   A tenor is a resonator on a rare note.
2. Done entirely by two-handed motions—
   A duck loves a watery locality for a family life.
3. Done entirely in Home Row—
   The senator sits on his leather seat in the national interest.

12. KEYBOARD PERFORMANCE RATINGS

Although similar percentile ratings are properly comparable as among the several keyboards, there is no evident way to combine these various percentile ratings significantly into a single all-inclusive performance rating for each keyboard. This is because the gradings of motions by difficulty are merely relative, and no satisfactory measure of absolute difficulty is available. Ideally, each keyboard should have a single rating, and a superiority of rating should be translatable into savings of time and money.

However, consideration of a keyboard performance rating immediately leads beyond the scope of this paper to a broader need. Operating a typewriter is a complicated human activity that baffles analysis by any ordinary methods. The keyboard is just one of several factors that act and react to influence typewriting performance: the operator, the material, the surroundings, the keyboard, and the machine. Further, all these factors are unknown variables, as illustrated by this extreme example:

> If an official typewriting speed record is set by an operator using a QWERT YUIOP keyboard on a motorized typewriter, who can say how much the performance is due to the skill of the operator, and how much to the feather touch and lightning response of the motorized machine? And who will attempt to estimate how much better the record might have been if the operator had devoted equal effort to developing skill on a MINIMOTION keyboard?

Nevertheless, all the typewriting factors can be evaluated conclusively by systematic collection of many thousands of observations on hundreds of typewriters under actual service conditions. The obser-

vational method of establishing ratings is slow and costly, but the validity of the resulting statistical indications is beyond question. In this connection, the Bell Telephone System has found that the observational method affords the only satisfactory basis for rating the transmission performance of telephones under actual conditions of use (14). Nothing remotely comparable to the telephone observations has been collected for the typewriter. As the telephone and the typewriter share the principal responsibility for commercial communications, similar methods of rating their service performance seem appropriate. The cost of a project to establish sound ratings for typewriter performance will be substantial, but small in comparison to the possible future savings.

The expert operator is characterized by a pace that slows momentarily to accommodate hard motions and speeds through easy motions like an automobile picking the way along a poor road at 10 m.p.h. on the rough spots and 30 m.p.h. on the smooth stretches. For example, chronagraphic investigation by Lahy showed that one-handed motions consumed on the average 73 per cent more time than two-handed motions; investigation by another on operators averaging 130 words per minute found the hardest motions being done at an instantaneous rate of 70 words per minute, while the easiest attained an instantaneous rate exceeding 200 words per minute. Hence a typewriter keyboard that reduces the proportion of hard motions and correspondingly increases the proportion of easy motions inherently produces a greater average speed and output for the same skill and effort; conversely, it requires less time and effort for a fixed output. The data from Lahy and another, plus a few reasonable assumptions, are sufficient to assign an individual speed to each coded motion which immediately enables computation of weighted and relative speeds and times inherent in the several keyboards. The results of an auxiliary study on this plan are as follows:

Performance ratings

Per Cent Ratings of Typewriter Keyboards

For Expert Operators of Equal Skill and Proficiency	Inherent Relative Speed (Fixed Effort)	Inherent Relative Time (Fixed Output)
QWERT YUIOP	101	99
RANDOM	100	100
DVORAK–DEALEY	111	90
MINIMOTION	112	89

13. THE MINIMOTION KEYBOARD

Obviously the cost of manufacturing typewriters is independent of the keyboard arrangement, so the MINIMOTION Keyboard involves no ultimate cost. Conversion of machines having QWERT YUIOP keyboards as part of major reconditioning, would make machines having MINIMOTION Keyboards available on trade-in transactions.

The MINIMOTION Keyboard is not only the easiest to use, but

it should be the easiest to learn. At any rate, the DVORAK–DEALEY Keyboard has amply demonstrated ease of learning (18). Most convincing is the report of tests in 1939 at the University of Chicago School of elementary education, where a group of young beginners using DVORAK–DEALEY keyboards developed proficiency more than twice as fast as a control group using QWERT YUIOP keyboards (18).

Operators already proficient on the QWERT YUIOP keyboard will find learning the MINIMOTION keyboard easier than might be expected. Again the best evidence has been collected by the DVORAK–DEALEY keyboard. The indications are that changing to a MINIMOTION keyboard will reduce the output of a commercial operator not longer than two weeks; thereafter the advantage will continue indefinitely.

So introduction of the minimotion keyboard is no problem at all for new typewriters and new operators, and only a minor problem during conversion. Thereafter the advantage of the MINIMOTION keyboard will continue as long as there is no major change in the usage and spelling of English words.

14. FUTURE OF THE MINIMOTION TYPEWRITER KEYBOARD

As the MINIMOTION keyboard is not now in use, its general acceptance is entirely an expectation for the future. Acceptance will be of three kinds: Scientific, popular, and commercial.

Scientific acceptance is based entirely on excellence of design to meet a need. This paper submits the design of the MINIMOTION keyboard for scientific examination.

Popular acceptance is often determined by prejudice. Generally people will accept a cautious succession of minor improvements but may set up a stubborn wall of prejudice against a major innovation, particularly if it affects a familiar thing and has the taint of reform.

> An extreme example of popular resistance to reform is the experience of the proposals for "Simplified Spelling." Notwithstanding the support of eminent philologists and educators on both sides of the Atlantic (9), these proposals have made little progress in the last 50 years. Accordingly, we in the United States still prefer the spelling *neighbor*, the British continue to insist on *neighbour*, and we both consider *nabor* quite indecent. (Incidentally, as stated in Part 6, the MINIMOTION keyboard is designed to write the "customary USA spelling" represented by *neighbor*.)

The MINIMOTION keyboard is a major innovation that can hardly escape being classed as a reform, yet it does have one important advantage over Simplified Spelling: It affects only the writer, not the reader—using an efficient keyboard does not at all change the appearance of the final typewritten words.

Commercial acceptance is based ultimately on demonstrated savings of time and money, though prejudice is sometimes an initial factor. Convincing demonstration of such savings requires the firm establishment of keyboard performance ratings. Finally, only advertising and salesmanship can bring the obtainable savings effectively to the attention of those who pay for typewriting service. In the long run, the MINIMOTION keyboard will benefit not only those who buy typewriting service but all who have any part in furnishing it.

Whatever the future, the author is gratified to have been able to complete this development that has such great possibilities of reducing the aggregate of typewriting effort.

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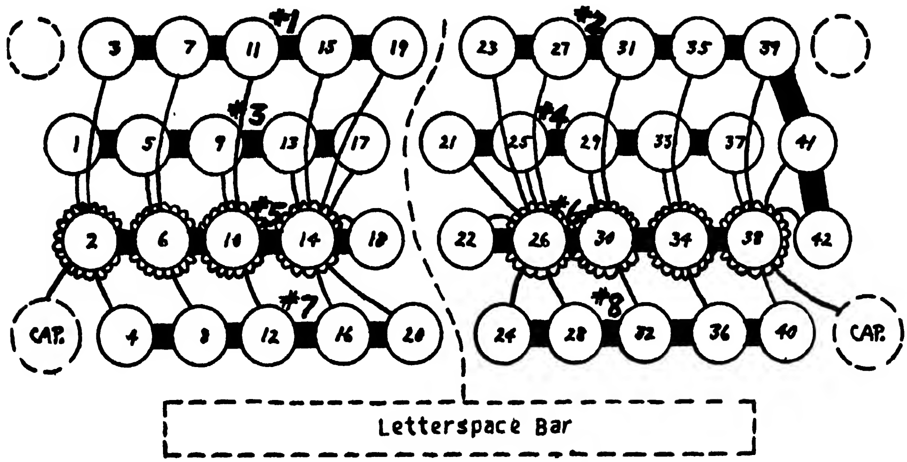
APPENDIX 1. TERMINOLOGY OF KEY GROUPS AND MOTION CODES

Composition and numbering of Key Groups #1 to #8, also key numbers corresponding to typebar numbers 1 to 42:



LEFT Key Groups are odd numbered; RIGHT Key Groups are even numbered. "HOME ROW" is Key Groups #5 and #6 (key 42 being in Key Group #2); "Reference keys" are scalloped and emanating curved lines show fingering, with the index fingers in "home position" on keys 14 and 26.

Key Group plan:

#1	#2
#3	#4
#5	#6
#7	#8

Number of possible different motions:
among 42 keys = $\Sigma 42 = (42)(43)/2 = 903$;
among 8 Key Groups = $\Sigma 8 = (8)(9)/2 = 36$,
as classified below by motion codes formed from terminal Key Group numbers.

CLASSIFICATION OF MOTIONS
by use of hands, by motion codes:

1-handed motions Left	1-handed motions Right	2-handed motions	
11	22	12	
13	24	14	23
H15	H26	16	25
H17	H28	18	27
33	44	34	
35	46	36	45
H37	H48	38	47
55	66	→56	
57	68	58	67
77	88	78	

The antecedent H marks the Hurdles.

Subsequent code letters to show the FINGERING of 1-handed motions:
- K = same Key (repeat stroke)
- S = Same finger (different keys)
- A = Adjacent fingers
- R = Remote fingers

CLASSIFICATION OF MOTIONS
by relative difficulty, by motion codes:

EASIEST–
2-handed without reaches
→56 only

EASY–
2-handed with reaches
12 14 16 18 23 25 27
34 36 38 45 47 58 67 78

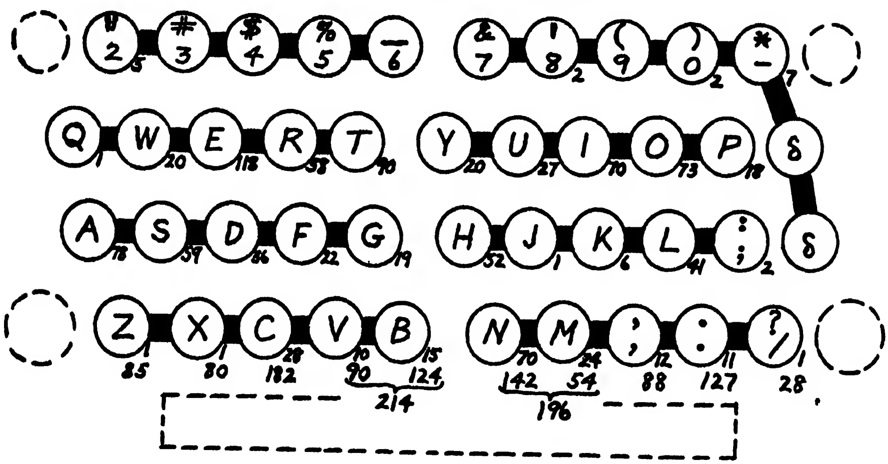
HARD–
1-handed without hurdles
11 13 33 35 55 57 77
22 24 44 46 66 68 88

HARDEST–
1-handed with hurdles
H15 H17 H37
H26 H28 H48

APPENDIX 2. THE *QWERT YUIOP* TYPEWRITER KEYBOARD—1872

Keyboard Diagram and Statistical Distribution of Strokes and Motions

Distribution of strokes by keys and fingers, per 1000 strokes



DISTRIBUTION OF STROKES by Key Groups:

Key Group Plan-

#1	#2
#3	#4
#5	#6
#7	#8

Per 1000 strokes-

5	11	16
287	208	495
214	102	316
55	118	173
561	439	1000

% of strokes-

1	1	2%
28	21	49%
22	10	32%
5	12	17%
56%	44%	100%

DISTRIBUTION OF MOTIONS
by motion codes, per 1000 strokes:

1-handed motions		2-handed motions	
11= 0	22= 0	12= 0	34=109
13= 3	24= 4	14= 2	36= 85
33= 75	44= 34	16= 1	38= 61
35=123	46= 42	18= 2	45= 88
55= 38	66= 7	23= 7	47= 32
57= 21	68= 9	25= 6	58= 70
77= 1	88= 7	27= 1	67= 10
H15= 3	H26= 2		78= 8
H17= 0	H28= 2	56= 41	
H37= 35	H48= 71		
299	178		523

299 + 523/2 = 561 Left strokes
178 + 523/2 = 439 Right strokes
477 + 523 =1000 Total strokes

CLASSIFICATION OF MOTIONS
by relative difficulty,
per 1000 strokes and %:

2-handed motions without reaches (EASIEST) . . .	41=	4%
2-handed motions with reaches (EASY).	482=	48%
1-handed motions without hurdles (HARD).	364=	37%
1-handed motions with hurdles (HARDEST) . . .	113=	11%
Total motions=	1000=	100%

FINGERING of 1-handed motions
per 1000 strokes:

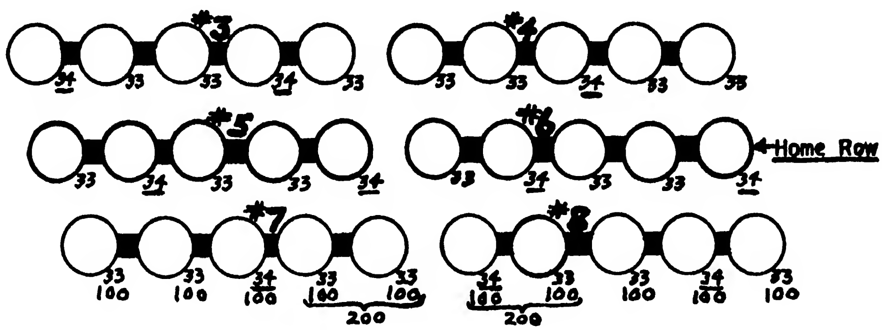
Same key	30	-	30	
Same finger	47	26	73=	7%
Adjacent fingers	150	54	204	
Remote fingers	137	33	170	
Total 1-handed	364	113	477=	48%

APPENDIX 3. THE RANDOM TYPEWRITER KEYBOARD—1942

Keyboard Diagram and Statistical Distributions of Strokes and Motions

Strokes are assumed to fall at random on the 30 keys in 6 Key Groups, hence 1000 strokes and motions distribute as follows: Strokes per key = 1000/30 = 33 + 1/3 = 33 or 34 (to avoid fractions); Strokes per Key Group = 1000/6 = 166 + 2/3 = 167 or 166; Number of different motions among 30 keys = Σ30 = (30) (31)/2 = 465; Number of different motions among 6 Key Groups = Σ6 = (6) (7)/2 = 21. Motions within any one Key Group = 1000/6/6 = 27 + 7/9 = 28 or 27; Motions between any two Key Groups = 1000/3/6 = 55 + 5/9 = 56 or 55.

Distribution of Strokes by keys and fingers, per 1000 strokes



DISTRIBUTION OF STROKES by Key Groups:

Key Group Plan—

#3	#4
#5	#6
#7	#8

Per 1000 strokes—

167	166	333
167	167	334
166	167	333
500	500	1000

% of strokes—

17	16	33%
17	17	34%
16	17	33%
50%	50%	100%

DISTRIBUTION OF MOTIONS by motion codes, per 1000 strokes:

1-handed motions		2-handed motions	
33= 28	44= 28		34= 55
55= 28	66= 28		36= 56
77= 27	88= 27		38= 55
35= 56	46= 56		45= 56
57= 56	68= 55		47= 55
H37= 55	H48= 56	56= 56	58= 56
			67= 55
			78= 56
250	250		500

250 + 500/2 = 500 Left strokes
250 + 500/2 = 500 Right strokes
500 + 500 =1000 Total strokes

FINGERING of 1-handed motions per 1000 strokes:

Same key	30*	—	30*
Same finger	77	31	108= 11%
Adjacent fingers	125	35	160
Remote fingers	157	45	202
Total 1-handed	389	111	500= 50%

CLASSIFICATION OF MOTIONS by relative difficulty, per 1000 strokes and %:

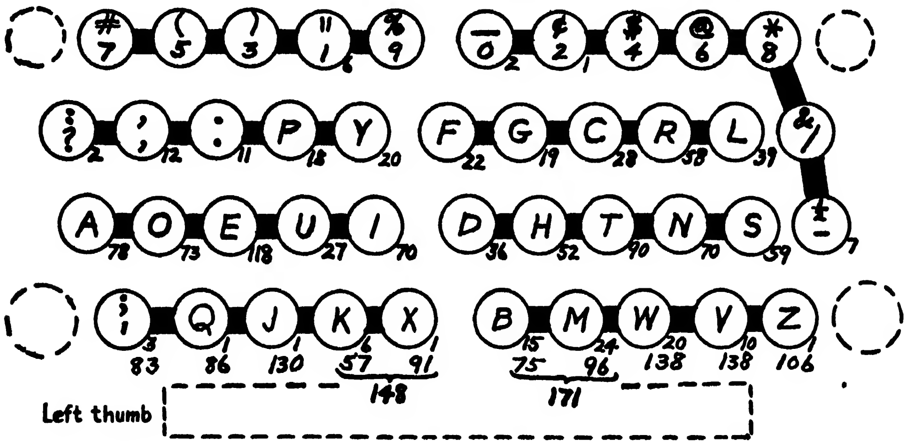
2-handed motions without reaches (EASIEST) . . .	56=	6%
2-handed motions with reaches (EASY).	444=	44%
1-handed motions without hurdles (HARD).	389=	39%
1-handed motions with hurdles (HARDEST) . . .	111=	11%
Total motions=	1000=	100%

*adjusted for average English

APPENDIX 4. THE DVORAK-DEALEY* "SIMPLIFIED" TYPEWRITER KEYBOARD—1932

Keyboard Diagram and Statistical Distributions of Strokes and Motions

Distribution of strokes by keys and fingers, per 1000 strokes



DISTRIBUTION OF STROKES by Key Groups:

Key Group plan—

#1	#2
#3	#4
#5	#6
#7	#8

Per 1000 strokes—

6	10	16
63	166	229
366	307	673
12	70	82
447	553	1000

% of strokes—

1	1	2%
6	17	23%
37	30	67%
1	7	8%
45%	55%	100%

DISTRIBUTION OF MOTIONS by motion codes, per 1000 strokes:

1-handed motions		2-handed motions	
11= 0	22= 0	12= 0	34= 23
13= 2	24= 3	14= 2	36= 33
33= 5	44= 21	16= 4	38= 11
35= 44	46= 63	18= 1	45=186
55= 53	66= 95	23= 1	47= 4
57= 12	68= 21	25= 6	58= 86
77= 0	88= 2	27= 0	67= 7
H15= 3	H26= 7		78= 1
H17= 0	H28= 2	56=288	
H37= 2	H48= 12		
121	226.		653

121 + 653/2 = 447 Left strokes

226 + 653/2 = 553 Right strokes

347 + 653 =1000 Total strokes

FINGERING of 1-handed motions per 1000 strokes:

Same key	30	–	30	
Same finger	33	6	39=	4%
Adjacent fingers	133	6	139	
Remote fingers	125	14	139	
Total 1-handed	321	26	347=	35%

CLASSIFICATION OF MOTIONS by relative difficulty, per 1000 strokes and %:

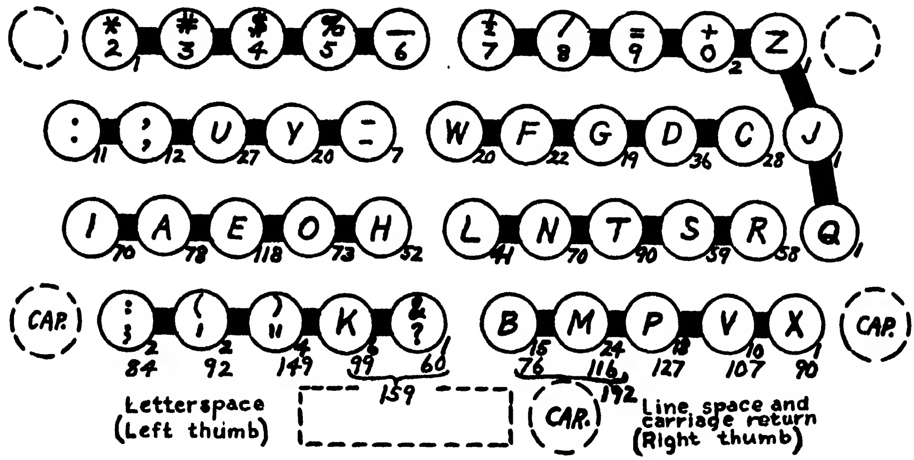
2-handed motions without reaches (EASIEST) . . .	288=	29%
2-handed motions with reaches (EASY).	365=	36%
1-handed motions without hurdles (HARD)	321=	32%
1-handed motions with hurdles (HARDEST) . . .	26=	3%
Total motions=	1000=	100%

* U. S. Patent 2,040,248, issued in 1936.

APPENDIX 5. THE MINIMOTION TYPEWRITER KEYBOARD—1941

Keyboard Diagram and Statistical Distributions of Strokes and Motions

Distribution of strokes by keys and fingers, per 1000 strokes



DISTRIBUTION OF STROKES by Key Groups:

Key Group plan—

#1	#2
#3	#4
#5	#6
#7	#8

Per 1000 strokes—

1	5	6
77	125	202
391	318	709
15	68	83
484	516	1000

% of strokes—

0	1	1%
8	12	20%
39	32	71%
1	7	8%
48%	52%	100%

DISTRIBUTION OF MOTIONS by motion codes, per 1000 strokes:

1-handed motions		2-handed motions	
11= 0	22= 0	12= 0	34= 22
13= 0	24= 1	14= 0	36= 59
33= 4	44= 8	16= 1	38= 15
35= 46	46= 68	18= 0	45=132
55= 87	66= 68	23= 2	47= 5
57= 11	68= 25	25= 4	58= 78
77= 0	88= 5	27= 0	67= 11
H15= 1	H26= 2		78= 2
H17= 0	H28= 0	56=335	
H37= 2	H48= 6		
151	183		666

151 + 666/2 = 484 Left strokes

183 + 666/2 = 516 Right strokes

334 + 666 =1000 Total strokes

FINGERING of 1-handed motions per 1000 strokes:

Same key	30	—	30	
Same finger	38	3	41=	4%
Adjacent fingers	138	4	142	
Remote fingers	117	4	121	
Total 1-handed	323	11	334=	33%

CLASSIFICATION OF MOTIONS by relative difficulty, per 1000 strokes and %:

2-handed motions without reaches (EASIEST) . . .	335=	34%
2-handed motions with reaches (EASY).	331=	33%
1-handed motions without hurdles (HARD).	323=	32%
1-handed motions with hurdles (HARDEST) . . .	11=	1%
Total motions=	1000=	100%

APPENDIX 6. STATISTICAL USAGE OF 37 SINGLE STROKES IN AVERAGE ENGLISH

Single strokes	Usages per 510,000	Usages per 1000	QWERT YUIOP	DVORAK–DEALEY	MINIMOTION
			Typewriter keyboards and Key Group numbers		
e	59,888	118	#3	#5	#5
T	46,095	90	3	6	6
a	39,819	78	5	5	5
o	37,366	73	4	5	5
N	35,718	70	8	6	6
i	35,679	70	4	5	5
S	29,980	59	5	6	6
R	29,809	58	3	4	6
H	26,514	52	6	6	5
L	19,788	39	6	4	6
D	18,116	36	#5	#6	#4
C	14,359	28	7	4	4
u	13,890	27	4	5	3
M	12,190	24	8	8	8
F	11,476	22	5	4	4
Y	9,969	20	4	3	3
W	9,947	20	3	8	4
G	9,741	19	5	4	4
P	9,143	18	4	3	8
B	7,448	15	7	8	8
,	6,000	12	#8	#3	#3
.	5,900	11	8	3	3
V	5,053	10	7	8	8
–	3,600	7	2	2	3
K	3,237	6	6	7	7
"	2,200	4	1	1	7
'	1,136	2	2	7	7
0# (zero)	1,000	2	2	2	2
1# (one)	1,000	2	6	1	6
X	798	1	7	7	8
J	651	1	#6	#7	#2
2# (two)	500	1	1	2	1
;	500	1	6	7	7
Q	421	1	3	7	2
?	400	1	8	3	7
Z	369	1	7	8	2
:	300	1	6	3	7
1–37	510,000	1000			

Strokes by Key Groups:

	QWERT YUIOP per 510,000	QWERT YUIOP per 1000	DVORAK–DEALEY per 510,000	DVORAK–DEALEY per 1000	MINIMOTION per 510,000	MINIMOTION per 1000
#1	2,700	5	3,200	6	500	1
#3	146,160	287	31,712	63	39,359	77
#5	109,132	214	186,642	366	199,266	391
#7	28,027	55	6,743	12	7,773	15
Left	286,019	561	228,297	447	246,898	484
#2	5,736	11	5,100	10	2,441	5
#4	106,047	208	85,173	166	63,639	125
#6	51,990	102	156,423	307	162,390	318
#8	60,208	118	35,007	70	34,632	68
Right	223,981	439	281,703	553	263,102	516
Total	510,000	1000	510,000	1000	510,000	1000

APPENDIX 7. STATISTICAL USAGE OF 544 ADJACENCIES IN AVERAGE ENGLISH

Adjacencies	Usages per 510,000	QW*	DD**	MM***	Adjacencies	Usages per 510,000	QW*	DD**	MM***
TH + HT	15,952	36	66A	*56*	ai + ia	2,712	45	55R	55A
eR + Re	14,603	33A	45	*56*	oW + Wo	2,706	34	58	45
He + eH	13,135	36	*56*	55A	Lo + oL	2,567	46S	45	*56*
iN + Ni	11,052	H48A	*56*	*56*	Mi + iM	2,559	H48A	58	58
aN + Na	10,000	58	*56*	*56*	NS + SN	2,551	58	66A	66R
iT + Ti	9,428	34	*56*	*56*	uT + Tu	2,518	34	*56*	36
aT + Ta	9,144	35R	*56*	*56*	Ho + oH	2,493	46R	*56*	55S
eN + Ne	9,109	38	*56*	*56*	Wa + aW	2,468	35A	58	45
eT + Te	9,073	33A	*56*	*56*	TT	2,329	33K	66K	66K
oN + No	8,918	H48R	*56*	*56*	uN + Nu	2,315	H48S	*56*	36
eS + Se	8,531	35A	*56*	*56*	Ge + eG	2,284	35A	45	45
oR + Ro	7,805	34	45	*56*	SS	2,257	55K	66K	66K
aR + Ra	7,483	35R	45	*56*	Fi + iF	2,226	45	45	45
To + oT	7,144	34	*56*	*56*	Po + oP	2,222	44A	35R	58
eD + De	7,098	35S	*56*	45	cc	2,215	33K	55K	55K
ST + TS	7,057	35R	66R	66A	Do + oD	2,203	45	*56*	45
oF + Fo	6,747	45	45	45	RS + SR	2,196	35R	46A	66A
iS + Si	6,735	45	*56*	*56*	CH + HC	2,174	67	46A	45
ND + DN	6,214	58	66R	46R	TY + YT	2,107	34	36	36
aS + Sa	5,846	55A	*56*	*56*	eF + Fe	2,078	35A	45	45
Le + eL	5,782	36	45	*56*	SH + HS	2,050	*56*	66R	*56*
Me + eM	5,463	38	58	58	aY + Ya	1,986	45	35R	35R
aL + La	5,427	*56*	45	*56*	Yo + oY	1,968	44R	35R	35S
NT + TN	4,919	38	66A	66A	WH + HW	1,958	36	68A	45
ea + ae	4,881	35R	55R	55A	Pa + aP	1,937	45	35R	58
Ha + aH	4,770	*56*	*56*	55R	eo + oe	1,889	34	55A	55A
ou + uo	4,737	44R	55R	35A	LY + YL	1,879	46R	34	36
Ve + eV	4,572	H37A	58	58	iG + Gi	1,833	45	45	45
Ce + eC	4,509	H37S	45	45	Wi + iW	1,831	34	58	45
Li + iL	4,501	46A	45	*56*	iV + Vi	1,803	47	58	58
NG + GN	4,441	58	46A	46A	uL + Lu	1,762	46R	45	36
Ri + iR	4,376	34	45	*56*	CT + TC	1,737	H37A	46S	46R
RT + TR	4,000	33S	46A	66R	LD + DL	1,721	*56*	46R	46R
Ca + aC	3,792	57R	45	45	NC + CN	1,688	78	46A	46R
iC + Ci	3,706	47	45	45	PR + RP	1,681	34	34	68R
Hi + iH	3,642	46A	*56*	55R	aV + Va	1,650	57R	58	58
Co + oC	3,528	47	45	45	Ga + aG	1,594	55R	45	45
oM + Mo	3,514	H48R	58	58	LT + TL	1,587	36	46R	66A
Be + eB	3,349	H37A	58	58	DT + TD	1,561	35A	66A	46A
aD + Da	3,337	55R	*56*	45	eY + Ye	1,561	34	35A	35A
So + oS	3,331	45	*56*	*56*	.T + T.	1,552	38	36	36
eW + We	3,323	33A	58	45	,T + T,	1,538	38	36	36
Ma + aM	3,208	58	58	58	aB + Ba	1,526	57R	58	58
ei + ie	3,118	34	55A	55R	RD + DR	1,403	35A	46R	46A
Di + iD	3,085	45	*56*	45	Fa + aF	1,403	55R	45	45
uS + Su	3,056	45	*56*	36	RY + YR	1,367	34	34	36
io + oi	2,992	44A	55R	55R	DS + SD	1,352	55A	66R	46S
LL	2,871	66K	44K	66K	e. + .e	1,328	38	35S	35R
uR + Ru	2,751	34	45	36	e, + ,e	1,318	38	35A	35A
Pe + eP	2,734	34	35A	58	Bu + uB	1,314	47	58	38
					51–100	+96,957			
1–50	298,789 = 58.6%				1–100	395,746 = 77.6%			

* QWERT YUIOP
** DVORAK-DEALEY
*** MINIMOTION

APPENDIX 7. STATISTICAL USAGE OF 544 ADJACENCIES IN AVERAGE ENGLISH—Continued

Adjacencies	Usages per 510,000	Motion Codes QW*	DD**	MM***
GH + HG	1,306	56	46S	45
GR + RG	1,306	35S	44R	46R
FT + TF	1,300	35S	46A	46A
oo	1,276	44K	55K	55K
FR + RF	1,260	35S	44R	46R
Bo + oB	1,250	47	58	58
PL + LP	1,244	46A	34	68A
SP + PS	1,231	45	36	68A
Ke + eK	1,190	36	57A	57A
Go + oG	1,187	45	45	45
uC + Cu	1,095	47	45	34
BL + LB	1,092	67	H48R	68S
SC + CS	1,064	57A	46R	46A
uP + Pu	1,045	44R	35S	38
S. + .S	1,040	58	36	36
S, + ,S	1,030	58	36	36
YS + SY	1,011	45	36	36
TW + WT	1,006	33R	68S	46A
SW + WS	992	35S	68R	46R
RC + CR	979	H37A	44A	46S
RN + NR	979	38	46S	66R
au + ua	978	45	55R	35A
LS + SL	977	56	46S	66R
RM + MR	976	38	H48R	68R
uG + Gu	960	45	45	34
–T + T–	947	23	H26R	36
Pi + iP	942	44R	35S	58
Mu + uM	925	H48S	58	38
oa + ao	908	45	55A	55R
SM + MS	907	58	68R	68R
NW + WN	884	38	68A	46S
BY + YB	883	47	38	38
oV + Vo	827	47	58	58
ue + eu	826	34	55A	35S
e– + –e	810	23	25	35A
.a + a.	805	58	35R	35A
,a + a,	796	58	35A	35S
NY + YN	782	H48S	36	36
MP + PM	775	H48R	38	88A
BR + RB	766	H37S	H48R	68R
aK + Ka	757	56	57R	57R
MY + YM	753	H48S	38	38
D. + .D	720	58	36	34
D, + ,D	714	58	36	34
Du + uD	713	45	56	34
HR + RH	706	36	46R	56
CK + KC	703	67	47	47
TM + MT	698	38	68A	68A
GT + TG	697	35S	46A	46S
.o + o.	695	H48S	35A	35R
101–150	+47,713			
1–150	443,459 = 87.0%			

Adjacencies	Usages per 510,000	Motion Codes QW*	DD**	MM***
,o + o,	688	H48A	35S	35R
eX + Xe	681	H37A	57A	58
SF + FS	675	55R	46R	46R
TP + PT	656	34	36	68S
FF	650	55K	44K	44K
N. + .N	649	88R	36	36
KN + NK	646	68A	67	67
N, + ,N	644	88A	36	36
SB + BS	636	57R	68R	68R
S– + –S	634	25	H26S	36
RW + WR	633	33R	H48A	46R
Fu + uF	630	45	45	34
LF + FL	625	+56	44R	46S
CL + LC	623	67	44R	46R
RL + LR	621	36	44A	66R
iB + Bi	619	47	58	58
NL + LN	606	68R	46A	66S
GS + SG	603	55R	46R	46A
DW + WD	584	35A	68A	44R
NF + FN	583	58	46R	46S
Ki + iK	579	46S	57S	57R
Yi + iY	571	44A	35S	35R
NN	544	88K	66K	66K
RR	542	33K	44K	66K
ui + iu	532	44A	55S	35R
“T + T“	531	13R	16	67
KS + SK	530	+56	67	67
NH + HN	518	68S	66R	+56
NM + MN	511	88S	68R	68S
R. + .R	502	38	34	36
R, + ,R	500	38	34	36
.i + i.	499	H48A	35A	35S
YH + HY	498	46S	36	35S
TB + BT	495	H37S	68A	68A
–a + a–	495	25	25	35R
,i + i,	494	H48S	35R	35A
Y. + .Y	494	H48R	33A	33R
DM + MD	493	58	68S	H48R
Y, + ,Y	486	H48A	33R	33R
DY + YD	483	45	36	34
F. + .F	476	58	34	34
F, + ,F	471	58	34	34
RK + KR	468	36	47	67
PP	468	44K	33K	88K
‘T + T‘	459	23	67	67
DH + HD	451	+56	66S	45
e” + ”e	449	13R	H15A	57S
.W + W.	444	38	38	34
D– + –D	441	25	H26R	34
,W + W,	440	38	38	34
151–200	+27,550			
1–200	471,009 = 92.3%			

APPENDIX 7. STATISTICAL USAGE OF 544 ADJACENCIES IN AVERAGE ENGLISH—Continued

Adjacencies	Usages per 510,000	Motion Codes QW*	DD**	MM***
DF + FD	439	55A	46S	44R
MM	434	88K	88K	88K
.H + H.	434	68R	36	35R
,H + H,	431	68A	36	35R
-o + o-	426	24A	25	35S
YW + WY	424	34	38	34
Qu	421	34	57R	23
DC + CD	402	57S	46A	44A
DB	398	57A	68S	H48R
N- + -N	397	H28R	H26A	36
YC + CY	395	47	34	34
GL + LG	394	+56	44R	46A
oK + Ko	378	46A	57R	57S
YP + PY	372	44R	33S	38
NB + BN	371	78	68R	68S
DG + GD	371	55A	46S	44A
PH + HP	357	46R	36	58
S" + "S	357	H15A	16	67
LW + WL	355	36	H48R	46S
DD	350	55K	66K	44K
," + ",	350	18	13R	H37A
.M + M.	336	88R	38	38
,M + M,	333	88A	38	38
L. + .L	325	68S	34	36
MB + BM	323	78	88S	88S
L, + ,L	322	68A	34	36
-i + i-	307	24R	25	35R
YF + FY	306	45	34	34
R- + -R	306	23	24A	36
DP + PD	305	45	36	H48A
Y- + -Y	303	24R	23	33S
.C + C.	300	78	34	34
,C + C,	299	78	34	34
NP + PN	296	H48R	36	68A
F- + -F	293	25	24R	34
.B + B.	292	78	38	38
GY + YG	290	45	34	34
,B + B,	290	78	38	38
FW + WF	289	35R	H48A	44S
G. + .G	286	58	34	34
G, + ,G	284	58	34	34
FH + HF	283	+56	46S	45
N' + 'N	283	H28S	67	67
"a + a"	281	H15S	H15R	57A
Ju	277	46S	57A	23
-W + W-	276	23	H28R	34
RV	273	H37S	H48S	68A
CC	270	77K	44K	44K
I#T+ TI#	268	36	16	66A
Ze + eZ	267	H37A	58	25
201–250	+16,819			
1–250	487,828 = 95.7%			

Adjacencies	Usages per 510,000	Motion Codes QW*	DD**	MM***
Hu + uH	266	46S	+56	35A
-H + H-	264	H26R	H26R	35S
.P + P.	247	H48A	33A	38
,P + P,	246	H48R	33R	38
O#T+ TO#	246	23	H26A	H26A
D" + "D	245	H15R	16	47
LM + ML	240	68R	H48R	68S
"o + o"	240	14	H15R	57A
iZ + Zi	239	47	58	25
aa	235	55K	55K	55K
LV	226	67	H48A	68R
eI# + I#e	225	36	H15A	+56
'S + S'	219	25	67	67
FM + MF	218	58	H48S	H48S
N" + "N	218	18	16	67
Xi + iX	214	47	57S	58
XP	214	47	H37S	88R
NV	208	78	68S	68R
LH + HL	207	66R	46R	+56
-M + M-	206	H28R	H28R	38
eJ + Je	204	36	57S	25
eO# + O#e	204	23	25	25
L- + -L	198	H26A	24S	36
HM + MH	196	68S	68S	58
FC + CF	195	57A	44A	44R
Jo + oJ	194	46R	57A	25
KT + TK	189	36	67	67
e' + 'e	189	23	57R	57A
GW + WG	187	35R	H48A	44A
FB + BF	186	57S	H48S	H48S
-B + B-	182	27	H28R	38
-C + C-	181	27	24R	34
SI# + I#S	178	+56	16	66R
"I + I"	174	14	H15S	57R
G- + -G	174	25	24R	34
Za + aZ	172	57S	58	25
GF + FG	170	55S	44S	44A
R" + "R	169	13R	14	67
GM + MG	168	58	H48S	H48A
eQ	166	33R	57A	25
Y" + "Y	166	14	13S	H37A
F" + "F	164	H15R	14	47
"W + W"	164	13A	18	47
SO# + O#S	161	25	H26R	H26S
FP + PF	154	45	34	H48A
-P + P-	153	24S	23	38
"H + H"	149	16	16	57A
i' + 'i	138	24A	57R	57A
I#a + aI#	137	+56	H15R	+56
HB + BH	136	67	68S	58
251–300	+9,821			
1–300	497,649 = 97.6%			

APPENDIX 7. STATISTICAL USAGE OF 544 ADJACENCIES IN AVERAGE ENGLISH—Continued

Adjacencies		Usages per 510,000	Motion Codes QW*	DD**	MM***
2#T	+ T2#	136	13R	H26A	16
;T	+ T;	136	36	67	67
LK	+ KL	134	66A	47	67
O#a	+ aO#	129	25	25	25
GB		124	57S	H48S	H48A
XT		124	H37R	67	68R
DI#	+ I#D	123	+56	16	46R
GC		122	57A	44A	44R
Ja	+ aJ	122	+56	57R	25
o'	+ 'o	120	24R	57A	57R
I#o	+ oI#	119	46S	H15R	+56
DV		118	57A	68R	H48S
.u	+ u.	118	H48R	35A	33R
,u	+ u,	118	H48A	35R	33A
MW	+ WM	117	38	88A	H48S
"M	+ M"	117	18	18	78
e2#	+ 2#e	113	13R	25	H15R
e;	+ ;e	113	36	57R	57R
NI#	+ I#N	112	68R	16	66S
O#o	+ oO#	111	24S	25	25
DO#	+ O#D	111	25	H26S	24S
L"	+ "L	110	16	14	67
MC	+ CM	108	78	H48A	H48R
KW	+ WK	108	36	78	47
GG		108	55K	44K	44K
Yu	+ uY	107	44S	35S	33A
HH		106	66K	66K	55K
"C	+ C"	106	H17R	14	47
?T	+ T?	104	38	36	67
GP	+ PG	102	45	34	H48S
NO#	+ O#N	100	H28R	H26R	H26R
"B		98	H17R	18	78
Xa	+ aX	97	57A	57R	58
e?	+ ?e	97	38	35R	57A
G"	+ "G	96	H15R	14	47
XC		95	77A	47	H48S
'a	+ a'	93	25	57S	57S
D'	+ 'D	93	25	67	47
YY		92	44K	33K	33K
S2#	+ 2#S	92	H15A	H26R	16
S;	+ ;S	92	+56	67	67
K.	+ .K	90	68S	H37A	H37R
K,	+ ,K	90	68A	H37R	H37R
"P	+ P"	89	14	13S	78
RI#	+ I#R	86	36	14	66R
I#i	+ iI#	85	46A	H15S	+56
YI#	+ I#Y	85	46R	13S	36
FI#	+ I#F	83	+56	14	46S
:T	+ T:	81	36	36	67
O#i	+ iO#	80	24A	25	25

301–350 +5,310
1–350 502,959 = 98.6%

Adjacencies		Usages per 510,000	Motion Codes QW*	DD**	MM***
VY	+ YV	79	47	38	38
'L	+ L'	77	H26R	47	67
RO#	+ O#R	77	23	24R	H26A
YO#	+ O#Y	77	24R	23	23
I#W	+ WI#	76	36	18	46S
FO#	+ O#F	75	25	24S	24R
I#H	+ HI#	75	66R	16	+56
KF	+ FK	74	+56	47	47
NJ		74	68S	67	H26R
e:	+ :e	74	36	35R	57R
KY	+ YK	73	46A	H37S	H37S
–u	+ u–	73	24R	25	33A
R'	+ 'R	73	23	47	67
WC	+ CW	72	H37A	H48S	44R
S?	+ ?S	72	58	36	67
ii		71	44K	55K	55K
O#W	+ WO#	71	23	H28A	24R
'M	+ M'	70	H28S	78	78
BJ		69	67	78	H28R
PW	+ WP	68	34	38	H48A
O#H	+ HO#	68	H26R	H26S	25
DK	+ KD	67	+56	67	47
2#a	+ a2#	67	H15S	25	H15A
;a	+ a;	67	+56	57S	57A
KB		63	67	78	78
D2#	+ 2#D	63	H15R	H26S	14
D;	+ ;D	63	+56	67	47
SV		62	57R	68A	68S
K–	+ –K	60	H26R	27	H37S
SQ		59	35A	67	H26A
WW		59	33K	88K	44K
DJ		58	+56	67	24A
2#o	+ o2#	58	14	25	H15R
;o	+ o;	58	46A	57A	57R
I#M	+ MI#	58	68R	18	68S
N2#	+ 2#N	57	18	H26R	16
N;	+ ;N	57	68R	67	67
Y'	+ 'Y	57	24S	H37R	H37R
uW	+ Wu	56	34	58	34
LI#	+ I#L	56	66K	14	66K
'W	+ W'	56	23	78	47
WB	+ BW	55	H37R	88A	H48S
TV		55	H37S	68A	68A
?a	+ a?	55	58	35S	57R
F'	+ 'F	55	25	47	47
KH	+ HK	54	66A	67	57S
NQ		54	38	67	H26R
S:	+ :S	54	+56	36	67
'C		53	27	47	47
O#M	+ MO#	53	H28R	H28S	H28R

351–400 +3,227
1–400 506,186 = 99.3%

APPENDIX 7. STATISTICAL USAGE OF 544 ADJACENCIES IN AVERAGE ENGLISH—Continued

Adjacencies	Usages per 510,000	Motion Codes QW*	DD**	MM***
oX	51	47	57R	58
KM + MK	50	68A	78	78
BB	50	77K	88K	88K
?o + o?	50	H48A	35A	57S
D? + ?D	50	58	36	47
'H + H'	50	H26S	67	57R
LO# + O#L	50	H26S	24R	H26R
KG + GK	49	+56	47	47
I#B	49	67	18	68S
GI# + I#G	49	+56	14	46A
I#C	48	67	14	46R
I#P + PI#	46	46A	13S	68A
2#i + i2#	46	14	25	H15S
Y2# + 2#Y	46	14	23	13R
;i + i;	46	46R	57R	57S
Y; + ;Y	46	46R	H37R	H37R
N? + ?N	46	88R	36	67
SJ	45	+56	67	H26A
:a + a:	45	+56	35S	57A
" u + u"	44	14	H15S	H37S
O#B	44	27	H28S	H28R
O#C	44	27	24A	24A
GO# + O#G	44	25	24S	24A
R2# + 2#R	44	13R	24R	16
R; + ;R	44	36	47	67
'B	43	27	78	78
F2# + 2#F	43	H15R	24S	14
F; + ;F	43	+56	47	47
O#P + PO#	43	24A	23	H28A
TJ	41	36	67	H26R
XH	40	67	67	58
K" + "K	40	16	H17S	77A
.O# + O#.	40	H28S	23	23
O#O#	40	22K	22K	22K
Y? + ?Y	39	H48R	33R	H37S
.V	38	78	38	38
,V	38	78	38	38
D: + :D	38	+56	36	47
PB	37	47	38	88A
2#H + H2#	37	16	H26S	H15R
;H + H;	37	66R	67	57R
PC + CP	36	47	34	H48R
R? + ?R	36	38	34	67
:o + o:	36	46A	35A	57R
N: + :N	36	68R	36	67
2#W	34	13A	H28A	14
;W	34	36	78	47
G' + 'G	33	25	47	47
2#M + M2#	33	18	H28S	18
;M + M;	33	68R	78	78
401–450	+2,124			
1–450	508,310 = 99.7%			

Adjacencies	Usages per 510,000	Motion Codes QW*	DD**	MM***
KP	32	46R	H37S	78
F? + ?F	32	58	34	47
u' + 'u	31	24S	57R	H37A
TZ + ZT	30	H37R	68R	H26R
?i	30	H48R	35R	57R
?H + H?	30	68R	36	57S
.J	30	68R	H37S	23
,J	30	68A	H37A	23
,O# + O#,	30	H28A	23	23
L2# + 2#L	29	16	24R	16
L; + ;L	29	66A	47	67
R: + :R	29	36	34	67
YJ	28	46S	H37A	23
?M + M?	27	88R	38	78
RJ	26	36	47	H26S
G2# + 2#G	26	H15R	24S	14
G; + ;G	26	+56	47	47
F: + :F	25	+56	34	47
:H + H:	25	66R	36	57R
'P	25	24R	H37R	78
'V	25	27	78	78
?W	25	38	38	47
CB + BC	24	77A	H48A	H48R
2#B	24	H17R	H28S	18
2#C	24	H17R	24A	14
;B	24	67	78	78
;C	24	67	47	47
L? + ?L	24	68A	34	67
:M + M:	24	68R	38	78
–V	24	27	H28A	38
FV	23	57S	H48R	H48R
I#u + uI#	22	46R	H15S	36
:i	22	46R	35R	57S
G? + ?G	22	58	34	47
Y:	21	46R	33R	H37R
CQ	20	H37R	47	24S
NX	20	78	67	68R
uX	20	47	57S	38
–J	20	H26R	27	23
KO# + O#K	20	H26A	27	27
uO# + O#u	20	24R	25	23
L: + :L	20	66A	34	67
:W	20	36	38	47
.'	20	H28R	H37R	H37A
?B	19	78	38	78
FJ	18	+56	47	24R
2#P	18	14	23	18
;P	18	46S	H37R	78
?C	18	78	34	47
GV	16	57S	H48R	H48R
451–500	+1,209			
1–500	509,519 = 99.9%			

APPENDIX 7. STATISTICAL USAGE OF 544 ADJACENCIES IN AVERAGE ENGLISH—Continued

Adja- cencies	Usages per 51,0000	QW* QW*	Motion Codes DD**	MM***
TQ	16	33R	67	H26R
DQ	15	35R	67	24A
:B	15	67	38	78
HV	14	67	68R	58
LJ	14	66R	47	H26R
:C	14	67	34	47
?P	14	H48S	33R	78
"V	13	H17R	18	78
GJ	12	+56	47	24R
O#V	12	27	H28R	H28S
YQ	11	34	H37R	23
:P	11	46S	33R	78
FQ	10	35R	47	24R
HJ	10	66S	67	25
Ku	10	46A	57S	H37A
LQ	10	36	47	H26R
MJ	10	68S	78	H28R
MV	10	78	88R	88R
NZ	10	78	68A	H26R
oQ	10	34	57S	25
RQ	10	33R	47	H26S
SZ	10	57A	68S	H26A
uu	10	44K	55K	33K
WV	10	H37R	88A	H48R
XS	10	57S	67	68A
XW	10	H37S	78	H48R
Zo	10	47	58	25
G:	10	+56	34	47
K'	10	H26A	77R	77R
KI#	10	66A	H17S	67
X.	10	78	H37A	38
X,	10	78	H37R	38
.Q	10	38	H37A	23
,Q	10	38	H37S	23
-Q	10	23	27	23
"Q	10	13S	H17R	27
"J	10	16	H17A	27
O#J	10	H26R	27	22A
I#V	10	67	18	68R
2#u	10	14	25	13R
;u	10	46R	57R	H37R
..	10	88K	33K	33K
--	10	22K	22K	33K
?"	10	18	13R	77A
501–544	+481			
1–544	510,000 = 100.0%			

Summary by Motion Codes

Motions by motion codes:	QWERT YUIOP per 510,000	QWERT YUIOP per 1000	DVORAK-DEALEY per 510,000	DVORAK-DEALEY per 1000	MINIMOTION per 510,000	MINIMOTION per 1000
11	0	0	0	0	0	0
13	1,650	3	746	2	56	0
H15	1,434	3	1,776	3	321	1
H17	265	0	70	0	0	0
33	37,975	75	2,500	5	1,821	4
35	62,883	123	22,341	44	23,713	46
H37	18,004	35	804	2	1,107	2
55	19,390	38	27,380	53	44,443	87
57	10,603	21	6,236	12	5,626	11
77	439	1	10	0	60	0
Left-handed motions	152,643	299	61,863	121	77,147	151
22	50	0	50	0	50	0
24	1,964	4	1,608	3	478	1
H26	827	2	3,754	7	1,033	2
H28	1,199	2	959	2	231	0
44	17,405	34	10,736	21	4,268	8
46	21,405	42	32,145	63	34,475	68
H48	36,107	71	5,976	12	2,828	6
66	3,648	7	48,211	95	34,736	68
68	4,466	9	10,772	21	12,941	25
88	3,534	7	1,058	2	2,311	5
Right-handed motions	90,605	178	115,269	226	93,351	183
12	0	0	0	0	0	0
14	891	2	967	2	190	0
16	375	1	2,256	4	358	1
18	785	2	585	1	75	0
23	3,724	7	710	1	1,014	2
25	3,050	6	2,949	6	1,986	4
27	608	1	120	0	40	0
34	55,488	109	11,449	23	11,151	22
36	43,518	85	17,014	33	30,239	59
38	31,103	61	5,360	11	7,796	15
45	45,109	88	94,629	186	67,110	132
47	16,320	32	2,100	4	2,510	5
56	20,769	41	146,942	288	170,929	335
58	35,636	70	43,651	86	39,961	78
67	5,324	10	3,541	7	5,333	11
78	4,052	8	595	1	810	2
Two-handed motions	266,752	523	332,868	653	339,502	666
One-handed motions	243,248	477	177,132	347	170,498	334
Total Motions	510,000	1000	510,000	1000	510,000	1000
Left-handed motions	152,643	299	61,863	121	77,147	151
Two-handed motions/2	133,376	262	166,434	326	169,751	333
Left-handed strokes	286,019	561	228,297	447	246,898	484
Right-handed motions	90,605	178	115,269	226	93,351	183
Two-handed motions/2	133,376	261	166,434	327	169,751	333
Right-handed strokes	223,981	439	281,703	553	263,102	516
Total Strokes	510,000	1000	510,000	1000	510,000	1000

Continued on p. 436.

APPENDIX 8. STATISTICAL USAGE OF SINGLE STROKES AND ADJACENCIES IN AVERAGE ENGLISH, PER 510,000 STROKES

(With the two orders of each adjacency (as TH and HT) added together and entered twice for 1,020,000 total)

	a	e	i	o	u	vow	B	C	D	F	G	H	J	K	L	M	N	P	Q	R
a	470	4881	2712	908	978	9949	1526	3792	3337	1405	1594	4770	122	757	5427	3208	10000	1957		7483
e	4881	4430	3118	1889	826	15144	3549	4509	7098	2078	2284	15135	204	1190	5782	5463	9109	2734	166	14605
i	2712	3118	142	2992	532	9496	619	3706	3085	2226	1835	3642		579	4501	2559	11052	942		4576
o	908	1889	2992	2552	4737	13078	1250	3528	2205	6747	1187	2493	194	378	2567	3514	8918	2222	10	7805
u	978	826	532	4737	20	7093	1514	1096	713	650	960	266	277	10	1762	925	2315	1045	421	2751
vow	9949	15144	9496	13078	7093	54760	8058	16630	16436	13084	7858	24306	797	2914	20039	15669	41394	8880	597	37018
B	1526	3549	619	1250	1514	8058	100	24	398	186	124	136	69	63	1092	323	371	37		766
C	3792	4509	3706	3528	1096	16630	24	540	402	195	122	2174		705	623	108	1688	36	20	979
D	3337	7098	3085	2205	713	16436	398	402	700	439	371	451	58	67	1721	495	6214	305	15	1403
F	1405	2078	2226	6747	650	13084	186	195	439	1300	170	285	18	74	625	218	583	154	10	1260
G	1594	2284	1835	1187	960	7858	124	122	371	170	216	1306	12	49	394	168	4441	102		1306
H	4770	15135	3642	2493	266	24306	136	2174	451	285	1306	212	10	54	207	196	518	357		706
J	122	204		194	277	797	69		58	18	12	10			14	10	74			26
K	757	1190	579	378	10	2914	63	705	67	74	49	54			154	50	646	32		468
L	5427	5782	4501	2567	1762	20039	1092	623	1721	625	394	207	14	154	5742	240	606	1244	10	621
M	3208	5463	2559	3514	925	15669	323	108	495	218	168	196	10	50	240	868	511	775		976
N	10000	9109	11052	8918	2315	41394	371	1688	6214	583	4441	518	74	646	606	511	1088	296	54	979
P	1957	2734	942	2222	1045	8880	37	36	305	154	102	357		32	1244	775	296	856		1681
Q		166		10	421	597		20	15	10					10		54			10
R	7483	14605	4576	7805	2751	37018	766	979	1403	1260	1306	706	26	468	621	976	979	1681	10	1084
S	5846	8531	6735	3331	3056	27499	636	1064	1352	675	605	2050	45	530	977	907	2551	1231	59	2196
T	9144	9073	9428	7144	2518	37307	495	1737	1561	1300	697	15952	41	189	1587	698	4919	656	16	4000
V	1650	4572	1805	827		8852			118	23	16	14			226	10	208			273
W	2468	3323	1831	2706	56	10384	55	72	584	289	187	1958		108	355	117	884	68		633
X	97	681	214	51	20	1063		95				40					20	214		
Y	1986	1561	571	1968	107	6193	883	395	483	306	290	498	28	73	1879	753	782	372	11	1367
Z	172	267	259	10		688											10			
CON	66719	99712	59941	59053	20241	305666	5758	10977	17135	8108	10574	27122	405	3240	18297	7421	27443	8496	205	20734
,	796	1518	494	688	118	3414	290	299	714	471	284	451	30	90	322	333	644	246	10	500
.	805	1528	499	695	118	3445	292	300	720	476	286	434	30	90	325	336	649	247	10	502
-	495	810	307	426	73	2111	182	181	441	295	174	264	20	60	198	206	397	153	10	308
"	281	449	174	240	44	1188	98	106	245	164	96	149	10	40	110	117	218	89	10	169
'	93	189	138	120	31	571	45	53	95	55	33	50		10	77	70	283	25		78
0#	129	204	80	111	20	544	44	44	111	75	44	68	10	20	50	53	100	43		77
1#	137	225	85	119	22	588	49	48	123	85	49	75		10	56	58	112	46		86
2#	67	113	46	58	10	294	24	24	63	43	26	37			29	33	57	18		44
;	67	113	46	58	10	294	24	24	63	43	26	37			29	33	57	18		44
?	55	97	30	50		232	19	18	50	32	22	30			24	27	46	14		36
:	45	74	22	36		177	15	14	38	25	10	25			20	24	36	11		29
P+N	2970	4920	1921	2601	446	12858	1080	1111	2661	1760	1050	1600	100	320	1240	1290	2599	910	40	1866
2Σ	79638	119776	71358	74732	27780	373284	14896	28718	36232	22952	19482	53028	1302	6474	39576	24380	71436	18286	842	59618

	S	T	V	W	X	Y	Z	CON	,	.	-	"	'	0#	1#	2#	;	?	:	P+N		ΣΣ
a	5846	9144	1650	2468	97	1986	172	66719	796	805	496	281	95	129	137	67	67	55	45	2970	a	79658
e	8551	9073	4572	3525	681	1561	267	99712	1318	1528	810	449	189	204	225	113	113	97	74	4920	e	119776
i	6735	9428	1805	1851	214	571	239	59941	494	499	507	174	138	80	85	46	46	30	22	1921	i	71558
o	3331	7144	827	2706	51	1968	10	59053	688	695	426	240	120	111	119	58	58	50	36	2601	o	74732
u	3056	2518		56	20	107		20241	118	118	73	44	31	20	22	10	10			446	u	27780
VOW	27499	37307	8852	10584	1063	6193	688	305666	3414	3445	2111	1188	571	544	588	294	294	232	177	12858		373284
B	636	495		55		883		5758	290	292	182	98	43	44	49	24	24	19	15	1080	B	14896
C	1064	1737		72	95	395		10977	299	300	181	106	53	44	48	24	24	18	14	1111	C	28718
D	1352	1561	118	584		483		17135	714	720	441	245	93	111	123	63	63	50	38	2661	D	36232
F	675	1300	23	289		306		8108	471	476	293	164	55	75	83	43	43	32	25	1760	F	22952
G	603	697	16	187		290		10574	284	286	174	96	33	44	49	26	26	22	10	1050	G	19482
H	2050	15952	14	1958	40	498		27122	431	434	264	149	50	68	75	37	37	30	25	1600	H	53028
J	45	41				.28		405	30	30	20	10		10						100	J	1302
K	530	189		108		73		3240	90	90	60	40	10	20	10					320	K	6474
L	977	1587	226	355		1879		18297	322	325	198	110	77	50	56	29	29	24	20	1240	L	39576
M	907	698	10	117		753		7421	333	336	206	117	70	53	58	33	33	27	24	1290	M	24380
N	2551	4919	208	884	20	782	10	27443	644	649	397	218	283	100	112	57	57	46	36	2599	N	71456
P	1231	656		68	214	372		8496	246	247	153	89	25	43	46	18	18	14	11	910	P	18286
Q	59	16				11		205	10	10	10	10								40	Q	842
R	2196	4000	273	633		1367		20734	500	502	306	169	73	77	86	44	44	36	29	1866	R	59618
S	4514	7057	62	992	10	1011	10	28532	1050	1040	634	357	219	161	178	92	92	72	54	3929	S	59960
T	7057	4658	55	1006	124	2107	30	48885	1538	1552	947	531	459	246	268	136	136	104	81	5998	T	92190
V	62	55		10		79		1094	38	38	24	13	25	12	10					160	V	10106
W	992	1006	10	118	10	424		7870	440	444	276	164	56	71	76	34	34	25	20	1640	W	19894
X	10	124		10				513	10	10										20	X	1596
Y	1011	2107	79	424		184		11925	486	494	303	166	57	77	85	46	46	39	21	1820	Y	19958
Z	10	30						50													Z	738
CON	28532	48885	1094	7870	513	11925	50	264784	8206	8275	5069	2852	1681	1306	1412	706	706	558	423	31194		601644
,	1030	1538	38	440	10	486		8206				350		30						380	,	12000
.	1040	1552	38	444	10	494		8275		20			20	40						80	.	11800
-	634	947	24	276		303		5069			20									20	-	7200
"	357	531	13	164		166		2852	350									10		360	"	4400
'	219	459	25	56		57		1681		20										20	'	2272
0#	161	246	12	71		77		1306	30	40				80						150	0#	2000
1#	178	268	10	76		85		1412													1#	2000
2#	92	136		34		46		706													2#	1000
;	92	136		34		46		706													;	1000
?	72	104		25		39		558				10								10	?	800
:	54	81		20		21		423													:	600
P+N	3929	5998	160	1640	20	1820		31194	380	80	20	360	20	150				10		1020		45072
ΣΣ	59960	92190	10106	19894	1596	19958	738	601644	12000	11800	7200	4400	2272	2000	2000	1000	1000	800	600	45072		1020000

Fingering of one handed motions:

		QWERT YUIOP per 510,000	QWERT YUIOP per 1000	DVORAK-DEALEY per 510,000	DVORAK-DEALEY per 1000	MINIMOTION per 510,000	MINIMOTION per 1000
Non-hurdles, left	K	9,065	18	4,377	9	4,025	8
	S	17,766	34	7,268	14	9,195	18
	A	56,296	111	17,975	36	36,330	71
	R	49,813	98	29,593	57	26,169	51
		132,940	261	59,213	116	75,719	148
Non-hurdles, right	K	5,988	12	10,620	21	11,028	22
	S	6,556	13	9,869	19	10,134	20
	A	19,894	39	49,706	97	34,309	67
	R	20,034	39	34,385	68	33,788	66
		52,472	103	104,580	205	89,259	175
Non-hurdles, total	K	15,053	30	14,997	30	15,053	30
	S	24,322	47	17,137	33	19,329	38
	A	76,190	150	67,681	133	70,639	138
	R	69,847	137	63,978	125	59,957	117
		185,412	364	163,793	321	164,978	323
Hurdles, left	S	6,456	13	734	2	262	1
	A	12,106	23	852	1	644	1
	R	1,141	2	1,064	2	522	1
		19,703	38	2,650	5	1,428	3
Hurdles, right	S	6,471	13	2,108	4	1,100	2
	A	15,957	31	2,375	5	1,315	3
	R	15,705	31	6,206	12	1,677	3
		38,133	75	10,689	21	4,092	8
Hurdles, total	S	12,927	26	2,842	6	1,362	3
	A	28,063	54	3,227	6	1,959	4
	R	16,846	33	7,270	14	2,199	4
		57,836	113	13,339	26	5,520	11
Total one-handed	K	15,053	30	14,997	30	15,053	30
	S	37,249	73	19,979	39	20,691	41
	A	104,253	204	70,908	139	72,598	142
	R	86,693	170	71,248	139	62,156	121
		243,248	477	177,132	347	170,498	334
Total, two-handed		266,752	523	332,868	653	339,502	666
Total Motions		510,000	1000	510,000	1000	510,000	1000

NOTES FROM THE NATIONAL BUREAU OF STANDARDS*

SUBMINIATURIZATION OF INTERMEDIATE FREQUENCY AMPLIFIERS

In military applications of miniature electronic systems, ease of mass production is as important as reduction in size. This requirement has motivated the search for new techniques and materials to use in the manufacture of electronic equipment. The National Bureau of Standards, under the sponsorship of the Bureau of Aeronautics, Department of the Navy, is engaged in adapting these techniques to the production of more complicated *subminiature* electronic devices, such as broad-band, high-gain, intermediate-frequency amplifiers for aircraft and missiles.

The intermediate-frequency amplifier chosen for miniaturization embodies a type of critical circuit layout that presents the most typical problems. The miniaturized amplifier was designed to have (*a*) eight stagger-tuned intermediate-frequency stages, a detector, a video amplifier, and a cathode-follower output circuit; (*b*) more than 95 decibels gain from the intermediate-frequency input to output of the detector; (*c*) manual and automatic gain control; (*d*) a 60-megacycle center frequency and a band-width of 10 megacycles; and (*e*) an assembly readily adaptable to mass-production methods.

Two methods of fabrication were employed in the construction of miniaturized intermediate-frequency amplifiers. One unit was developed using a maximum of miniature component parts based on standard design, whereas a second assembly used a maximum of printed circuits.

In the development of high-temperature printed resistors, simple high-capacity printing equipment for establishing the circuitry on the special cylindrical ceramic parts has been designed and constructed. Conductive decalcomanias have been developed as an alternate method of applying the required multiple circuit elements.

Other production jigs and techniques to facilitate the production of the intermediate-frequency amplifier have been developed for greater mechanization of printed circuit processing. Experience gained on this development program points the way toward even smaller and lighter printed and miniature component assemblies.

HELIUM DISAPPEARANCE IN ARC DISCHARGE TUBES

Electron tubes filled with a noble gas such as argon, neon, or helium are now widely employed in many kinds of electronic equipment, and

* Communicated by the Director.

the problem of gas disappearance in these tubes is of considerable importance. This tendency of filler gases to disappear from the tube volume is known as "clean-up" and is generally thought to result from the bombardment of negatively charged electrodes by high-velocity positive gas ions that penetrate the metal surfaces and become permanently trapped.

The National Bureau of Standards has been carrying on a study of clean-up phenomena in a specially designed discharge tube with a replaceable tantalum probe wire to collect the positive ions. During operation, the rate of gas clean-up is measured for various negative direct-current potentials on the probe in the presence of an arc discharge. In addition to the probe wire, this helium-filled discharge tube employs an oxide-coated cathode and a nickel anode.

With a known tube volume, the total number of ions trapped during a test run may be computed from the observed drop in pressure, whereas the number of ions striking the probe can be found directly from the observed probe current. These data have been used to construct a graph that illustrates the dependence of clean-up on probe voltage.

A PRIMARY HIGH-FREQUENCY VOLTAGE STANDARD

The National Bureau of Standards is developing primary standards of voltage for radio frequencies up to several hundred megacycles. Precision voltage standards are urgently needed by communications services and research groups in the adjustment of many laboratory and field instruments operating at high frequencies.

The Bureau has concentrated on methods of measuring single-frequency voltages directly in terms of a standard direct-current cell. Specific techniques have been chosen for best time-efficiency and accuracy over the widest range of voltage and frequency, without using frequency corrections. Reliability has been achieved by cross-checking the results of several independent methods based on different principles. Reproducibility of results and agreement between individual primary-standard methods is required within ± 1 per cent, since measurements to that accuracy are considered to be of good precision in the high-frequency region.

One of the techniques developed by the Bureau that satisfactorily meets the basic requirements for a primary standard is the voltage-measuring bolometer bridge, utilizing the dependence of bolometer resistance on power dissipation. In this method, a direct-current bridge with a bolometer in one of its legs is first balanced on direct current. Radio-frequency power is then substituted for some of the direct-current power and the bridge is rebalanced. The amount of radio-frequency power equals the difference in direct-current power required for balance in each case.

EFFECTS OF ACID TREATMENT ON ABRASION AND ACID RESISTANCE OF PORCELAIN ENAMELS

Recent work at the National Bureau of Standards has revealed the effects of acid pretreatment of porcelain enamels upon their acid and abrasion resistance. Studies were made to compare the relative effects of hydrochloric, acetic, butyric, and citric acids on a number of enamels. These studies indicate that acetic acid is much less corrosive than hydrochloric and citric acids; moreover, treatment with acetic or butyric acid, which produces only minor visible attack, strongly inhibites further attack when the specimens are subsequently treated with citric acid. By comparison, citric acid severely attacked the untreated areas of the same enameled specimens. In other tests, the resistance of certain enamels to abrasion was appreciably reduced after they had been treated with any of the acids.

Both effects, the increase in acid resistance and the decrease in abrasion resistance, may be explained by the hypothesis that a silica-rich film is formed on the enameled surface by the acetic and butyric acids. This layer is then resistant to further attack even by citric or stronger acids. However, it is less resistant to abrasion than the original surface.

The increased use of titanium-type enamels and the sensitivity of these enamels to attack by abrasion after treatment with acid, as compared with antimony-type enamels, indicates that an abrasion treatment should be incorporated in the standard test for acid resistance.

IMPROVEMENT OF LABORATORY GAS BURNERS

Marked improvement in the performance of laboratory-type gas burners with natural gas has been accomplished through design changes worked out at the National Bureau of Standards. Basic principles of gas-burner design and analysis of the operating characteristics of commercial and experimental burners were applied.

The difference in flame characteristics between natural and manufactured gas necessitates the alteration of all gas-burning appliances when a change in supply is made. The exacting requirements of Bunsen and similar types of laboratory burners, however, have made it much more difficult to convert these devices to the use of natural gas than has been true of domestic appliances. Even those burners advertised as designed for natural gas have not proven satisfactory. Eight Meker type and eight straight-tube Bunsen burners were examined in the Bureau's gas chemistry laboratory and the changes required for satisfactory operation determined.

These changes included an increase in the primary air opening to $2\frac{1}{4}$ times the port area and, in the Meker burner, enlargement of the

port area to 2½ times the throat area. The orifice should also be of sufficient size to give the maximum gas rate when fully opened and the needle valve used only to reduce the flow of gas. Only after these adjustments had been made did the burners reach their maximum heats of 10,000 Btu. per hour for Meker types and 5,000 Btu. for straight-tube types. As a result of this research, several manufacturers are redesigning their burners and the improved appliances will be available in the near future.

STABILITY TEST FOR ADDITIVE-MOTOR OILS

A new evaluation test for lubricating oils with additives, which simulates the characteristic deposits found in engine tests of motor oils, has been developed in the National Bureau of Standards' lubrication laboratory. The method has several advantages over the present testing procedure, which requires disassembly of the test engine and an examination of its parts. It employs simple laboratory apparatus, is time saving, is less expensive, and eliminates the fuel variable that results from contamination of the oil by the engine fuel. Most important, it gives a quantitative as well as visual rating of an oil.

The new test method consists essentially of circulating the oil sample over a heated steel strip in a thin film quite similar to the oil film on an engine cylinder wall. The difference in weight of the strip before and after the test provides a numerical indication of the amount of deposit formed.

In general, the oils used in the Bureau's investigation are in commercial production and have a considerable background of service performance. They include oils with no additives, premium-type additive oils containing inhibitors and/or detergents, and full heavy-duty oils containing additives. Data obtained with the new test, therefore, give a sensitive indication of the effectiveness of inhibitors, detergents, and their combinations.

SAMPLING PLAN REDUCES INSPECTION COSTS

Whenever industrial and agricultural products are purchased in quantity, the buyer must employ some sort of inspection procedure to determine whether or not the material meets the specified standard of quality. The extent of this inspection will depend on the nature of the material and may range all the way from a rough appraisal based on an examination of a few items to the complete inspection of each item. The decision to accept or reject a given lot of material is often determined by the average quality of a random sample taken from the lot. Whenever such a routine acceptance test is conducted, any way of reducing the amount of inspection, and therefore the cost, is worth exploring.

The Statistical Engineering laboratory of the National Bureau of Standards has developed a modified double sampling plan that makes possible a saving in the average amount of inspection from 20–40 per cent. In this new plan, the regular testing procedure is interrupted after a predetermined number of units have been examined. The material is accepted if the test results at the time of interruption fulfill the criteria of a simple rule that requires neither computation nor the use of tables. If the material does not pass this test, the inspection is completed in the usual manner. In either case, the inspection procedure itself remains unchanged.

The modified double sampling plan developed at the National Bureau of Standards is a substitute for the usual single sampling plan that does not alter the sampling itself or require additional computation. In this new plan the same number of units is drawn as in the single sample case, but the regular testing procedure is interrupted after a fixed number of items have been tested. If a predetermined number of these test results individually exceed the acceptance value of the original plan, the material is immediately accepted and testing stopped. If less than the required number meet the acceptance value, testing is completed in the usual way and the individual test results are averaged. Acceptance or rejection then depends on whether or not this average exceeds the acceptance value.

The chance that material of a given quality will be accepted under the modified double sampling plan is thus slightly larger than on the original single sample plan, since there will be some samples which have the required number of items exceeding the acceptance value at the point where inspection is interrupted, but which, if completely inspected, would have yielded an overall average below the acceptance value. This increase in the chance of acceptance is desirable when the general quality of the material under inspection is known to be good. It is undesirable when the quality is bad, and this slight increase in the chance of accepting bad material is the price to be paid for reducing the average amount of inspection without disturbing existing routines.

No sampling plan can guarantee the rejection of unserviceable material, but the chance of accepting such material can be made as small as desired by properly choosing the acceptance value and the number of units to be tested.

THE FRANKLIN INSTITUTE.

MEDAL DAY MEETING

WEDNESDAY, OCTOBER 19, 1949

The Annual Medal Day Exercises of The Franklin Institute were held on Wednesday, October 19, 1949 in the Benjamin Franklin Memorial Hall. There were approximately 300 persons who attended the dinner and the presentation of the awards. Twenty three of our former medalists were among the guests present.

Prior to the dinner, a reception in honor of the 1949 Medalists was held in the rooms adjoining the Memorial Hall.

The Medal Day Exercises opened with Mr. Guy Marriner, Director of Music at The Franklin Institute, playing the National Anthem. Mr. Richard T. Nalle, President, officiated at the ceremonies.

The Stated Monthly meeting of The Franklin Institute was held in conjunction with the Medal Day Exercises but the minutes of the previous meeting were not read since they had been printed in full in the JOURNAL. Mr. Nalle reported that Dr. Svedberg had been nominated by the Board of Managers for Honorary Membership in The Franklin Institute, he therefore presented his name to the membership for voting. A motion was made, approved and carried out in parlimentary procedure and Dr. Svedberg was elected to Honorary Membership in The Franklin Institute.

Two of our honored medalists, Dr. The Svedberg of Sweden, and Dr. William Hume-Rothery, of England were unable to be present and their awards were presented *in absentia.* Mr. Ingemar Hägglöf, Counselor of the Swedish Embassy in Washington, received the award for Dr. Svedberg and Dr. W. A. Macfarlane, Scientific Attaché of the British Embassy received the award for Dr. Hume-Rothery.

Mr. William Hulse Millspaugh had arrived in Philadelphia with the hope of being present at the presentation of his award but was stricken seriously ill and taken directly to the hospital. Dr. Henry B. Allen, Executive Vice President and Secretary of the Institute, acted as Mr. Millspaugh's proxy.

Dr. Allen presented a short paper on "The Franklin Institute in 1949" and Mr. J. Burton Nichols delivered Dr. Svedberg's address "Giant Molecules in Solution." These papers and the entire proceedings will be printed in full in a subsequent issue of the JOURNAL OF THE FRANKLIN INSTITUTE.

PROGRAM

Reception for Medalists.......... THE HOSTESS COMMITTEE
Assembly in Franklin Hall.......... Upon gong signal at seven-ten
National Anthem.......... GUY MARRINER
Dinner.......... MEDALISTS, OFFICIALS AND GUESTS
Toast to Benjamin Franklin.......... RICHARD T. NALLE, *President*
Greetings.......... THE PRESIDENT
October Stated Meeting of The Institute.......... THE PRESIDENT PRESIDING
Introduction of Former Medalists.......... THE PRESIDENT
The Institute in 1949.......... HENRY B. ALLEN, *Secretary*
Presentation of Awards.......... THE PRESIDENT
Presentation of Dr. Svedberg's paper on "Giant Molecules in Solution"
(Read by Dr. J. Burton Nichols, in the absence of Dr. Svedberg.)

MEDALISTS

To be Awarded Longstreth Medals (1890)

ARTHUR M. YOUNG
New York City, New York

WILLIAM H. MILLSPAUGH (*in absentia*)
President and Director, Centrifugal Steel, Inc.
Sandusky, Ohio

To be Awarded Potts Medals (1906)

JOHN WILLIAM MAUCHLY
President, Eckert-Mauchly Computer Corporation
Philadelphia, Pennsylvania

J. PRESPER ECKERT, JR.
Vice President, Eckert-Mauchly Computer Corporation
Philadelphia, Pennsylvania

CLINTON RICHARDS HANNA
Associate Director of Research
Westinghouse Electric Corporation
Pittsburgh, Pennsylvania

To be Awarded a Levy Medal (1923)

ALAN STEWART FITZGERALD
Consulting Engineer
San Francisco, California

To be Awarded a Henderson Medal (1924)

JOHN V. B. DUER
Pennsylvania Railroad (Retired)
Fairfield, Connecticut

To be Awarded Wetherill Medals (1925)

HARLAN D. FOWLER
Consulting Engineer
Whittier, California

THOMAS L. FAWICK
President, Fawick Airflex Company, Inc.
Cleveland, Ohio

EDGAR COLLINS BAIN
Vice President, Carnegie-Illinois Steel Corporation
Pittsburgh, Pennsylvania

To be Awarded a Clamer Medal (1943)

WILLIAM HUME-ROTHERY (*in absentia*)
Royal Society Warren Research Lecturer
Metallurgical Chemistry
University of Oxford, England

To be Awarded a Ballantine Medal (1946)

SERGEI A. SCHELKUNOFF
Research Mathematician
Bell Telephone Laboratories
Murray Hill, New Jersey

To be Awarded the Franklin Medal (1914) and a Certificate of Honorary Membership in The Franklin Institute

THE SVEDBERG (*in absentia*)
Director
The Gustav Werner Institute for Nuclear Chemistry
Upsala, Sweden

COMMITTEE ON SCIENCE AND THE ARTS

(*Abstract of Proceedings of Stated Meeting held Wednesday, October 12, 1949.*)

HALL OF THE COMMITTEE,
PHILADELPHIA, OCTOBER 12, 1949.

MR. W. LAURENCE LEPAGE in the Chair

The following reports were presented for final action:

No. 3196: Work of Merle A. Tuve on the Proximity Fuze Development.

This report recommended the award of a Howard N. Potts Medal to Merle A. Tuve, of Washington, D. C., "In recognition of the scientific insight exhibited in the conception of the VT Proximity Fuze and of the administrative ability and practical good judgment shown in the supervision of its development and engineering design."

No. 3200: 20,000,000-Volt Betatron.

This report recommended the award of a John Price Wetherill Medal to Donald William Kerst, of Urbana, Illinois, "In consideration of his contribution to the theory of the Betatron and the practical application of the theory to the construction of the first satisfactory workable machine, and his subsequent work in this field."

JOHN FRAZER,
Secretary to Committee.

MUSEUM NOTÈS

RADIOSONDES

The weather is always with us but beyond keeping an umbrella and rubbers within convenient range, few of us do much else about it. However, weather is a matter of primary importance to some people and their needs justify the development of methods for securing more comprehensive and accurate data. An interesting feature of modern meteorological investigation is the radiosonde, a combination of devices for obtaining information on conditions prevailing at altitudes up to more than 100,000 ft. The information thus obtained may have an immediate use or it may be required for comparative purposes in the assembly of data to provide a basis for forecasts.

The immediate uses for information of this nature will be recognized as of high importance for aviation, especially in relation to jet propelled aircraft. Not so obvious is the need for the artillerist whose projectiles may rise above surface conditions and who must know the conditions prevailing at the altitudes through which the projectile must travel. Knowledge of this nature is imperative to permit correction of gun sighting.

The Signal Corps of the United States Army is responsible for the collection and dissemination of meteorological data and the Evans Signal Laboratory at Fort Monmouth, N. J. has made some impressive improvements in the radiosonde, the assembly of measuring devices sent into the upper air. The Evans Laboratory has designed and built a special exhibit for the Museum which illustrates the progressive steps in the development of the instruments and the construction and use of the radiosonde now employed by the Signal Corps. The exhibit shows very clearly what rapid strides have been made in the improvement in the device during its twenty years of existence.

The dispatch of human agents into the upper air by means of balloon or other aircraft is an expensive undertaking not unaccompanied with danger. There is, too, a somewhat restricted ceiling. The purpose of the radiosonde is to render unnecessary any human flight into the air. The attainable ceiling is accordingly very much elevated.

The radiosonde now in use has been carried to altitudes exceeding 100,000 ft. The desirable ceiling is about 150,000 ft. and the additional distance presents a problem out of all proportion. When one realizes that the diminishing air pressure implies a proportional expansion of the gas within the balloon it will be seen that the greatest obstacle to increasing the altitudes to which a gas-filled balloon may ascend resides in the difficulty of finding an elastic fabric for the balloons.

Any radiosonde system dispatched into the air must convey its own means for making measurements and for transmitting the results obtained to a receiving station, since the chance of recovering the instruments uninjured is small. The device developed by the Evans Laboratory is a marvel of compactness and comprises the radiosonde (which is the assembly of measuring instruments), a power source, balloon, parachute and, under conditions of high or gusty winds, a launching wheel in addition to the component parts of the permanent station equipment. The latter comprises a meteorological direction-finder which tracks the radiosonde, and a recording apparatus for making a permanent record of the data emitted by the measuring instruments.

The instruments carried into the upper air by the radiosonde assembly are very tiny but they are capable of accurately measuring five meteorological quantities—atmospheric pressure, temperature, relative humidity, wind velocity, and direction. All of these data are collected on a single ascent, and transmitted by a battery-powered radio transmitter to the ground station where they are automatically recorded.

At the present time radiosondes are released twice daily from seventy different places in this country. Each one faithfully records the local conditions but all gathered together furnish data upon which more general pictures of the weather may be built and forecasts made.

THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

Abstract of **Measurement of the Spin of a Projectile in Flight.**[1]—HOWARD D. WARSHAW.[2] Projectile spin is measured by recording the voltage generated in a pick-up coil as the rotating field of a transversely magnetized projectile passes the coil.

A permanent magnet within a brass sleeve (for magnetic insulation) is inserted in a projectile of mild steel. Rotation of the projectile as it passes the pick-up coil generates a sinusoidal e.m.f. which is displayed on the screen of a cathode-ray tube and photographed for subsequent analysis. Figure 1 shows the laboratory setup and a block diagram of the as-

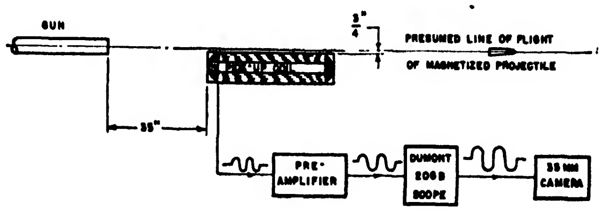


FIG. 1. Gun, pick-up coil, and recording equipment (pick-up coil viewed from above).

sociated equipment used for measuring and recording projectile spin. The pick-up coil consists of insulated copper wire, wound on a rectangular coil form which is positioned so that its long sides are parallel to the line of flight. As the rotating projectile passes the coil, a voltage is generated in that side of the coil nearest the line of flight. Any voltage generated in the opposite side of the coil will reduce the signal output, which is the difference in voltage generated in the two sides. However, the signal reduction is negligible since the intensity of the magnetic field due to the projectile is negligible at the far side of the coil.

The signal output of the coil is passed through a pre-amplifier and then to the vertical deflection amplifier of a Dumont, Type 208B, cathode-ray oscilloscope. The over-all voltage

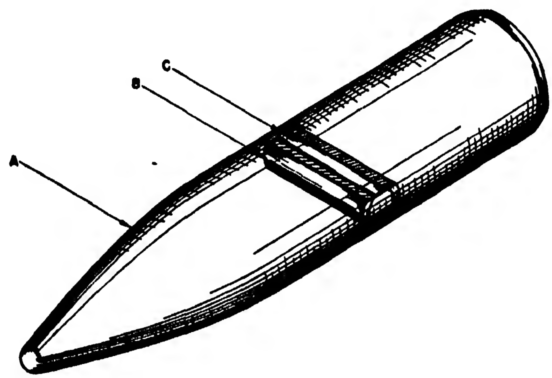


FIG. 2. Cut-away sketch of projectile. (A) Cold-rolled steel projectile. (B) Magnetic insulator (brass sleeve). (C) Magnetized insert (hardened drill rod).

amplification of the system is approximately 10,000. The oscilloscope, modified for single sweep, is triggered by a switch in the breech mechanism of the gun. Permanent records are obtained by photographing the trace that appears on the screen when the projectile passes the coil.

[1] Paper based on work done for the Office, Chief of Ordnance, Department of the Army. Published in *The Review of Scientific Instruments*, Vol. 20, No. 7, July, 1949.

[2] Research Engineer, The Franklin Institute Laboratories for Research and Development, Philadelphia 3, Pa.

The caliber 0.50 projectiles used in these tests were made of cold-rolled steel. This material has high magnetic permeability but low retentivity. A satisfactory design was evolved in which a hardened steel insert was magnetically insulated from the projectile, as shown in Fig. 2.

Figures 3 and 4 are photographic records obtained with the equipment described. Cal-

FIG. 3. Oscilloscope record of projectile spin.

FIG. 4. Oscilloscope record of projectile spin.

culations based on the linear velocity of the projectile and the rifling twist in the gun indicate that the projectile should execute two full revolutions while passing the pick-up coil. In Fig. 3, it may be seen that two complete sine waves of voltage were generated in the pick-up coil. However, due to the spurious signal that preceded the desired signal, the base line of the sine waves is not parallel to the zero-signal base line (at the extreme left end of the trace). The true base line is midway between, and parallel to the lines joining the peaks of the sine waves. Thus, the number of revolutions of the projectile as it passes the coil may be determined.

In Fig. 3, it should be noted that the signal started and ended at zero voltage. Apparently, the magnetized insert in the projectile was normal to the plane of the coil when passing the leading and trailing edges of the coil. Occasionally, this condition may be duplicated, provided the projectile is accurately positioned in the gun prior to firing. However, this is not a necessary prerequisite, since in Fig. 4 the number of revolutions was determined accurately, even though the insert was in a position to generate a voltage as it passed the leading edge of the coil. As shown on the photograph, it is necessary to extrapolate the signal to the point on the trace at which the insert passed the leading edge of the coil.

Although angular velocity was not measured, it may be obtained simply by the addition of time-markers to the oscillograph trace. This could be accomplished by modulating the intensity of the trace with pulses of a known frequency.

Engravings loaned through the courtesy of The Review of Scientific Instruments.

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in the JOURNAL within the next few months:

JACOBSEN, LYDIK, R. L. EVALDSON AND R. S. AYRE: Response of an Elastically Non-Linear System to Transient Disturbances.

MOON, PARRY AND DOMINA EBERLE SPENCER: A Modern Approach to Dimensions.

FANO, ROBERT M.: Theoretical Limitations on the Broadband Matching of Arbitrary Impedances.

FANO, ROBERT M.: A Note on the Solution of Certain Approximation Problems in Network Synthesis.
SILBERSTEIN, LUDVIK: Developable and Developed Silver Halide Grains.
MINIAMIOZI, K. AND H. OKUBO: A Note on the Notch Effect of Metals.
ROOP, WENDELL P.: The Part of Octahedral Theory in the Study of the Plasticity of Metals.
LEITNER, A., AND R. D. SPENCE: The Oblate Spheroidal Wave Functions.

LIBRARY

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. The average cost for a print 9 × 14 inches is thirty-five cents.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A.M. until 5 P.M.; Wednesdays and Thursdays from 2 P.M. until 10 P.M.

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BARTOL RESEARCH FOUNDATION

Abstract of **The Detection of Heavy Particles in the Primary Cosmic Radiation.***— M. A. POMERANTZ [1] AND F. L. HEREFORD.[1] A new technique, involving a coincidence train of Geiger-Mueller counters filled at low pressure, has revealed the presence of heavy nuclei in the primary cosmic radiation. The method exploits the dependence of the probability of discharge of a Geiger-Mueller counter upon the nature of the gas filling and renders it feasible to resolve the various components of cosmic radiation. By comparing results obtained under different conditions, it has been determined that the primary radiation at 52° N consists approximately of two-thirds proton, one-third α-particles, and a small amount of heavier nuclei. The experiments are still in progress and should lead ultimately to a complete quantitative resolution of the cosmic radiation throughout the atmosphere.

* Letter to the Editor in the *Physical Review*, October 1, 1949.
[1] Physicist, Bartol Research Foundation of The Franklin Institute, Swarthmore, Pa.

Abstract of **Summary of Bartol Cathode Work.***—W. E. DANFORTH.[1] The Bartol program, sponsored by the Bureau of Ships, consists in the carrying out of such investigations concerning thoria and similar cathode materials as are pertinent to magnetron development. Thermionic emission, pulsed and d-c., is continually being studied. One objective, among others, is to establish the picture regarding emission activation, a matter which has primary importance in the temperature region below 1500° C. Composite results for well-activated cathodes from several laboratories are in fair agreement with a Richardson curve in which A = 0.33 and $\varphi = 2.0$.

Direct current from thoria is limited to approximately 5 amp./cm.^{2} by decay, and investigations of this subject are continuing. No short time decay of 10^{-5} sec. time constant, as often observed with BaSr oxide, is present, the smallest time constant being some tenths of one second, a fact which recommends thoria for long pulse applications.

Rates of evaporation, and of disappearance of the material with passage of current, have been measured and the observations are continuing. On the basis of evaporation alone a coating of 1 mil thickness will completely evaporate in 350 hr. at 1800° C. (true), a temperature where the primary thermionic emission is 14 amp./cm.^{2} At 1650° true temperature, where the primary emission is about 5 amp./cm.^{2}, the evaporation life is 4000 hr. per mil thickness. The passage of high d.c. accelerates the disappearance, preliminary results indicating that for coatings the passage of 1 amp./cm.^{2} decreases the life by a factor of 2, a figure which is roughly independent of temperature.

Studies of electrical conductivity of sintered thoria at high temperatures and high vacua are being pursued. Present results are confined to moderate vacua of about $0.5\text{–}1 \times 10^{-5}$. Activation phenomena occur. A resistance as low as 20 ohm-cm. at 1000° C. has been obtained by high current activation. A sealed-off and gettered vacuum furnace for these studies has been built and preliminary tests are being made.

Thermionic emission studies have been made with thoria plus numerous admixtures, none of which, to date, has increased its emission. Studies have also been made with coated cathodes of ZrC, TaB, TaC, and TiC, among which ZrC is the most interesting, its emission exceeding that of thoria in the temperature region below about 1400° C.

A 10-cm. high powered pulsed magnetron, with a structure permitting cathode replacement, is being developed.

Abstract of **Zirconium Carbide as a Thermionic Emitter.***—R. E. HADDAD,[1] D. L. GOLDWATER,[1] AND F. H. MORGAN.[1] The thermionic emitting properties of zirconium carbide (one of several materials under investigation) are reported. Values obtained are

$$A = 0.2\text{–}0.5 \text{ amp./deg. k}^2\text{cm.}^2$$
$$= 2.1\ (6) \text{ volts.}$$

For the Richardson equation

$$I_s = AT^2e^{-b/T},$$
$$b = 11{,}605.$$

At 1500° C_b, stable emission of 0.6 amp./cm.^{2}, d.c. is obtained. At 1700° C_b, 6 amp./cm.^{2} currents are obtained with microsecond pulses.

The spectral emissivity of ZrC has been obtained, and is essential to interpreting pyrometer readings. The value obtained is 0.96 ± 0.04.

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* Paper given at ONR Conference in Washington, D.C., August 8, 1949.

[1] Assistant Director, Bartol Research Foundation of The Franklin Institute, Swarthmore, Pa.

* Letter to the Editor appearing in the *Journal of Applied Physics*, September, 1949.

[1] Junior Chemist, Electrical Engineer and Junior Physicist, respectively, Bartol Research Foundation of The Franklin Institute, Swarthmore, Pa.

MEMBERSHIP

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS, OCTOBER 19, 1949

HONORARY

Dr. The Svedberg

ACTIVE FAMILY

Benj. J. Baskin
Sylvan M. Cohen
David E. Connor
Wayne P. Conway
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Lester L. Lessig
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L. F. Bourgarde
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Albert Ferri
Albert Glazer
Robert Headley
E. M. Howard
Vladimir Hrubant
Alda R. Johnson
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Morris Shore
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Philip C. Wantz
Miss Anna C. Weber

ACTIVE NON-RESIDENT

Burt L. Newkirk
Sidney C. Smith

NECROLOGY

Charles MacBride '44

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

APPLICATION OF A SEMI-QUANTITATIVE ESTIMATION OF NITROGEN AND OTHER ELEMENTS IN ORGANIC MICROANALYSIS*

BY

WILLIAM G. BATT

When organic nitrogen samples are brought into the microchemical laboratory at least one of the following analyses is required: (1) qualitative; (2) semi-quantitative, whereby an approximation is arrived at rather quickly; (3) quantitative, whereby the amount of nitrogen is determined to the decimal point.

The qualitative determination of the presence or absence of nitrogen, sulfur, and the halogens is an important step in the identification of an unknown substance. Not only is this information useful in itself but other steps in the identification procedure depend upon it. This step is essential when dealing with unknown samples, for it is wasted effort to spend probably an hour on a quantitative determination when a simple screening test may eliminate additional work by proving the absence of nitrogen. Among the methods mostly used for the elementary detection are those developed by Lassaigne, using molten sodium; Castellana, using magnesium and potassium carbonate; and Emich using lime and copper powder.

When nitrogen is present it is not always expedient to run an analysis by Kjeldahl's method, because not all nitrogenous compounds are easily broken down, for example, the pyrimidines and nitro compounds, and because the ultimate analysis by Dumas' method requires nearly an hour. Analysis by ter Meulen's method of the hydrogenation of nitrogen compounds with a catalyst (nickel) requires a high degree of experience.

A semi-quantitative method makes it possible not only to determine the approximate amount of nitrogen present, which may be all the information required, but it also serves the purpose of enabling the analyst to estimate better the amount of sample that should be used if and when a fully quantitative result is necessary.

The practical value of semi-quantitative estimation in inorganic chemistry has been emphasized by Benedetti-Pichler (5) and Swift (9) but apparently no one heretofore has applied this technique to organic elementary analysis.

In some types of control work and in synthetic organic procedures, a rapid method is very useful in following certain reactions; and in

* Paper presented as part of the Symposium on Determination of Nitrogen in Organic Compounds. St. Louis Meeting American Chemical Society, September 6 to 10, 1948.

pharmaceutical preparations the presence of certain impurities can be ascertained and estimated by a rapid although not too accurate measurement of the nitrogen content. In the testing of biochemical products a simple approximation may be advantageous in following an extraction procedure.

The sodium fusion method seems to be favored by the majority of authors of textbooks on qualitative organic analysis. On the other hand Baker and Barkenbus (4) claim that although the sodium fusion method has been widely used, it is well known that nitrogen often escapes detection. Middleton (8) also dispenses with the use of sodium and uses instead sodium carbonate-sugar, or sodium carbonate-zinc

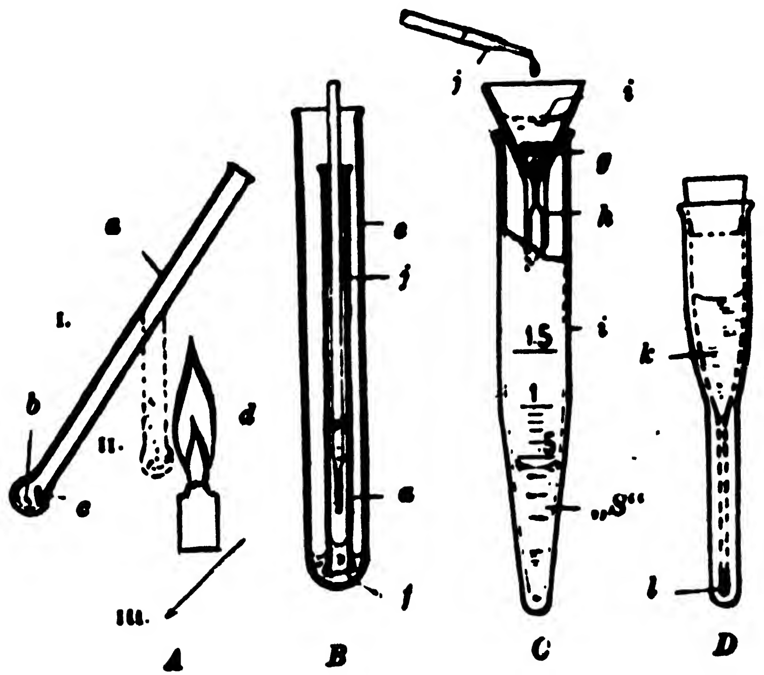

FIG. 1. Sodium fusion and semi-quantitative estimation of elements. Alber (3).

dust mixtures, and claims certain advantages of this method over the sodium procedure. Campbell (6) recently showed that a modified sodium fusion followed by detection of the cyanide ion by means of the Prussian blue test is favored for teaching inexperienced students. The advantages of this procedure have been substantiated by the number of analyses performed in this laboratory in which the sodium fusion method for elementary analysis is considered more reliable than other generally described procedures especially when other elements as well as nitrogen are to be detected. For the detection of nitrogen alone the method of Emich (7), in which air-slaked lime is used, has been found successful with as small an amount as 0.1 microgram of nitrogen. With

substances which may contain nitrogen in combination with oxygen, copper powder is added to the lime mixture.

The method to be emphasized in this paper is the one proposed by Alber (1–3), which allows the estimation to within 5 per cent (absolute) of the quantities of nitrogen, halides, sulfur, and phosphorus (arsenic) present, so that an unknown compound may be classified within a narrower range. This approximation also permits the study of the influence of interfering substances; it indicates whether one of the elements sought is present as a trace (below 1 per cent), and thus eliminates misinterpretations due to impurities, a condition which must be considered in testing biological substances.

A simple, inexpensive balance, modified according to the principle of Salvioni, may be used for the rapid weighing of samples from 0.2 to 1 mg. The sample is introduced to the bottom of a fusion tube with

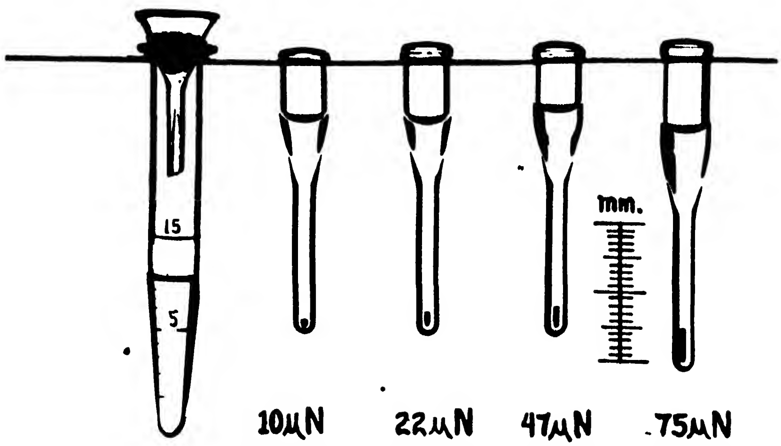

FIG. 2. Semi-quantitative estimation of nitrogen as Prussian blue precipitate.

two bulbs, both of which contain metallic sodium. By heating the upper bulb first, even liquid samples will be decomposed without loss, since the vapors have to pass the hot sodium. The heated mass is dissolved in water, with the usual precautions, and the soluble material is allowed to be thoroughly leached out by boiling slightly. The filtrate is separated from the insoluble material by passing through a Strzyzowski funnel, with asbestos as a filtering medium, and centrifuged by means of a hand model. The filtrate is then brought to a definite volume in a calibrated microcentrifuge cone. Aliquot portions of this slightly alkaline stock solution are used for subsequent tests. The individual tests are performed in straight-tip cones as used by Benedetti-Pichler and Spikes.

The test for nitrogen is made by the usual method of adding ferrous sulfate solution, mixing well and then acidifying with H_2SO_4. No particular advantage has been observed by adding a small amount of

potassium fluoride as recommended by Viehoever and Johns (10). The precipitate of Prussian blue is concentrated in the stem of the cone by hand centrifuging a definite number of revolutions or a prescribed length of time and compared with a standard compound which has been treated in a similar manner. The height of precipitate in the straight tip cone gives an approximation of the nitrogen present in the original sample. If the N to C ratio is high it may be found advisable to mix the compound with an equal weight of powdered sugar before the sodium decomposition and the fusing of this mixture with the molten metal will often assist in converting the nitrogen to sodium cyanide.

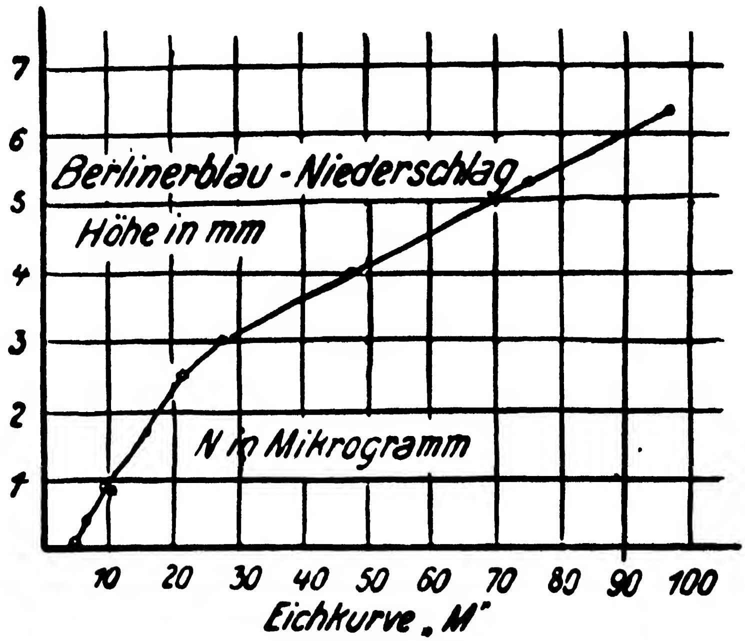

FIG. 3. Working curve for estimation of nitrogen content. Alber (3).

Sulfur may be estimated by acidifying the stock solution with acetic acid and adding a few drops of lead acetate solution. The black precipitate of lead sulfide is centrifuged in the previously described manner and compared with a standard. If a test of the stock solution in dilute HCl with ferric chloride solution produced a blood-red color the presence of thiocyanate is indicated. This is suggestive of insufficient sodium being used in the original fusion. Sodium thiocyanate first formed is frequently decomposed by excess metallic sodium into sodium cyanide and sodium sulfide.

The halides may be approximated by acidifying an aliquot portion of the stock solution with concentrated nitric acid and boiling gently for a few minutes to expel any hydrogen cyanide or hydrogen sulfide which may be present. A few drops of silver nitrate solution are then

added, and the solution again boiled to destroy any silver thiocyanate which may be present when both nitrogen and sulfur are found in the original compound. A heavy precipitate indicates the presence of chlorine, bromine, or iodine. If only a faint turbidity or opalescence is produced it is probably due to the presence of impurities in the reagents or in the glass of the tube used in the original decomposition. The differentiation and separation of chlorine, bromine, and iodine, which are present as silver halides after the estimation, are made possible by the differences in their solubilities in ammonium carbonate and ammonia, provided the ratio of the elements is approximately 1:1:1.

Phosphorus may be estimated by boiling a portion of the stock solution with concentrated nitric acid. The solution is cooled to about 60° C. and ammonium molybdate reagent added. The crystalline precipitate of ammonium phosphomolybdate (ammonium molybdiphosphate) is measured and compared with a known sample.

The estimations of the four elements described may be carried out easily after a little practice, and the time consumed will, on the average, not exceed one-half hour for all the tests. The information obtained is more conclusive and supporting than the mere reporting of the presence or absence of nitrogen or other elements.

SUMMARY

Several methods of decomposition of original sample for the detection of nitrogen and other elements are reviewed briefly. Emphasis is placed on a procedure in which aliquot portions of the stock fusion solution are used and the amount of constituent elements present is approximated. An accuracy of within 5 per cent (absolute) can be obtained within half an hour for nitrogen, halides, sulfur, and phosphorus. The method is useful as a screening test, and allows the selection of the correct amount of sample for a quantitative organic microanalysis.

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BOOK REVIEWS

INTRODUCTION TO RADIOCHEMISTRY, by G. Friedlander and J. W. Kennedy. 412 pages, illustrations, 14 × 21 cm. New York, John Wiley and Sons, Inc., 1949. Price, $5.00.

This book, written as a text for a college or graduate course in radiochemistry, is the first of its kind to appear and is certain to meet with a most cordial welcome. One of the most striking features is the large variety of topics discussed, some in brief and some in detail, and although many of them seem far afield from the subject of radiochemistry, the order and mode of presentation are such as to bring them neatly together into a logical picture of this extremely important branch of science. Included in the book are chapters on nuclear structure and properties, on high-energy particle accelerators and nuclear chain reactors, on the interactions of radiations with matter, on instruments for radioactivity measurements, on statistics of radioactivity measurements, and on the study of nuclear radiations, all in addition to chapters on the actual application of radioactivity to chemistry. In my opinion, one objection to the book is the relative lack of material on the subject of radiochemical techniques and procedures. Presumably this subject is to be treated in detail in a forthcoming reference book entitled "Radioactivity Applied to Chemistry," edited by Professor A. C. Wahl of Washington University, and one may hope that this companion volume to an "Introduction to Radiochemistry" will soon be published.

The problems, which are included at the end of each chapter, are well-chosen and do much to complete the understanding of the text. The references cited do not constitute a complete bibliography but rather serve as an introduction to the recent literature and to the standard reference works.

Both Friedlander and Kennedy have wide experience as radiochemists and as teachers of radiochemistry and they have put into their book many features which will make it a useful reference for anyone working with radioactive materials. They are to be commended for the excellent job they have done in assembling such a well-written text.

E. SHAPIRO

FROM EUCLID TO EDDINGTON, by Edmund Whittaker. 212 pages, 14 × 23 cm. Cambridge, University Press, 1949. Price, $4.00.

About three centuries before the beginning of the Christian era through a sustained effort by the early Greeks, a new revolutionary concept concerning the properties of space was enunciated. These early geometers building on the quicksands of their immediate knowledge began the development of scientific philosophy which culminated in the exposition of the theory of relativity early in the 20th century.

Between Euclid and Eddington there lived many expounders of natural philosophy whose names have become cornerstones in the Physics structure today. Scientist philosophers like Descartes, Kepler, Newton, Leibnitz, Kant, Einstein, and others flashed comet-like across the horizon leaving trails, or threads, tying together the building blocks of our knowledge. These giants were to be followed by still another intuitive genius, Sir Arthur Eddington, who undertook the monumental task of unifying philosophical thought and correlating the laws of nature from the macrocosmos to the microcosmos.

In "From Euclid to Eddington" Sir Edmund Whittaker has digested the history and development of scientific philosophy in a remarkably skillful and succinct manner and at the same time he has "disengaged the essential ideas from the intricacy of mathematical presentation." He has taken the philosophers of the past and present and filed them into niches which expose their contributions to the science. He has further shown how, with an almost mystical insight, Eddington has arrived at abstract numbers which linked the domain of the atoms and the universe. He has succeeded in taking a complex, difficult subject and with uncanny facility translating it into readable form. The reviewer finds a close analogy between

this book and "Stars and Atoms" by Eddington. "Stars and Atoms" is essentially a non-mathematical digest of portions of "The Internal Constitution of the Stars" and as such is a readable and delightfully pleasant book. Sir Edmund has in a way accomplished this feat in his book.

The history and development of scientific philosophy is a history of the evolution of concepts and principles. These concepts change and with their change results scientific progress. Man, being what he is, cannot stagnate, his thought processes cannot become static and because of this, new ideas, new concepts, new principles will appear on the horizon to relegate the present ones to the past. In this book the author has tried to bring the reader up-to-date in this revolution by a penetrating analysis of the profound changes which have taken place in our thinking even to the controversial concepts now under discussion.

Of especial and immediate interest is the imminent work of the 200-in. telescope. It may be safely assumed that within the next ten years many answers to controversial questions such as the red shift, the expanding universe, the size and structure of the universe may be obtained and it is to be regretted that Eddington is no longer alive to see whether his pioneering concepts in the new philosophy will be discarded, altered, or will prove a powerful influence to mould the course of natural philosophy in the future. The next few years should mark a new era in the history of natural philosophy and "From Euclid to Eddington" will go a long way in grounding one in the essentials to understand the changes which will inevitably take place in our thought and reasoning.

I. M. LEVITT

ENGINEERING DEVELOPMENTS IN THE GASEOUS DIFFUSION PROCESS, by Manson Benedict and Clarke Williams. 129 pages, illustrations, 16 × 23 cm. New York, McGraw-Hill Book Co., 1949. Price, $1.25.

As the annals of the atomic bomb are slowly presented in the form of the Manhattan Project Section of the National Nuclear Energy Series, a significantly larger, but still small, fraction of the public will become aware of the vast number of years of work which were crowded into the three actually involved. The declassifiable material available for the series will probably occupy 60 volumes, of which the above-titled is one of the first few published.

This book describes a few of the important engineering techniques born of the complications entailed in separating U^{235} in the form of corrosive uranium hexafluoride (UF_6). Since the UF_6, mixed with other gases, was diffused through porous membranes at definite pressures, instrumentation was required, after obtaining vacuum tight apparatus, to follow the pressure and percentage composition, and perform various other operations. The descriptions of the nine devices presented were apparently written by the inventors and edited by Benedict and Williams.

Continuous process-gas analysis was accomplished by means of recording mass spectrometers and ionization chambers. The former was used for analyses of all the components of the gas streams containing 0.1 per cent to 100 per cent UF_6, while the latter determined UF_6 present in quantities of 0.1 per cent to 1×10^{-5} as a function of the radioactivity of the three uranium isotopes. These instruments are described with the aid of schematic diagrams.

A "magnetic gear for torque transfer to a closed system" is discussed briefly and none too clearly.

One of the greatest engineering achievements of the diffusion process was the construction of thousands of vacuum tight diffuser units, necessitating a tremendous and immediate advance in vacuum pressure testing and leak-detecting. For this purpose, the mass spectrometer calibrated for helium detection, was most successfully applied. When helium was directed at suspicious points of the evacuated apparatus, the spectrometer quickly announced the entrance of the foreign gas. The instrument itself, together with related plant problems and specifications, are presented in two chapters.

Other sections include papers on UF_6 cold traps, absorption of UF_6 in oil and continuous fluorine disposal apparatus.

It is hoped that later books in this series will present further data and more complete descriptions of these and other engineering aspects of the diffusion process, which can provide

many pages of thrilling material. And in any case, the series will indicate experimentally the high probability that exists for science's solving critical problems such as cancer if given the proper material backing.

ALBERT L. MYERSON

PHILOSOPHY OF MATHEMATICS AND NATURAL SCIENCE, by Hermann Weyl. 311 pages, 15 × 23 cm. Princeton, Princeton University Press, 1949. Price, $5.00.

A large portion of this book is a translation of an article written in 1926 and published in 1927 under the title "Philosophie der Mathematik und Naturwissenshaft," appearing in R. Oldenbourg's *Handbuch der Philosophie.* The book has been brought up-to-date by a number of minor alterations, the inclusion of references to recent publications, and the addition in six appendices of brief essays based on recent developments in mathematics, physics, chemistry and biology, the latter having been excluded from the original work principally because of another specialized article in the same publication.

As indicated by the circumstances attending its original preparation, the book is concerned as much with the historical as with the systematic aspects of scientific philosophy. In his preface the author states that "one of the principal tasks of the book should be to serve as a critical guide to the literature listed in the references," much of which, as well as parts of the critical guide itself, will require a certain amount of preparation on the part of the reader.

Part One, Mathematics, discusses Mathematical Logic, Number, and Geometry; and Part Two, Natural Science, is concerned with Space and Time, Methodology, and The Physical Picture of the World, all from the point of view of the philosophically-minded mathematician and scientist. Ideas and concepts made definite by modern experimental science are compared with those of earlier philosophers and natural scientists. In the course of the discussion, the line of thought covers a wide range. It is intriguing, for example, to find in a single sentence, and presented in a single connection, references to Albert Schweitzer, Coleridge, and St. Anthony's sermon to the fishes.

Essays in the appendix, based on recent scientific work, include The Structure of Mathematics; *Ars Combinatoria* (with applications to biology and genetics as well as the other sciences); Quantum Physics and Causality; Chemical Valence; Physics and Biology; and finally Morphe and Evolution, with references to Bergson, Shaw's "Methuselah," H. F. Osborn, P. Jordan, Julian Huxley, and the Bible included in the closing paragraphs.

In the concluding sentences of Part Two, the author presents the following summary: "Exact natural science, if not the most important, is the most distinctive feature of our culture in comparison to other cultures. Philosophy has the task to understand this feature in its peculiarity and singularity. The ideas presented here . . . should be looked upon as endeavors toward this end—although I cannot but admit that the task is at present far from being completely accomplished."

C. T. CHASE

SKYSHOOTING, by R. Newton Mayall and Margaret Mayall. 174 pages, illustrations, and plates, 16 × 24 cm. New York, Ronald Press Co., 1949. Price, $3.75.

Many, many times in the observatory of The Franklin Institute the reviewer has been asked the question, "How do I take a photograph of the moon, the sun, or of the stars?" After the question there came the involved answer—dealing not only with the question but also the inquirer's ability as a photographer. Now along comes a book with the completely modest title of "Skyshooting" which answers all the questions one can raise regarding celestial photography.

The Mayalls have written a wholly intelligible book containing all the essentials necessary for the taking of sky pictures. This reviewer has been through the entire routine. He spent six years photographing the sun each clear day at the Cook Observatory. He has used the 10-in. astrographic and the 28-in. reflector at the same Observatory. He has taken planet pictures with the 10-in. Zeiss refractor and has photographed the moon in and out of eclipse with the 24-in. reflector of The Franklin Institute Observatory. With this as a background

he whole-heartedly endorses the book "Skyshooting." It contains all the information needed to use a camera on the sky.

There are some minor suggestions which could be offered and these not in the sense of criticism, but in hope that in a second edition these might be given serious consideration. The summary presented at the end of each chapter is an excellent digest of the content of the chapter. It would be desirable to have a single section in an appendix which would combine all this information where it may be immediately available. A listing of all the Amateur Astronomical Associations similar to the listing found in *Sky and Telescope* would be a welcome addition and more to be desired than the list of the professional observatories in the country This book may not appeal to the professional astronomer because in all probability his background includes most of the material contained in the book. This type of book will appeal to the layman who seeks a hobby or to the amateur astronomer who desires to do serious photographic work on the sky. A listing of organizations where this work may be furthered would be a valuable addition to the book.

The book is remarkably free of errors and only in one place is there any question as to the text. The authors state, ". . . the most valuable property of the photographic emulsion is its light-gathering power, ..." The word gathering is an unfortunate choice. The optical system determines the light-gathering power—a better word would have been light-storing.

The book is well written, beautifully illustrated with carefully selected amateur photographs, excellently compiled and highly recommended for the average individual who would turn his camera to the sky.

I. M. LEVITT

ELEMENTARY METALLURGY AND METALLOGRAPHY, by Arthur M. Shrager. 297 pages, tables, illustrations, 16 × 24 cm. New York, The Macmillan Company, 1949. Price, $4.75.

The increasing interest in metals and alloys used in manufacturing machines, tools, appliances and industrial products, makes it necessary for those dealing with metals to have a clear understanding and knowledge of the underlying principles and concepts of metallurgy and metallography. Many excellent books have been written as well as a vast number of technical papers on the general subject and specialized fields. However, most of these are treated on or above the engineering level. This book treats the subject matter from an elementary approach for individuals below the engineering level, requiring metallurgical training. The subject matter is presented well in a clear, concise manner which is easy to follow yet combines theory and practice in a well ordered progressive sequence, with sufficient theoretical background to afford anyone interested in the subject of metals and alloys ample opportunity to gain a useful working knowledge and understanding of the subject. The book consists of 24 chapters dealing with structure of metals, slip and plastic deformation, recrystallization, mining, ore reduction, constitutional diagrams. The structure of iron and steel in the solid state is well illustrated. The allotropic changes and transformations which take place upon heating and cooling are included. Typical constitution diagrams are shown illustrating their application and use in connection with the various processes such as heat treating. The correlation of microstructure, and physical properties with the constitution diagrams is useful.

Most metals occur in nature as ores containing a great deal of other material and must be separated and purified to be put in useful form as metals or alloys for specific purposes. The processes involved in winning metals from their ores are described, and the history of steel is covered showing how advancement in the art and science has progressed from the sword of Damascus to our present alloy steels used for highly stressed parts. The function and use of alloying elements together with laboratory methods of control are described briefly. The principles underlying heat treatment, working and fabricating metals are covered including the more recently developed isothermal transformation diagrams, hardenability tests, Austempering, Martempering, induction hardening, and surface hardening. The mechanical properties can be altered by suitable alloying, working and heat treatment resulting in more satisfactory performance of steel parts for industrial use.

Chapters covering copper, aluminum, magnesium, and zinc together with typical alloys and properties are included. Useful tables are included as well as an excellent glossary of metallurgical terms.

This book should prove useful to a large number of individuals working with or dealing with metals and alloys. Presented in such a clear, concise manner, it fills a real need in the metals industries for sound information about metals.

STUART S. KINGSBURY

STRENGTH OF MATERIALS, by Gerner I. Olsen. 442 pages, illustrations, 16 × 24 cm. New York, Prentice-Hall, 1949. Price, $4.25.

Another book has been added to the already long list of books on the subject of strength of materials. However, this differs from most of the others in the method of approach to the various subjects. As the author indicates in the preface, the book is for use in technical institutes, college extension courses, trade schools and colleges where the subject is presented without the prerequisite of calculus. Consequently, the material is presented very simply and directly, requiring no more mathematics than algebra. Refreshingly, too, the author emphasizes the practical use of the material. Unfortunately, the scope of the various subjects treated is somewhat limited, but then it isn't written for the advanced student.

The book contains the usual strength of materials subject matter. The initial three chapters discuss the principles of mechanics and fundamental stress and strain relationships. Then follow chapters on thin-walled cylinders and spheres and riveted and welded joints, torsion, shear and bending moment diagrams, design and deflection of beams, statically indeterminate beams, stresses due to eccentrically applied loads, columns, combined stresses, and fatigue strength and stress concentrations. Excellent illustrations throughout the volume aid greatly in the explanation of various concepts. Sometimes it is difficult to "see" forces or stresses in three-, or even two-, dimensional objects; the student should have little or no such difficulty upon studying this book. Examples are used frequently to explain theories, and each chapter contains problems whose solution will aid further in the understanding of the strength of materials.

E. W. HAMMER, JR.

COMMUNICATION CIRCUITS, by Lawrence A. Ware and Henry R. Reed. Third edition, 403 pages, illustrations, plates, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1949. Price, $5.00.

Having used the first and second editions of this book for classroom instruction, this reviewer finds the present third edition a welcome addition to that group of textbooks especially written for communications engineering students taking a first course in this subject. It is presupposed that the reader has a knowledge of calculus and a-c. theory. Sufficient vector and electromagnetic theory to develop Maxwell's equations in forms useful to the modern communications engineer is included in an appendix, as are the elements of Bessell functions.

The elements of transmission circuits are common to many branches of electrical engineering. A first chapter, completely rewritten from the earlier editions, treats of wire transmission line parameters, their physical interpretation and means of computation for the entire frequency range of interest. The ideas of shunt conductance and capacitance lead to the development of the short line with lumped constants and the standard transformations for T and π sections.

In a clearly written style the authors carry the reader through the basic ideas of characteristic impedance, propagation constants, and distortion, first for the line composed of finite sections; then the corresponding theory is developed for the line having uniformly distributed parameters. These concepts are finally used to consider the transmission line which is terminated first in its characteristic impedance resulting in unidirectional propagation, then in a general terminating impedance which causes reflections. The open and short circuited line are given the usual "special consideration" because of the importance of these cases in line tuning. Questions of line matching and impedance transformations are treated separately because of their importance.

As a natural development from the frequency dependency of T and π sections, consideration is given to the fundamental properties of reactive networks, which serve as a basis for the design of the common varieties of constant-K and M-derived filters.

Circular and cylindrical wave guides and coaxial cable are today commonly used transmission means. An elementary but thorough treatment of introductory problems is included in this edition. These subjects are treated by field theory based upon Maxwell's equations with appropriate boundary conditions. One chapter is devoted to each of the major transmission means noted above, each time starting with the basic differential equations to derive the important parameters. One chapter is devoted to an outline of twelve transmission line experiments which illustrate basic principles.

This volume should find the same wide acceptance in educational circles as did the earlier editions. The illustrative examples worked out in the text help the reader grasp the principles presented and the rather complete set of problems, many of them new in this edition, are sufficient in number to keep any student busy for quite some time. Practicing electrical engineers who wish a quick review of some basic communications engineering would do well to read this book in their leisure moments.

S. CHARP

ATOMIC ENERGY LEVELS, Vol. I, ^{1}H–^{23}V. National Bureau of Standards Circular 467. 309 pages, 24 × 30 cm. Washington, National Bureau of Standards, 1949. Price, $2.75.

A critically evaluated compilation of all known data on the energy levels of elements of atomic number 1 through 23 has recently been published by the National Bureau of Standards and is now available from the U. S. Government Printing Office. Designed to meet the needs of workers in nuclear and atomic physics, astrophysics, chemistry, and industry, the publication is an up-to-date compendium of all energy levels for these elements exclusive of those due to the hyperfine structure ascribed to atomic nuclei.

The present volume is the first of a series being prepared at the Bureau as part of a general program on atomic energy levels derived from observations of the optical spectra of atoms and ions. Section 1 of this volume (hydrogen through fluorine) has previously been published separately. The new series represents the cooperative effort of scientists throughout the world and will constitute the first compilation of atomic energy levels in the last 18 years, during which time the number of known energy levels has increased by a factor of 4 or 5.

In this series, spectra are presented in order of increasing atomic number, and under a given atomic number they are listed in order of increasing stages of ionization. For each spectrum a selected bibliography covering the analysis is given. The energy levels are tabulated in the related groups that form spectroscopic terms, counting upward from the lowest as zero. Electron configurations are also given in the tables, together with term intervals, Lande g-values, and term designations in a uniform notation. For the more complex spectra arrays of observed terms and their electron configurations are included. Similar arrays of the terms predicted by theory for important isoelectronic sequences are given in the introduction.

Volume 1 (containing Sections 1–3) of the National Bureau of Standards Circular 467, entitled *Atomic Energy Levels*, by Charlotte E. Moore, 352 large 2-column pages, cloth-bound, can be obtained from the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., at a cost of $2.75 a copy. Remittances from foreign countries must be in U. S. exchange and must include an additional sum of one-third the publication price to cover mailing costs.

ERRATUM: "Photoelectricity and Its Applications," by V. K. Zworykin and E. G. Ramberg, reviewed on p. 267, Vol. 248, of this JOURNAL, was erroneously listed as "Photoelasticity and Its Applications."

BOOK NOTES

TECHNICAL DRAWING, by Frederick Giesecke, Alva Mitchell and Henry Cecil Spencer. Third edition, 851 pages, illustrations, 16 × 24 cm. New York, The Macmillan Co., 1949. Price, $4.50.

This standard textbook on engineering drawing has been considerably revised and amplified in the present edition. The chapters on Fasteners, Shop Processes and Aeronautical Drafting are among those which show extensive revision. In addition to including over one hundred new illustrations, many of the old ones have been redrawn to make them clearer. The text throughout has been brought into accordance with the 1946 revision of "American Standard Drawings and Drafting Room Practice." The material contained in the appendix adds much to the reference features of the work.

BIBLIOGRAPHY OF RESEARCH ON HEAVY HYDROGEN COMPOUNDS, compiled by Alice H. Kimball and edited by Harold C. Urey and I. Kirshenbaum. 350 pages, 15 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1949. Price, $3.25.

This volume of the National Nuclear Energy Series presents a listing of nearly two thousand references on heavy hydrogen and its compounds. The list is alphabetical, arranged with cross references from variant spellings of names and co-authors. A code system has been established to indicate the general scope of each article. A subject index arranged by these code symbols is included indicating the articles coded under each classification. In addition, there is an index by compounds, subdivided by the code symbols.

The list was compiled primarily from *Chemical Abstracts* 1932 to 1945, with additions from other sources. A useful feature is the inclusion of the *Chemical Abstracts* citation in nearly all cases. With its various aids, the bibliography will serve a number of purposes and should prove valuable for anyone concerned with the early work on deuterium.

TV PICTURE PROJECTION AND ENLARGEMENT, by Allan Lytel. 179 pages, illustrations, 14 × 22 cm. New York, John F. Rider Publisher, Inc., 1949. Price, $3.30.

This book offers a presentation of the basic optical principles involved in television picture projection, with a description of the different methods employed. Particular attention is paid to the commercial applications of the modified Schmidt and the refractive projection systems. Suggestions are made as to adjustments. Brief consideration is given to newer systems and possible developments in television projection. The work should prove useful for both the serviceman and the student.

CURRENT TOPICS

Aerocar Prototype to Begin Flight Tests This Month (*American Aviation*, Sept. 15, 1949).—A new development in the roadable airplane field is scheduled to unfold this month at Longview, Wash., where the Aerocar prototype is slated to begin flight tests.

Officials of Aerocar, Inc., report that to date the auto portion of the craft has traveled about 1000 miles; that driving it is routine—except for the problem of handling the crowds of people whenever it's parked in town.

Tests have shown the mileage to be 28 miles to the gallon, equal to the best performing automobiles in the light auto bracket. The craft attained speeds of about 60 on the highway with best performance determined at about 45 miles an hour. On a sod surface runway it accelerated to 75 mph. in 900 ft., plenty of speed for a take-off run. It appears that the Aerocar can perform about like a Crosley or an Austin on the highway.

The early-scheduled flight tests should go smoothly, say the engineers, since wind tunnel tests in 1948 indicated that design characteristics were satisfactory. Tests were run off at the University of Washington with a one-fourth scale model which preceded the development of the prototype. In the wind tunnel the model indicated the design competed favorably with that of conventional lightplanes.

The prototype incorporates the spin-proof, two-control feature and full provision for cross wind landings, with one steering gear for operation in the air and on the highway. It has front wheel drive from a 100 hp. Franklin engine which furnishes power for the more or less conventional prop at the tail of the flight component. The shaft to the prop was one of the most troublesome design problems, but officials say it has been solved to their satisfaction and to the "fair satisfaction," so far, of CAA. Full throttle operation of the drive shaft under stringent conditions showed that it should prove satisfactory in the air. Engineers expect top speed in the air to be about 109 mph., normal cruise about 100. Rate of climb should be over 500 fpm. and service ceiling 12,000 ft. Cruising range should be "about 300 miles;" it should land at 53 mph. with flaps in 300 ft. to full stop. It should take off and clear a 50 ft. obstacle in 1250 ft. Fuel consumption is estimated at 6 gal./hr. in the air.

The Aerocar has all of the equipment needed to meet state vehicle laws for operation on the highway. Four wheel brakes, three speeds forward, one in reverse are controlled with conventional foot pedals and gear shift. Excellent ground stability is designed into the auto component built entirely of metal, employing separate keel box structure. Hydraulic shock suspension ride control and roll-down windows should insure comfortable riding on the highway.

Fort Benning Housing Project Uses TECO Trussed Rafters.—Battle Park Homes, Inc., the Army's new 200-unit housing project for service personnel near Fort Benning, Georgia, may well serve as an economical prototype for the long-range plans of the National Military Establishment to alleviate the short-

age of quarters in all branches of the service, says Homer Maxwell of Maxwell and Hitchcock, Atlanta, who have worked closely with the designers and builders in effecting new economies in the project.

When completed, Battle Park Homes will make available 88 modern 4½ room and 112 5½ room residences for service families stationed at the big Georgia base. James W. Biggers and Associates, Columbus, Georgia were the architects for the project, which is being built by the Jordan Construction Company.

Expansion and improvement in quarters at bases in all branches of the service has long been sought by the various departments. The Army alone estimates its needs at 150,000 new units in the next few years. Many service families have been occupying quarters which were built as temporary structures during the first World War. It now appears that, coincidental with the passage of the new general housing bill, military families will also find relief through Congressional authorization for extensive construction of multi-family rental units at scores of posts.

The new Fort Benning units are high-efficiency, low-cost buildings, says Maxwell. They are F.H.A. Type 608 built with a wood frame covered with brick veneer on concrete slabs. In addition to the low-cost wood frame an important new economy has been added in the form of fabricated trussed rafters joined with split ring connectors. The new rafters cut roof framing material about 25 per cent over conventional methods and reduce labor accordingly.

Maxwell explains that the trussed rafters, based on designs developed by the Timber Engineering Company, Washington, D. C., are simple four-member assemblies composed of standard 2 × 4 and 2 × 6 lumber. Extra strength gained through the use of the timber connectors reduces the size of the lumber required to do the job and permits the trusses to be spaced 24 in. on centers rather than the usual 16 in. used with ordinary rafters.

The new system which has already been used in nearly 20,000 units in privately built garden type apartments, does away with the need for bearing partitions and thus saves labor on most interior mechanical operations. Trusses are usually built on the job by the contractor through a simple fabrication system employing common labor. The rafters are spaced and held in place by another new development—TRIP-L-GRIP framing anchors, easily installed lightweight metal clips providing reliable ties between roof and walls.

Three other new Georgia projects, the Poole Creek Apartments, Atlanta, the University Court Apartments, Athens, and a housing project of the Mangham-Butler Company in Atlanta, have also adopted the new roof framing system.

Cortisone Research.—The necessity for providing financial support for further clinical research on cortical and related hormones was highlighted recently at the Cortisone Conference sponsored by Research Corporation.

Dr. Robert R. Williams, director of grants for Research Corporation, outlined for key scientists and representatives of private foundations and government services three steps that had been taken to speed the vital research on the hormone which has been widely heralded for its promise in treatment of rheumatoid arthritis, rheumatic fever and other diseases:

1. An agreement is about to be effected with owners of patents on many of the 30-odd steps in the present method of producing cortisone whereby each will be free to use the patents of the others. Research Corporation will also be free to license other firms under any of the patents. This is regarded as an important step in assuring future production of cortical hormones for public use.

2. Research Corporation and Merck & Co., Inc., present sole producer of cortisone, have agreed to abide by the decisions of the committee appointed by the National Academy of Sciences to allocate the currently meager supply of cortisone. This agreement assures that the quantities of the hormone available in the next few months will be employed only for experimental use by qualified scientists whose requests are approved by the National Academy.

3. In addition to grants of some $50,000 made since 1941 to Dr. Kendall of Mayo Clinic, discoverer of cortisone, Research Corporation is contributing about $85,000 this year in grants to scientists in American academic institutions who may be able to contribute better synthetic approaches to cortisone and related substances.

Pointing out the immediate need for additional research funds, Dr. Williams declared, "The entire output of bile acids (present raw material for cortisone) from our slaughter houses will not make enough cortisone to provide continuous treatment for more than one-tenth of one per cent of all the arthritics in this country, should such treatment prove necessary or desirable.

"The remedy is not to be found in minor improvements of present process. A synthesis must be developed either from simple artificial substances or from some much more abundant natural material. Either will require extended research, but important clues to processes on other raw materials exist and are being explored."

Enormous Coal Reserves of Montana Appraised.—Coal reserves in Montana of 221,779,000,000 tons in place recently appraised by the U. S. Geological Survey constitute an assured and abundant source of fuel that will be available when needed to supply synthetic fuel plants, Director W. E. Wrather has announced. More than 28,000,000,000 tons of lignite and sub-bituminous coal, or about 12 per cent of the total reserve is known to be present in beds 10 feet or more thick, lying under less than 2000 feet of overburden, and less than 2 miles horizontally from the outcrop.

The new appraisal is the first of a series of state summary studies being undertaken by the Geological Survey as part of a program to reappraise the coal reserves of the United States. During the period since 1928, when the last coal reserve estimate for the United States was prepared, much new work has been done by the Geological Survey and State agencies on the thickness and distribution of coal. Similarly, new estimates have been prepared for several eastern states, and for many individual mining districts. But for most States, including Montana, and for the United States as a whole, no comprehensive re-appraisal has been undertaken.

The new appraisal of Montana coal reserves was prepared in greater detail than the earlier estimates by the Geological Survey, and presents the data by counties, by rank of coal, by classes of coal according to the reliability of the

information on which the calculations were based, and by coal beds in several thickness groups. The total reserve of 221,779,000,000 tons is somewhat smaller than the earlier estimate of the Geological Survey, but the present estimate was made on a much more conservative basis, and is believed to represent a minimum figure. As new information is obtained it is expected that the estimate will be substantially increased.

Combating Cosmetic Allergy (*Manufacturing Chemist*, Vol. XX, No. 8).— However strictly a cosmetic manufacturer controls the processing of his products he must always reckon on the mischance of his lipstick or face powder being used by a person allergic to one or other of the ingredients. Unless the person is aware of her idiosyncrasy and seeks medical help the chance is she will blame the product in question and tell her friends, to the detriment of the product. The point is, assuming medical advice is sought, is it always at hand? That doctors are not always properly informed is indicated by a note in the "Any Questions" section of a recent issue of the *British Medical Journal.* The questioner asked if anything was known about allergy to lipsticks and quoted a case of a patient coming to him with raw patches and swelling of the lips occurring some two hours after using a lipstick. The answer explains that such a condition may be due to hypersensitiveness not only to eosin—the most frequent cause—but also to the perfume or fatty base used. It is also stated that there may be a superadded factor of induced light-sensitiveness owing to the fact that eosin is fluorescent. A change to a lipstick not containing eosin is recommended and the enquirer is advised to seek information from a recognized cosmetic house. It seems that in view of the prevalence of lipstick allergy manufacturers might circulate doctors with an explanatory leaflet and, where applicable, point out that a non-eosin lipstick is available. This would surely be a piece of sales promotion of great value to both manufacturer and doctor. Incidentally, it should be noted that the *B. M. J.* expert states that the application to the lips of a barrier cream, of the kind employed as a protective against industrial irritants, has been known to permit the use of a lipstick without injury.

Portable Spray Drier (*Chemical and Engineering News*, Vol. 27, No. 38).— A unique, portable spray-drying unit of Danish design is being made available in this country through the Niro Corp.

This compact laboratory unit is particularly designed for experimental work in universities, laboratories and small scale production in pharmaceutical and industrial organizations. It may be easily wheeled from one department to another. Its performance is parallel with that of the Niro units now in use. This makes it possible to determine, on a small scale, the practicality and the technique of spray-drying various substances on a large scale.

The Niro Portable Laboratory Spray features the specially designed, high-speed atomizer head, rotating at 50,000 rpm. and ejecting the liquid product as an extremely fine mist into the drying chamber, where circulating warm air instantaneously evaporates the water content. The dry product falls to the bottom of the chamber as a fine powder, where it is funneled out as a finished, easily soluble product.

The new portable unit is constructed of stainless steel and glass, making it

suitable for conducting test runs on a wide variety of materials. The unit is equipped with both electrical and gas heating elements, and either may be used. Its capacity is 2 to 15 lb. of water evaporated per hour.

Melamine Dinnerware Promotion (*India Rubber World*, Vol. 120, No. 6).—In order to make Melmac melamine plastic dinnerware available to the general public, the plastics department of American Cyanamid has launched a comprehensive advertising and sales promotion program designed to expand consumer markets. Melmac dinnerware has been thoroughly tested and widely accepted by restaurants, hotels, and institutions, and now a number of large molding companies will offer their lines of the product through retail stores. American Cyanamid will run a series of advertisements in consumer pubilcations, and will also cooperate extensively with the actual producers of Melmac dinnerware and with all retail outlets in sponsoring sales to consumers.

The main features of the campaign will stress that this dinnerware is a new type and style of plastic dinnerware unlike the so-called picnicware known in the past. Specifically, breakage, resistance, attractive color and design, food insulating properties, resistance to food staining, ability to withstand hot water, and similar characteristics will be pointed out to consumers.

An Electrostatic Clutch (*Discovery*, Vol. X, No. 8.—Among the exhibits which Metropolitan-Vickers Electrical Co. Ltd. showed at the fourth annual exhibition held by the Institution of Electronics in Manchester on July 19–21 was an electrostatic clutch. A polished disc of high-resistance semi-conductor and a polished metal disc, both optically flat to 10^{-4} cms., are pressed lightly together. When a potential is applied between them, a force of attraction is developed approximately proportional to the square of the potential across the discs. This effect was first observed by Johnson and Rahbek using moist lithographic stone, and ionic semi-conductor. An electronic semi-conductor, unlike lithographic stone, is not dependent on moisture content and has, therefore, been substituted in the model.

A small clutch operating on this principle was exhibited driving a dynamometer and transmitting a torque of several hundred g.cm. The power consumption of the clutch is very low.

Truck Able to Lift 110,000 lb.—A huge trailer truck and four automobiles were lifted 5 ft. from the ground by the world's largest industrial truck in a demonstration in Chicago.

The powerful truck was built for the Oldsmobile division of General Motors Corp., by the Automatic Transportation Company, 149 West 87th Street, Chicago. The show of strength took place at the Automatic factory.

Designed to lift and carry dies weighing 80,000 lb., it actually can handle loads up to 110,000 lb.

The die handling machine carries its load on a platform 93 in. long and 69 in. wide. From a starting position 26 in. off the ground, the load can be lifted to a height of 60 in.

The truck is estimated to weigh 40,000 lb., so in normal factory operation the carrier and load will total 120,000 lb. The demonstration was a mild workout requiring the lifting of a mere 15 tons.

A 72-volt battery powers the machine, enabling it to lift its massive load at the rate of 2.7 ft. per minute.

At its broadest point the truck is 85 in. wide and it has an over-all length of 184 in.

Despite the great weight involved, simple pushbutton controls guide its operation. Maneuvering dies on or off the press is a delicate task for which a winch attachment is used. This can be controlled by buttons on either side of the truck, giving the operator perfect vision.

Following the demonstration, the die handler was shipped to Lansing, Mich., where Oldsmobile's plant is located.

Designer of the record-breaking machine is Edward W. Gammell, of Automatic's engineering staff.

Automatic has been the leading producer of huge materials handling machinery since 1906. It is the producer of the Skylift Giant series of ram trucks, able to handle huge coils of steel weighing up to 30 tons. The Giants are now being used in many mills in this country and abroad.

The George Eastman House, Inc., a public educational institute to further the knowledge of photography's means, accomplishments and potentialities in every field, will be opened to the public this month.

It is fitting that the home which George Eastman built in Rochester, N. Y., should thus become an educational focus in the field which was so greatly advanced by his genius, by placing on exhibit the technological developments and the applications of photography both in the historical past and the present progress in the entire field of photography.

The institute, with facilities for exhibitions, demonstrations, lectures and the showing of moving pictures, will be a national and international center for conferences on photography; for research in its history, science and application; for the education of school children by guided tours; for meetings of camera clubs and photographic organizations, for changing exhibitions of photographic work; for demonstrations of the latest apparatus and processes; for the screening of both historical and modern motion pictures; and for other related purposes.

Such an institute should become the mecca not only of all interested in picture-making—from the specialist to the amateur snapshooter and tourist—but to the ever-increasing number of writers, magazine editors, and historians who use pictures.

Thus the purpose of the institute will be to demonstrate the technique of photography, to show the manifold part it plays in nearly every branch of human activity, and to facilitate the research of those interested in the world's most facile medium of communication.

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GIANT MOLECULES IN SOLUTION *

BY

THE SVEDBERG [1]

In a primeval sea, life probably originated from further development of giant molecules in solution. How these were formed we do not know—as a possible cause we might assume some chance combination of amino acids. In the solid state well-defined, closed molecules are rare and we are, therefore, led to believe that such entities as protein molecules were first built up in a solution. At present we find them only as synthesis products in living beings. The viruses occupy an intermediate position between dead and living matter, between protein molecules and bacteria and they are known to exist in a liquid or semi-solid state.

Such giant molecules as we know from the solid, crystalline state and which are incapable of directly going into solution without the action of chemical agents are not so well-defined. They mostly represent the repetition of a basic unit in various degrees of polymerization.

It was through physico-chemical studies of protein solutions that the existence of well-defined giant molecules was found. The colloid character of proteins was recognized at an early date—as a matter of fact the term "colloid" was coined from observations on a protein, gelatin—but the monodispersity or paucidispersity of these substances could not be proved by means of the methods in use at that time.

Convection-free sedimentation in the ultracentrifuge combined with refined diffusion measurements gave the surprising result that most of the proteins in solution and some of those in the condensed state were built up of well-defined molecular units. An enthusiastic team of young research men—rather international in character—carried out the first observations of this kind in Upsala (the American Nichols, the Austra-

* Talk presented at the Medal Day Meeting at The Franklin Institute, October 19, 1949. (Read by Dr. J. Burton Nichols, in the absence of the author.)

[1] Director, The Gustaf Werner Institute for Nuclear Chemistry, Upsala, Sweden.

lian Lewis, the Swede Fahraeus, the Romanian Chirnoaga). We had happened to hit upon representatives of quite a few different classes of proteins for our first experiments, ranging in molecular weights from about 40,000 (egg albumin), about 70,000 (hemoglobin), about 150,000 (phycocyan), about 300,000 (phycoerythrin) to about 9,000,000 (hemocyanin). A rapid development followed, especially in America, England and Sweden, both with regard to experimental tools and the investigation of substances. At present sedimentation constants can be measured with an accuracy better than 1 per cent.

In principle the sedimentation experiments are carried out in the following way. A small sample of the solution to be studied is enclosed between quartz windows in a sector-shaped cell, put into a suitable rotor (in Upsala we use steel) and spun at high speed in low pressure (the Upsala procedure) or in a vacuum. Observations of the sedimenting boundary in the solution are carried out by means of refractive index or light absorption measurements. From the data thus obtained the sedimentation constant is computed and, by combination with the diffusion constant, the molecular weight can be found.

Among the results on protein molecules, existing spontaneously in solution, may be mentioned the following ones. The respiratory proteins are exceedingly well-defined. Their molecules possess one or a few prostetic groups, often containing a metal atom (iron, copper). The lowest molecular weights observed are those of cytochrom (12,000) and myoglobin (17,000). The red pigment of the nitrogen bacteria is 34,000 and the hemoglobin of the vertebrates 68,000 with the exception of the lowest class, the Cyclostomata, the members of which have a blood pigment of molecular weight 17,000. The copper-containing hemocyanines of the Gastropodes and the Crustaceans present a number of quite interesting features. By raising or lowering the pH or, in certain cases, simply by the addition of salt, the molecule often dissociates into halves and eighths. The hemocyanin of the vineyard snail contains two kinds of molecules in the proportion 3:1. Both types are dissociated by certain pH-changes but the more abundant type also by salts. As an other example of reversible pH-dissociation one could mention the phycocyan, the blue pigment of the algae, the molecule of which splits into halves already at a pH of 6.8.

The serum proteins represent a very important but also very complex group. Especially the globulins are numerous. In pathological sera new constituents often appear, or the proportions of the normal ones are changed. In the serum of the fetus and the new-born of certain animals (ox, horse, sheep) a new protein, named the fetuin, was recently demonstrated. With age this constituent disappears. A comparative study of the different vertebrate classes shows that the serum of the birds closely resembles that of the mammals. The reptiles, the amphibians and the fishes possess sera of similar, though slightly different,

constitution, while the serum of the lowest class of the vertebrates, the Cyclostomata, is entirely different from all other vertebrate sera.

Among the proteins occurring naturally in the condensed state, but extractable by salt solutions, may be mentioned the vegetable globulins and the muscle proteins. The former ones are quite well-defined and often characteristic of their respective families; the latter ones, which (in contradistinction to most of the proteins mentioned above) have thread-like molecules, are not quite monodisperse. It seems to be the rule that globular macromolecules represent closed entities, while thread-like molecules can vary in size within certain limits, probably because of the ease with which sub-units may be added or subtracted.

The polysaccharides which play such an important part within the vegetable kingdom as support or reserve substances are mostly built up of thread-like macromolecules (cellulose, pectines). Only a few polysaccharides, such as the arabogalactane of the larch tree, has globular molecules. Also in the case of the polysaccharides the substances possessing thread-like molecules (such as cellulose) are more or less polydisperse, while those composed of globular molecules are monodisperse. The polydispersity of cellulose is, however, less pronounced than would be the case if its molecules had been formed by chance polymerization of glucose units. This shows that the formation of cellulose in the plants follows ways quite different from those which we are used to observe when high polymers are synthesized in vitro. The distribution curve for the molecular weights of native cellulose, as recently determined in Upsala, possesses several maxima, indicating the accumulation of certain molecular sizes in distinct groups.

The cellulose as it occurs in the wood is closely bound to lignin and it takes a certain chemical treatment to liberate it. If this is performed with care the product appears to be equally well-defined as the much less firmly bound cellulose of the cotton-seed hairs. Both in the case of wood cellulose and cotton cellulose the distribution curve shows maxima at a degree of polymerization of about 100, 200 and 300 glucose units. It is now of considerable interest that it has been possible, quite recently, to prepare by partial hydrolysis of native cellulose a sort of cellulose colloid built up of fairly equally sized rods possessing a length corresponding roughly to 100 glucose units as determined in the electron microscope. Electron diffraction studies on these cellulose rods have shown that they possess a very regular crystal lattice of the same general type but better defined than, according to previous X-ray studies, the cellulose fibers. Apparently the micelles as they occur in the fibers are partially hidden by amorphous material and thus less accessible to X-ray studies than in the isolated state. All this tends to convince us that in these rods we are confronted with the real elementary cellulose particles. These consist of bundles of parallel molecules of the type observed by means of the ultracentrifuge in cellulose solutions.

Ultracentrifugal and diffusion studies on synthetic polymers such as polymethacrylates and polychloroprene have demonstrated the chance distribution of their molecular weights. It has further been possible to show, in the case of chloroprene, that bulk polymerization produces chain-like, free-draining molecules, while in emulsion polymerization the molecules produced resemble matted coils.

The importance of high-molecular substances, both natural and synthetic, in biochemistry, biology, medicine and technology is becoming increasingly evident. When I started work along the lines mentioned above 25 years ago, interest in these things was rather limited. Largely through the encouragement I received in the United States during a stay at the University of Wisconsin in 1923 and later on through efficient financial help from the Rockefeller Foundation and through the enthusiastic collaboration with numerous colleagues and students from many countries, but foremost from the United States, I was led to continue my work on high molecular substances during many years. It is for me a great satisfaction that my endeavors have now been rewarded by this high distinction conferred upon me, The Franklin Medal.

RESPONSE OF AN ELASTICALLY NON-LINEAR SYSTEM TO TRANSIENT DISTURBANCES

BY

RUNE L. EVALDSON,[1] ROBERT S. AYRE,[1] AND LYDIK S. JACOBSEN[1]

ABSTRACT

The problem relates to an undamped, elastically non-linear, single-degree-of-freedom system subjected to three forms of ground motion pulses (rectangular, cosine and "skewed" cosine); each form consists of a single pulse, the duration of which has been varied over a wide range. The maximum relative displacements (maximum distortions) have been plotted in dimensionless form as functions of the pulse durations. Under some circumstances the non-linearity results in reduced maximum distortion, but in other cases it acts to increase it. Simple quantitative conclusions cannot be drawn. Analytical, graphical and experimental methods were employed; the graphical method has been presented in detail.

INTRODUCTION

The problem of a single-degree-of-freedom mechanical vibratory system subjected to a transient disturbance is found in many fields of engineering. When the restoring elements are linear the solution is straight forward but when they are non-linear it is often necessary to use experimental, graphical and approximate analytical methods. While the effects of *steady-state* disturbances on the non-linear case are fairly well known (**1, 2, 3, 4**)[2] the *transient state* has not received much attention (**6, 7**).

The investigation[3] consists of an experimental and graphical determination of the response to transient *ground motions*. The non-linearity of the restoring element is symmetrical with respect to the instantaneous position of the moving ground and involves an abrupt but constant increase in stiffness at a given relative displacement between the ground and the mass. The relative displacement at which the increase in stiffness becomes effective and the ratio of the two constant stiffnesses have been varied over wide ranges. The study has been limited to the case of zero damping in order that primary emphasis could be placed on the effect of the non-linearity.

The ground motions were of various single-pulse types, including the rectangular, the cosine and the "skewed" cosine shapes. The periods or durations of the pulses were varied widely. For comparative

[1] Graduate Student in Mechanical Engineering, Acting Associate Professor of Mechanical Engineering, and Professor of Mechanical Engineering, respectively, Stanford University, Stanford, Calif.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

[3] Sponsored by ONR at the Vibration Laboratory, Stanford University.

purposes the theoretical responses of the limiting linear systems to the same pulses are given.

The experimental portion of the program made use of a translational mechanical vibrating system which was subjected to the pulses by means of a cam-driven mechanical linkage. The graphical work employed a stepped approximation to the actual ground motion and a method (5, 6) which is related to the phase-plane method of non-linear mechanics.

THE VIBRATORY SYSTEM

The idealized vibratory system is shown schematically in Fig. 1, and the restoring force-relative displacement function in Fig. 2.

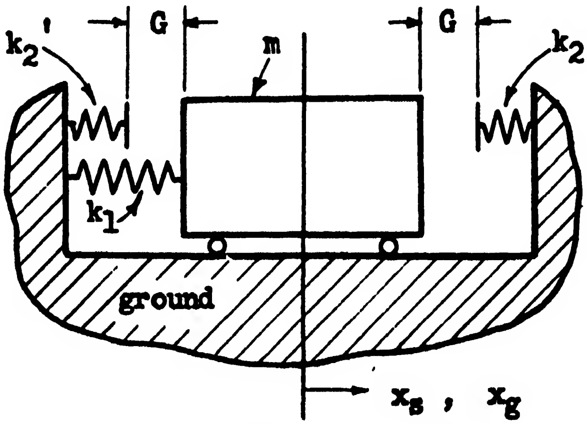

FIG. 1. The idealized vibratory system.

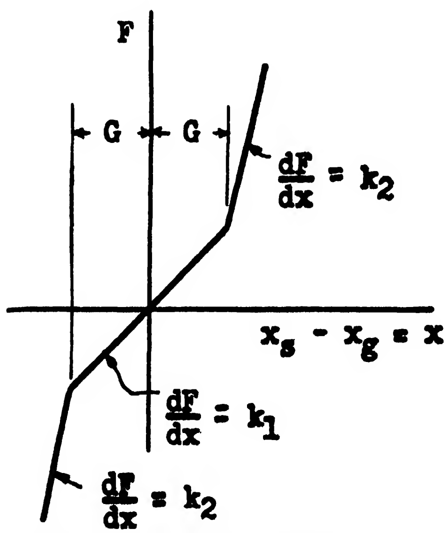

FIG. 2. Restoring force—relative displacement function.

The differential equation of motion is

$$m\ddot{x}_s + Kx_s = Kx_g, \tag{1}$$

where the notation is as follows:

x_s = absolute displacement of the mass of the system,
x_g = absolute displacement of the ground,
X_g = maximum value of x_g,
t = time, measured from start of the ground pulse,
t_g = period or duration of the ground pulse,
t_m = time of occurrence of X_g,
σ = degree of "skewness," defined as t_m/t_g,
ω = ground pulse frequency = $2\pi/t_g$,
$x = x_s - x_g$ = relative displacement of mass and ground (distortion),
X = absolute value of the maximum of the maxima of x for a particular system when subjected to a particular ground pulse,
X_m = the maximum X for a particular system when subjected to pulses of like form and amplitude but where t_g is varied,
m = mass of the system,
G = a distance (spring-gap),
k_1 = spring constant of a restoring element which is in action at all values of x,
k_2' = spring constant of a restoring element ("snubbing" spring) which acts only when $|x| > G$,
$k_2 = k_1 + k_2'$,
T = natural period of a linear system,
T_1, T_2 = natural periods of the limiting linear systems of a bi-linear family,
τ = degree of bi-linearity, defined as $\sqrt{\dfrac{k_2}{k_1}}$, that is, $\dfrac{T_1}{T_2}$,
F = restoring force, and
$K = \dfrac{F}{x}$ = instantaneous spring constant.

The natural period of the system shown in Fig. 1 is in general a function of displacement. However, the system will be linear in the two limiting cases, namely:

$$1.\ \text{When } \frac{X}{G} \lesseqgtr 1,\ K = k_1 \quad \text{and} \quad T_1 = 2\pi\sqrt{\frac{m}{k_1}}. \tag{2a}$$

$$2.\ \text{When } \frac{X}{G} \rightarrow \infty,\ K = k_2 \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{m}{k_2}}. \tag{2b}$$

Under each of conditions (2*a*) and (2*b*) the system is linear for all values of x. These limiting systems provide points of departure in the study of the non-linear family lying between them.

Since the restoring force-relative displacement curve is composed of three straight lines with two slopes, k_1 and k_2, the system will be termed

bi-linear. It approximates physical systems incorporating snubbing- or stop-springs. Furthermore, spring characteristics of continuous non-linearity may be reduced without excessive error to sequences of linear elements, of which the bi-linear characteristic is the simplest.

The degree of bi-linearity will be denoted by τ, with the definition:

$$\tau = \frac{T_1}{T_2} = \sqrt{\frac{k_2}{k_1}}. \tag{2c}$$

THE GROUND MOTIONS

The system was subjected to three forms of single pulse ground motion:

Form 1. Rectangular Pulse, (Fig. (3a)):

$$x_g = X_g \quad \text{for} \quad 0 \gtreqless t \gtreqless \frac{t_g}{2}, \tag{3}$$

$$x_g = 0 \quad \text{for} \quad \frac{t_g}{2} \gtreqless t \gtreqless t_g.$$

Form 2. "Skewed" Cosine Pulse, (Fig. (3b)):

$$x_g = X_g \frac{e^{(2\pi\sigma - \omega t)\cot \pi\sigma}}{1 - \cos 2\pi\sigma}(1 - \cos \omega t) \tag{4}$$

for $$0 < \sigma < \tfrac{1}{2}.$$

Form 3. Cosine Pulse, (Fig. (3c)):

$$x_g = \tfrac{1}{2} X_g (1 - \cos \omega t). \tag{5}$$

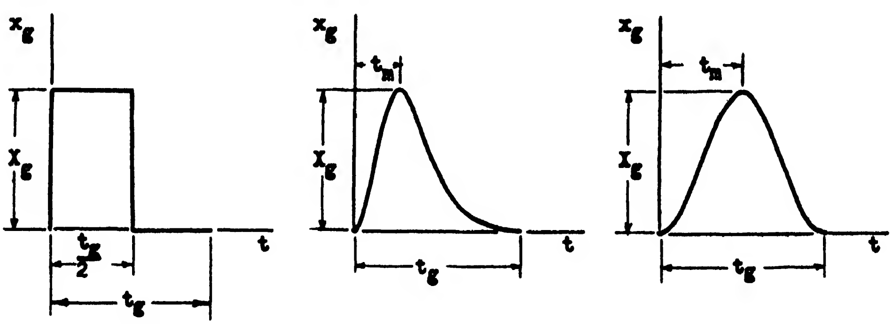

FIG. 3. The ground motion pulses (a) Rectangular pulse (form 1). (b) Skewed cosine pulse; $0 < \sigma < \frac{1}{2}$ (form 2). (c) Cosine pulse; $\sigma = \frac{1}{2}$ (form 3).

Form 2 represents a family of motions obtainable by variation of σ, the degree of "skewness." In the range $0 < \sigma < \frac{1}{2}$ the family provides examples of the common type of ground motion which progresses to a maximum and then recedes less rapidly to its original position. The portion of the family obtained by letting $\frac{1}{2} < \sigma < 1$ is of less interest. Form 3 is a special case of Form 2 where $\sigma = \frac{1}{2}$.

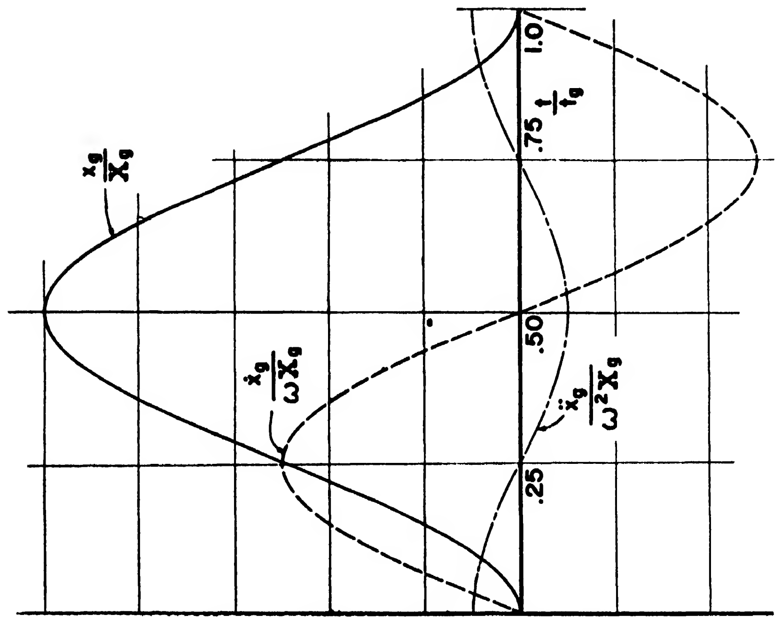

FIG. 5. Displacement, velocity and acceleration of ground; cosine ground pulse; $\sigma = \frac{1}{2}$.

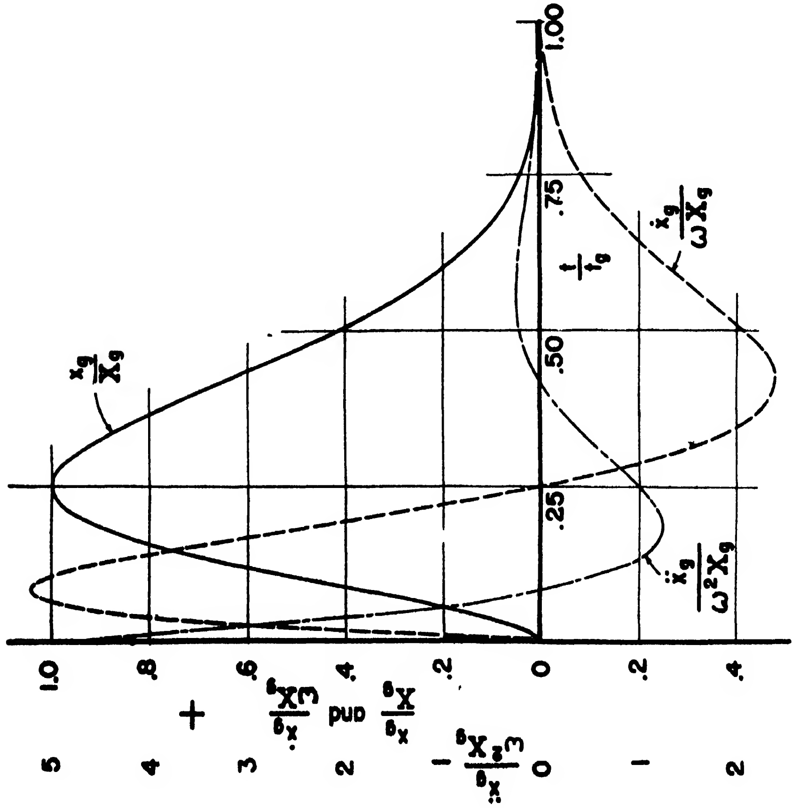

FIG. 4. Displacement, velocity and acceleration of ground; skewed cosine ground pulse; $\sigma = \frac{1}{4}$.

The displacement, velocity and acceleration of the ground for pulses of Forms 2 and 3 have been shown in Figs. 4 and 5. It should be noted that the effect of "skewing" the displacement cosine pulse results in a greater effect on the corresponding velocity pulse and in a still greater effect on the corresponding acceleration pulse.

RESULTS

Values of X have been obtained in two manners: first, by changing the ratio τ which defines the bi-linear family of systems; and second, by varying the ratio X_g/G which defines the bi-linearity *within* each family of systems.

In the experimental procedure τ was varied by holding T_1 and consequently k_1 constant while the "snubbing spring" k_2' and hence k_2 and T_2 were changed (see Figs. 6 and 7).

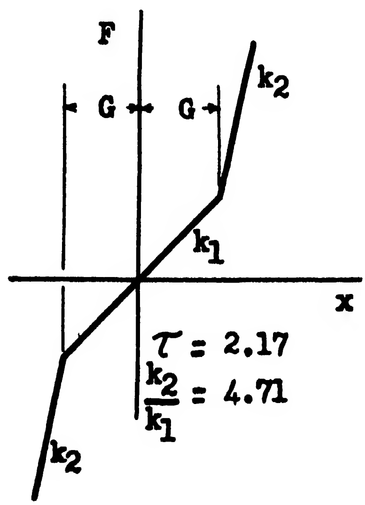

FIG. 6. System with "soft" snubbing springs.

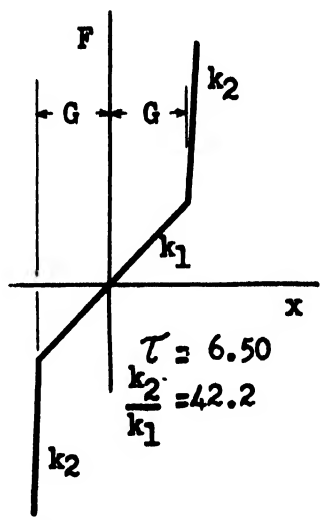

FIG. 7. System with "stiff" snubbing springs.

Nine diagrams follow in which X/X_g has been plotted as a function of $t_g/T = 2\pi/\omega T$ for the general linear case or t_g/T_1 for the bi-linear systems. Since the phenomenon is a transient one it becomes necessary to identify the time or order of occurrence of X, otherwise the discontinuities which appear in the diagrams are not readily explainable. In Fig. 8, which is a typical time-distortion curve for the linear system, the maxima have been labeled x_1, x_2, etc. In this particular case the greatest is x_3, which is then X; it occurs within the duration of the ground motion. Maxima which occur during the ground motion will be termed *forced vibration maxima* in order to distinguish them from the *free vibration maxima* which occur afterward.

The use of the identification is illustrated by Fig. 11a which gives the response of the linear system to the cosine pulse. The curve labeled

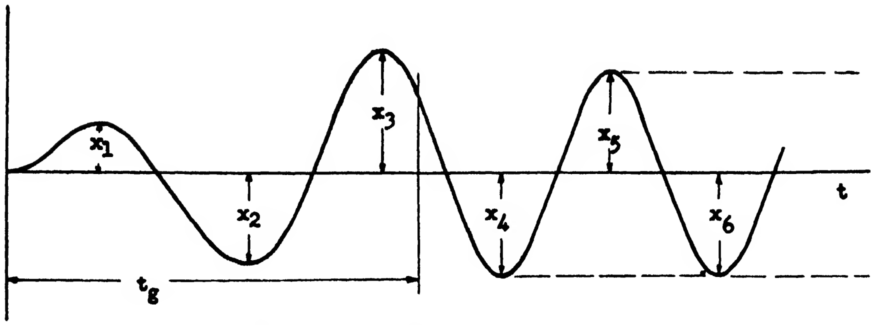

FIG. 8. Identification of the maxima.

1, 1, 1, $\cdots$ is the curve of x_1/X_g which always occurs during the ground motion and which is X/X_g for values of t_g/T from 0 to 0.30. The curve labeled 2, 3, 4, 5, 6, $\cdots$ is the curve of free vibration maxima, x_2/X_g, etc., for t_g/T from 0 to 0.48; moreover, for values of t_g/T from 0.30 to 0.48 it is also the curve of X/X_g. When t_g/T is greater than 0.48, x_2/X_g occurs during the ground motion and is less than X/X_g, except that for t_g/T from 1.67 to 2.42 it again becomes the maximum. The envelope of the curves for x_1/X_g, etc., is the curve of X/X_g. The envelope for a bi-linear system may be constructed in the same general

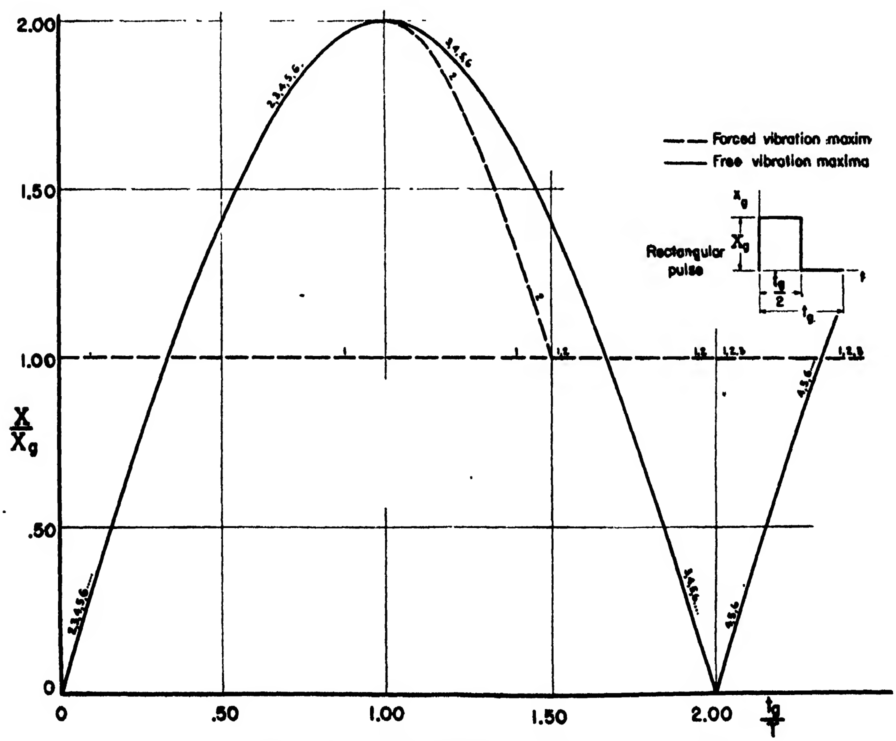

FIG. 9(a). Distortion of linear system; rectangular ground pulse.

The numbers on the curves refer to consecutive maxima (see Fig. 8). The peak of the envelope is symmetrical and repeats itself at regular intervals; X_m/X_g occurs at $t_g/T = 1, 3, 5$, etc.

manner. Since we are interested in the order of occurrence of the maxima only as it aids in locating and explaining the discontinuities in the envelopes, no further identification will be made.

The Linear System

The response of the linear system to the three forms of ground motion is shown in Figs. 9(*a*), 10(*a*), and 11(*a*). Since the linear case is

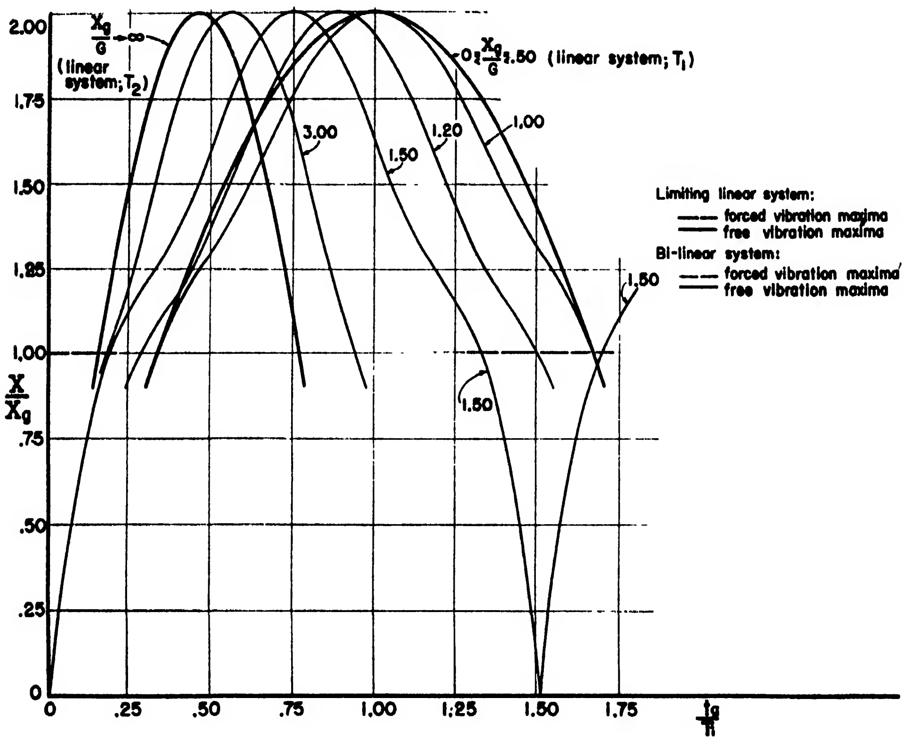

FIG. 9(*b*). Distortion of bi-linear systems, $T_1/T_2 = \tau = 2.17$; rectangular ground pulse.

Note that the first peak of the curve of free vibration maxima for one example, $X_g/G = 1.50$, has been shown in complete form. The peak is symmetrical and repeats itself at regular intervals. The curve of forced vibration maxima (horizontal dashed line) has not been shown completely. X/X_g is never less than 1.00 in the case of the rectangular pulse.

trivial in itself and serves only as an introduction to the non-linear family the following remarks will be limited:

1. X/X_g may occur during either the forced vibration or the ensuing free vibration, depending on t_g/T and the form of the pulse.
2. $X/X_g = 1.00$ at $t_g/T \to 0$, for any form of pulse.
3. X/X_g never falls below 1.00, for the rectangular pulse.
4. For the three forms of pulse investigated X_m/X_g occurs during the free vibration and at values of t_g/T as follows: rectangular pulse, 1.00, 3.00, 5.00, etc.; quarter skewed cosine pulse, 1.05; cosine pulse, 0.84.

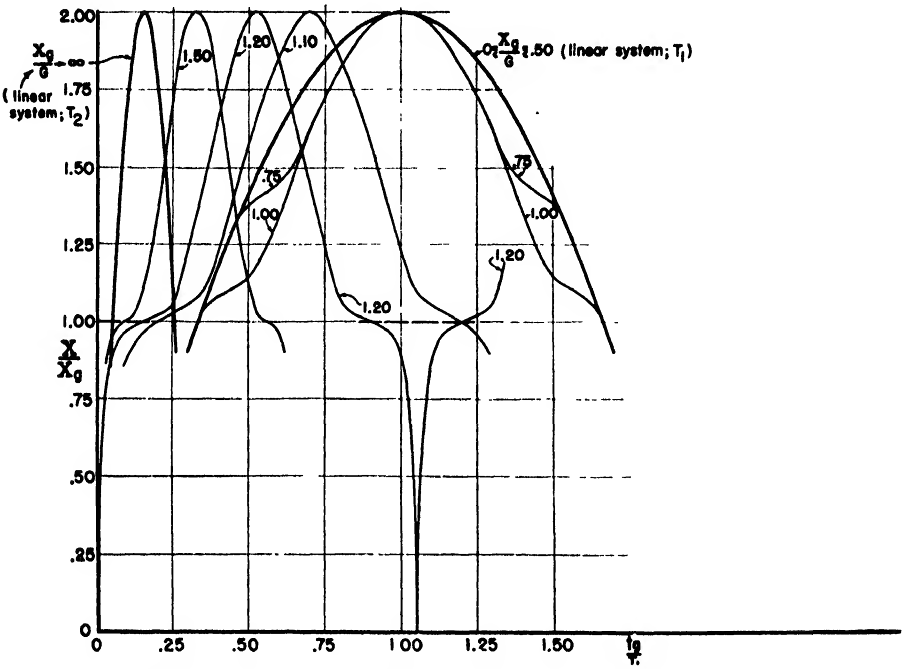

FIG. 9(c). Distortion of bi-linear systems, $T_1/T_2 = \tau = 6.50$; rectangular ground pulse.

In this case the first peak of the curve of free vibration maxima has been shown in complete form for the example, $X_g/G = 1.20$. The peak is symmetrical and repeats itself at regular intervals.

5. The magnitudes of X_m/X_g are: rectangular pulse, 2.00; quarter skewed cosine pulse, 1.52; cosine pulse, 1.64. The value 2.00 is of course the greatest that may be attained with any single pulse.

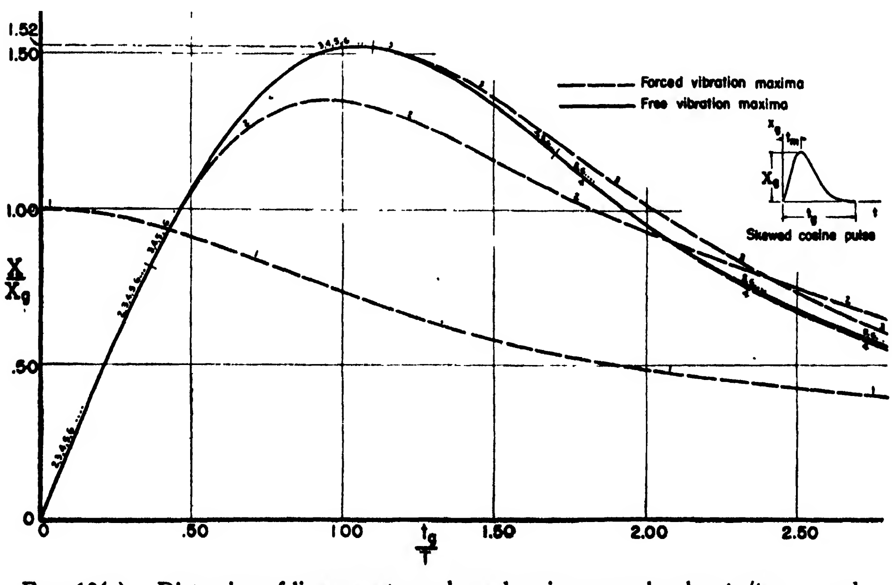

FIG. 10(a). Distortion of linear system; skewed cosine ground pulse, $t_m/t_g = \sigma = \frac{1}{4}$.

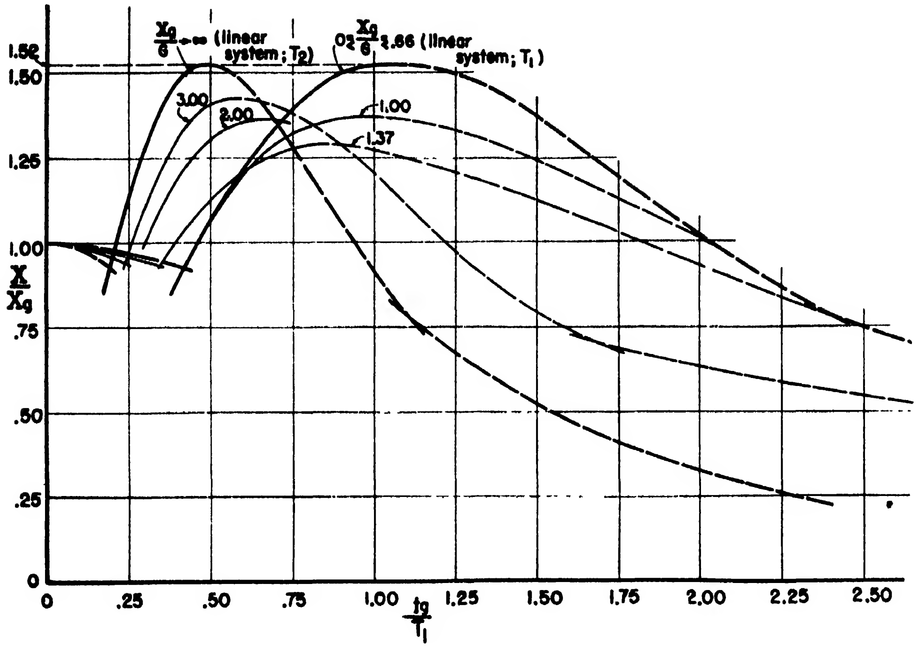

FIG. 10(b). Distortion of bi-linear systems, $T_1/T_2 = \tau = 2.17$; skewed cosine ground pulse, $t_m/t_g = \sigma = \frac{1}{4}$.

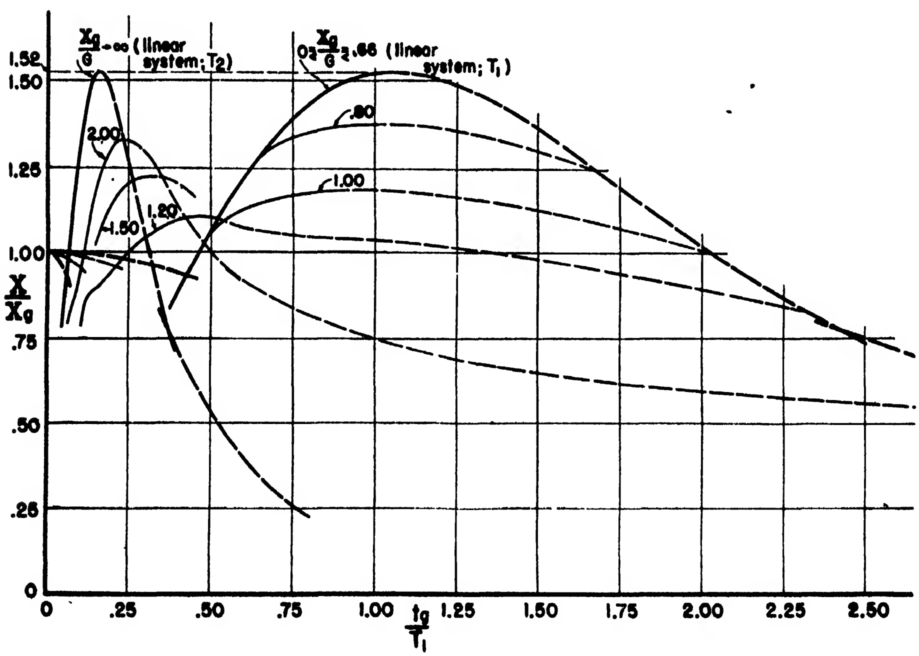

FIG. 10(c). Distortion of bi-linear systems, $T_1/T_2 = \tau = 6.50$; skewed cosine ground pulse, $t_m/t_g = \sigma = \frac{1}{4}$.

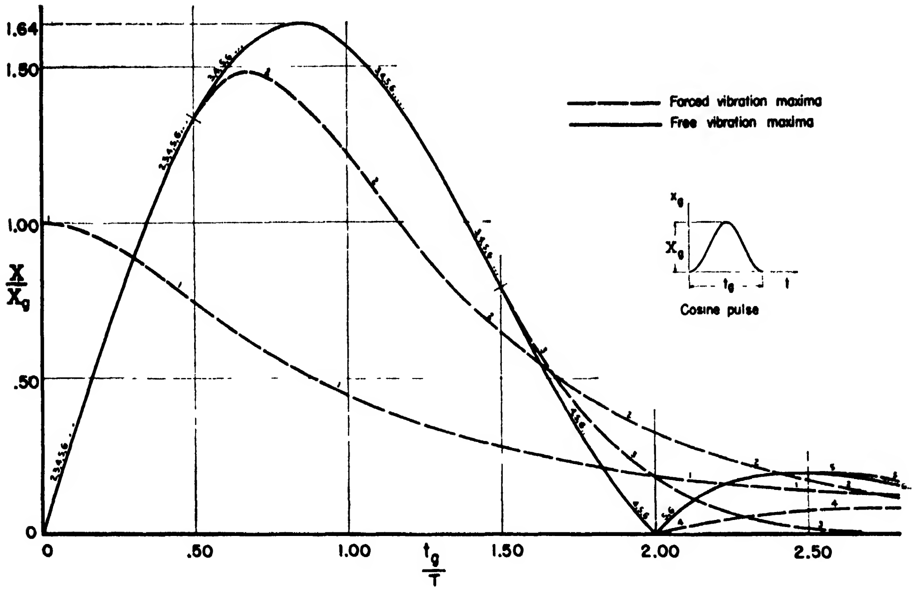

FIG. 11(*a*). Distortion of linear system; cosine ground pulse.

The Families of Bi-Linear Systems

The envelopes of maximum distortion are shown in Figs. 9(*b*), 9(*c*), 10(*b*), 10(*c*), 11(*b*), and 11(*c*). For reasons of clarity some of the enve-

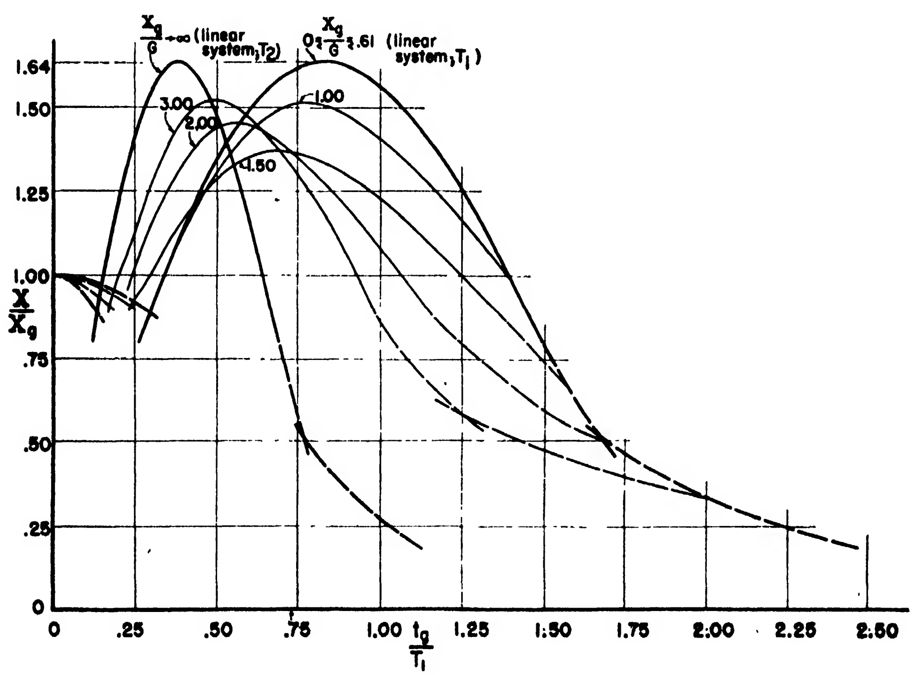

FIG. 11(*b*). Distortion of bi-linear systems, $T_1/T_2 = \tau = 2.17$; cosine ground pulse.

lopes are not shown completely, especially in the case of the rectangular pulse for which only the first peak of each envelope is included.

The most obvious remark relating to a particular family and particular type of pulse is that, starting with the limiting linear system of period T_1 and introducing bi-linearity by means of the parameter X_g/G, there is a gradual change in the shape of the envelope and in general in the magnitude of X_m/X_g until the limiting linear system with period T_2 is reached. Figures 10(*b*) and 11(*b*), which are for the family $\tau = 2.17$, show rather smooth transitions from one limit to the other. When the pulse is of Form 2, X_g/G must reach a value of 0.66 before the system starts to become bi-linear. As X_g/G is increased beyond this value the

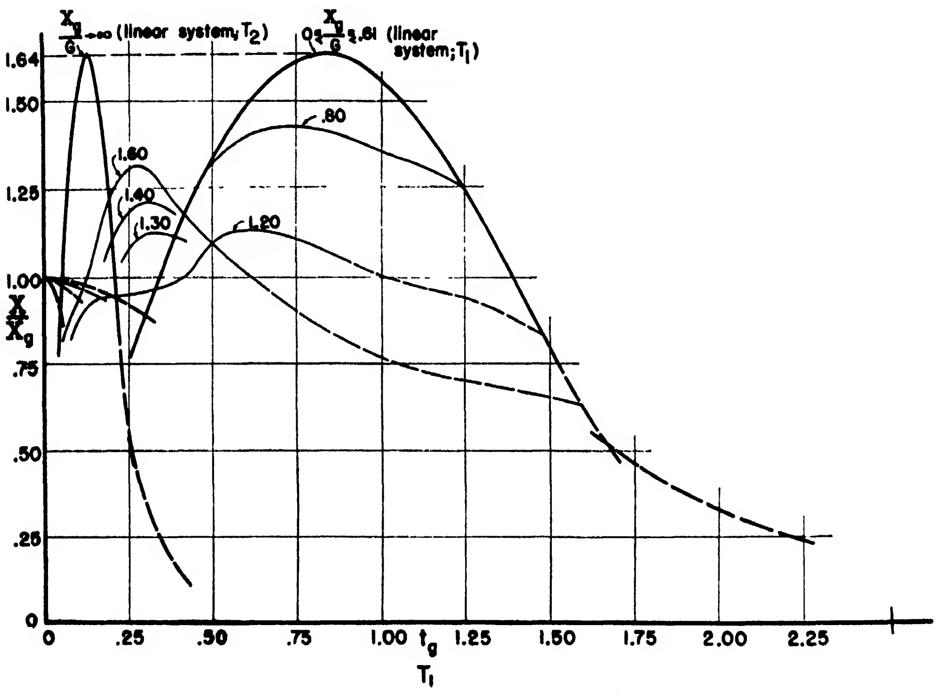

FIG. 11(*c*). Distortion of bi-linear systems, $T_1/T_2 = \tau = 6.50$; cosine ground pulse.

envelopes for members of the bi-linear family branch off from the envelope of the linear system of period T_1. When the pulse is of Form 3, X_g/G must reach 0.61 before bi-linearity appears; in this case the branching of the bi-linear envelopes from the linear is even more evident. The progression in the family with $\tau = 6.50$, Figs. 10(*c*) and 11(*c*), is less uniform.

Figures 9(*b*) and 9(*c*) relate to the rectangular pulse. The same characteristics as indicated above are evident, except for the pronounced difference that X_m/X_g is always 2.00, regardless of X_g/G.

The effect on X_m of variation in the degree of bi-linearity has been summarized in Fig. 12 where the ratio $(X_m)_{\text{bil}}/(X_m)_{\text{lin}}$ has been plotted

as a function of $G/(X_m)_{\text{lin}}$. Several generalizations may now be made based on Fig. 12:

1. When $G/(X_m)_{\text{lin}} \geqq 1$ the system is obviously linear for all values of t_g/T_1.
2. When $G/(X_m)_{\text{lin}} = 0$ the system is linear for all values of t_g/T_1. This is exactly true if $G = 0$, and is sensibly true if X_g is large in comparison with G.
3. There is, in general, some optimum value of $G/(X_m)_{\text{lin}}$ for which the absolute maximum distortion of the bi-linear system will be a minimum. The case of the rectangular pulse is an exception.
4. As τ approaches unity the system approaches linearity, but the approach to linearity is rather slow.

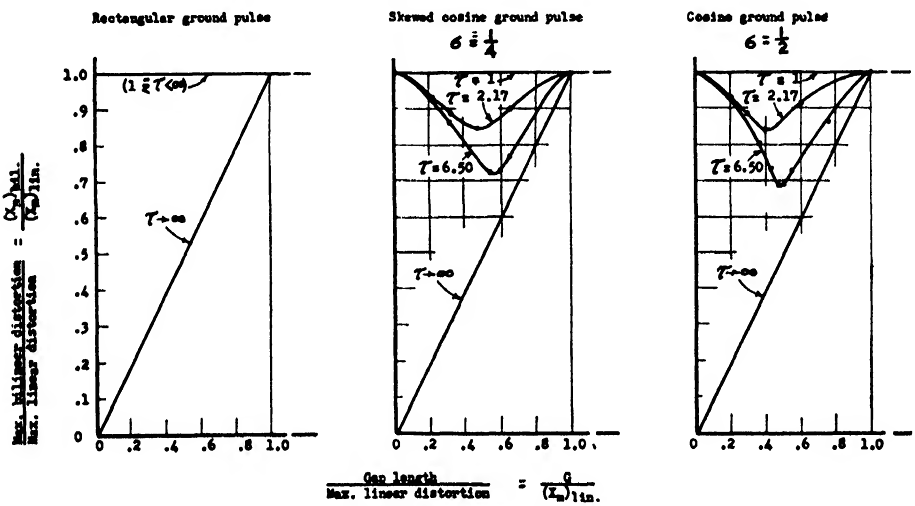

FIG. 12. Effect of bi-linearity τ and of gap length on maximum distortion; for three types of ground pulses.

(a) Rectangular ground pulse.
(b) Skewed cosine ground pulse, $\sigma = \frac{1}{4}$.
(c) Cosine ground pulse, $\sigma = \frac{1}{2}$.

5. Another extreme is possible if we make the snubbing springs k_2' infinitely stiff. This case is represented by the straight line passing through the origin and through the point (1.0, 1.0). This limiting condition indicates in Fig. 12(b) and (c) that curves for any value of τ must stay on the left side of the limiting straight lines.

The following general remarks can now be made about the bi-linear systems:

1. X/X_g may occur either during the forced vibration or the ensuing free vibration, depending on t_g/T_1, the form of the pulse, and the degree of bi-linearity.
2. $X/X_g = 1.00$ at $t_g/T_1 \to 0$, for any form of pulse and for any system.

3. X/X_g never falls below 1.00, for the rectangular pulse.

4. X_m/X_g for the linear systems of period T_1 and T_2 occur at two specific values of t_g/T_1, which depend on the form of the ground pulse. X_m/X_g for a member of the bi-linear family occurs at some value of t_g/T_1 lying between the two specific values for the limiting linear systems. Moreover, when the pulse is rectangular, X_m/X_g for a bi-linear system occurs at many values of t_g/T_1 in a manner similar to that for the linear system.

Examples

(*a*) More startling effects of non-linearity appear if we take some particular values of t_g/T_1. For example, assume that a linear system of natural period T_1 is subjected to a cosine pulse for which $t_g/T_1 = 0.84$; the resulting maximum distortion is then $1.64X_g$. Next we suppose that the system is made bi-linear by the introduction of snubbing springs and a gap such that $\tau = 6.50$ and $X_g/G = 1.60$. The maximum distortion will now be $0.83X_g$, or about 50 per cent of that for the original linear system (see Fig. 11(*c*)).

(*b*) A similar situation arises if the original linear system has a natural period T_2 and if the springs k_1 are then made softer for distortions smaller than the gap. If we take $\tau = 6.50$, $X_g/G = 1.60$, and $t_g/T_1 = 0.13$, the maximum distortion of the new bi-linear system will be near $1.00X_g$ or about 60 per cent of that of the linear (see Fig. 11(*c*)).

(*c*) On the other hand, non-linearity may act to increase the distortion. Referring again to Fig. 11(*c*), we assume that t_g/T_1 equals 0.25. The original linear system has a period T_2 with a corresponding maximum distortion of $0.54X_g$. If it is *softened*, as described above, then the maximum distortion of the bi-linear system for $X_g/G = 1.60$ will be $1.32X_g$ or about two and one-half times that of the linear case. Similarly, if the original linear system has a period T_1 and if *snubbing* springs are added, the maximum distortion of the bi-linear system for $T_g/G = 1.60$ will be nearly 70 per cent greater than that for the linear. The possibility of increased distortion due to non-linearity and particular values of t_g/T_1 is also apparent in the case of the rectangular pulse (see Fig. 9(*c*)).

Restoring Force and Acceleration

The foregoing discussion has been related to relative displacement or distortion. Criteria which often may be of equal or greater importance in judging the effect of a pulse are restoring force and acceleration.

The variation of maximum restoring force with duration of ground motion has been shown for one bi-linear family and for one type of ground pulse in Fig. 13. The absolute maximum restoring force in the limiting linear system of natural period T_2 is $(T_1/T_2)^2$, or k_2/k_1, times

the absolute maximum for the limiting system of period T_1. The absolute maxima for the intervening bi-linear systems lie between those for the linear systems. The bi-linearity results in *either* increased or decreased restoring force and acceleration, depending on particular values of t_g/T_1.

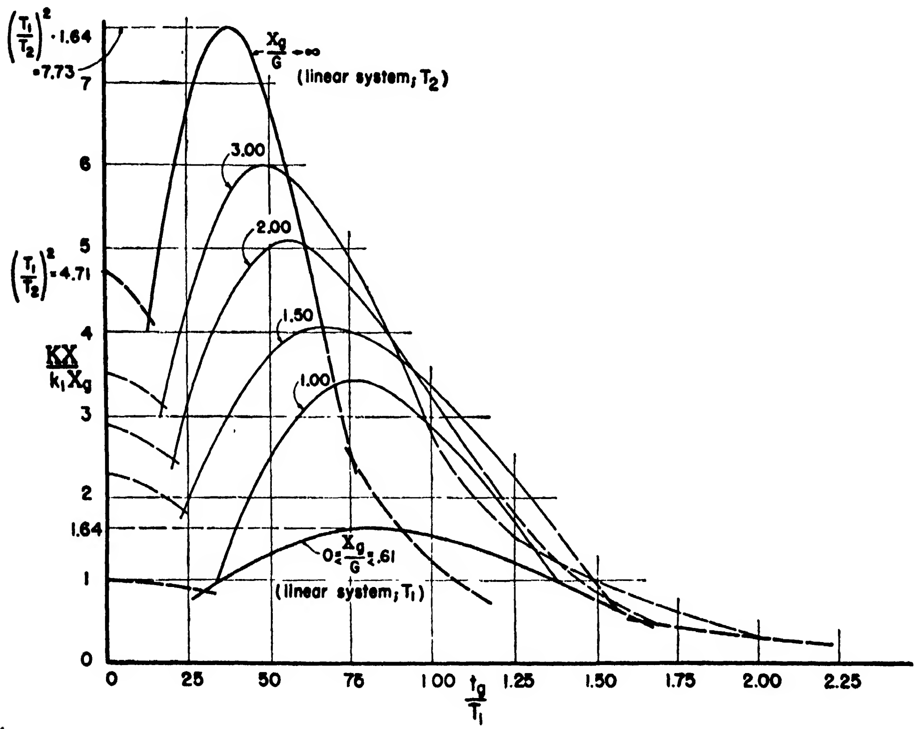

FIG. 13. Restoring force in bi-linear systems, $T_1/T_2 = \tau = 2.17$; cosine ground pulse.

SUMMARY AND CONCLUSIONS

The quantitative results of the investigation have been presented mainly in the form of envelopes of maximum distortion plotted as a function of forcing period, where the variables have been expressed in dimensionless form. Each envelope includes a number of discontinuities in slope which arise from the transient nature of the phenomenon. More specifically, for some values of forcing period the maximum distortion is the first of the maxima of the distortion-time function; for other forcing periods it is the second, and so on; this change in the order of occurrence of the maximum results in the discontinuities. The manner of determining the envelopes has been shown in detail for the linear system.

Corresponding to each family of linear and bi-linear systems there is, for a particular form of forcing function, a family of envelopes of maximum distortion. As X_g/G is increased beyond a limiting finite value,

the envelopes for the bi-linear members of the family branch off from the envelope for the limiting linear system of period T_1 and gradually approach the envelope for the limiting linear system of period T_2. The absolute maximum values of the two limiting envelopes are, of course, equal. The absolute maximum distortion of a bi-linear system, however, is in general less than the absolute maximum for the limiting linear cases, and for some optimum value of X_0/G the absolute maximum distortion attains a minimum value. An exception is provided by the rectangular ground pulse. In this case the absolute maximum distortions for the bi-linear systems are equal to those for the limiting linear systems.

The above is based on the assumption that the forcing period may take *any* value and indicates, except for the rectangular pulse, that bi-linearity may be advantageous if it is desirable to reduce the absolute maximum distortion. However, for particular values of forcing period bi-linearity may be either advantageous or disadvantageous. The essence of the matter lies in the relationship of the forcing period to the instantaneous natural period of the system. Since the natural periods for the bi-linear systems are in general functions of displacement one cannot rely on assumed simple relationships.

It has been pointed out that maximum distortion is not the only criterion for judging the effect of bi-linearity. Other criteria as *maximum restoring force* and *maximum acceleration*, which are related directly to each other and to maximum distortion through the instantaneous spring constant, may be of importance.

While the symmetrical bi-linear system is perhaps the simplest of elastically non-linear systems, its response, even to simple forcing functions, is complicated. One must be wary in estimating what the effect of bi-linearity will be. If many problems are to be solved an experimental apparatus is advantageous and can be constructed without great effort. The particular form of apparatus used in this investigation is not necessarily recommended, although it has proved satisfactory. If only a few specific problems are to be solved the graphical method is superior to the experimental.

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APPENDIX A, GRAPHICAL METHOD

The graphical method is an adaptation of the rotating vector method of representing the vibrations of single degree of freedom systems (5, 6). The adaptation relates to ground motions and involves the consideration of:

(1) The trajectory of the system in the $\dot{x}_s/p$, x_s phase plane.

(2) An approximation of the actual ground motion by a sequence of rectangular steps.

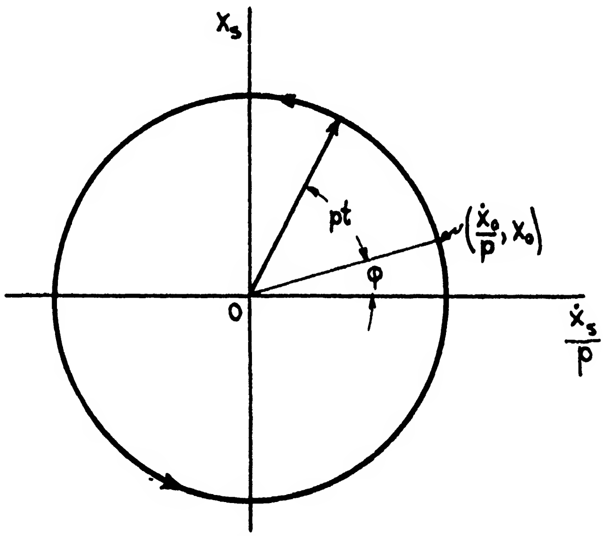

FIG. A-1. Free vibration of a linear system.

(3) The changes in the position of static and/or temporary points of equilibrium about which the system will oscillate as a result of the ground motion and/or the changes in the incremental spring constant of the non-linear systems.

(4) The sudden changes in the coordinate $\dot{x}_s/p$ as the incremental spring constant (and hence the effective natural frequency p) vary.

The presentation of the method for the bi-linear system will be facilitated by considering some simpler cases first.

Free Vibration of a Linear System

Let the initial displacement and velocity be x_0 and $\dot{x}_0$. The vibration of the system may be represented by a circle passing through the

point $\dot{x}_0/p$, x_0 and with center at the origin. The circle is generated by a vector of length equal to the amplitude of the vibration, rotating with constant velocity p. (Fig. A-1.)

Free Vibration of a Bi-Linear System

The spring characteristic of Fig. A-2(a) is similar to that of Fig. 2 except that it has been rotated through a positive angle of 90°. Let $x_0 < G$ and $\dot{x}_0 > 0$ (Fig. A-2(b)). During the first stage the system has a natural frequency p_1 and moves about the static position of equilibrium 0. After a time t_1, x_s becomes equal to G and the system starts to oscillate with an effective natural frequency p_2 about the temporary position of equilibrium O_R. Due to the sudden change in effective natural frequency the abscissa at $x_s = G$ must change suddenly from $\dot{x}_1/p_1$ to $\dot{x}_1/p_2$ or an amount $1 - 1'$. The ensuing vibration is described by a new vector rotating about O_R with an angular velocity p_2.

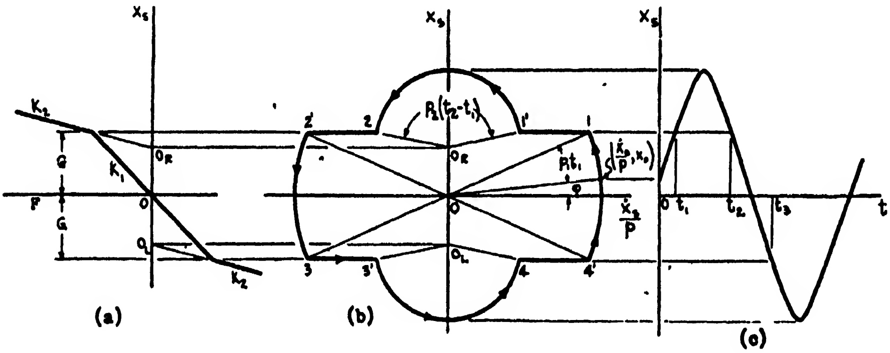

FIG. A-2. Free vibration of a bi-linear system.

After a time interval $(t_2 - t_1)$, x_s returns to the value G, the effective natural frequency again becomes p_1, and the system again oscillates about the static equilibrium position O. The process continues, the rotating vectors describing a symmetrical closed diagram as a consequence of the symmetrical spring characteristic and zero damping of the system. The representation is exact. Displacement-time and velocity-time diagrams may be constructed from the $\dot{x}_s/p$, x_s diagram as illustrated in Fig. A-2(c).

Linear System Subjected to a Rectangular Ground Pulse

Assume that the system is initially at rest and unstrained. The motion of the system until the time t_1 is represented by the circular arc $0, 0' - 1, 1'$ described by a vector of length X_g rotating (through an angle pt_1) about the displaced position of static equilibrium 0_1 (Fig. A-3(b)). At the time t_1 the position of equilibrium is returned to its

original location, and the ensuing free vibrations are described by the circle with center at 0. The ground motion and the displacement-time diagrams appear in Fig. A-3(c). The method is exact.

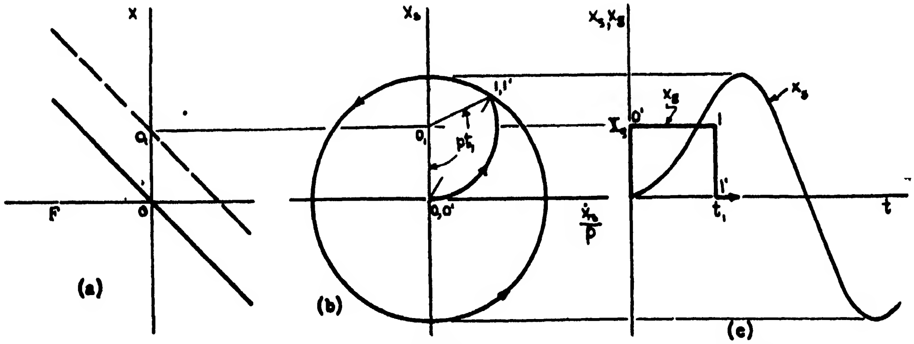

FIG. A-3. Linear system subjected to a rectangular ground pulse.

Ground Motion of General Form

The method may be applied to any ground motion by the use of a stepped approximation, as illustrated in Fig. A-4, where for clarity the number of steps has been limited to three. The vibration during the ground motion is represented by the arcs $0, 0' - 1, 1'$, $1, 1' - 2, 2'$ and $2, 2' - 3, 3'$ with centers at the temporarily displaced static positions of equilibrium 0_1, 0_2, 0_3. The ensuing free vibration is described by the circle with center at 0 and passing through the point $3, 3'$.

Bi-Linear System Subjected to Ground Motion of General Form

The first step results in the arc $0, 0' - 1$ of Fig. A-5(b). The ground displacement x_{g1} is small enough so that the resulting distortions are less than G. During the instantaneous ground displacement occurring at the start of the second step ($t = t_1$) the distortion is forced to a value numerically greater than G, the effective natural frequency becomes p_2 and the abscissa of the $\dot{x}_s/p$, x_s diagram at $t = t_1$ decreases from $\dot{x}_1/p_1$ to $\dot{x}_1/p_2$ or by an amount $1 - 1'$. The system starts to oscillate about the effective position of equilibrium 0_{2L} and continues so until the distortion has decreased to a value equal to G (at $t_1 < t < t_2$). At this instant the effective natural frequency changes from p_2 to p_1, the abscissa increases correspondingly, and the system starts to oscillate about the displaced position of static equilibrium 0_2; it continues so until $t = t_2$.

The vibration from time t_2 until time t_3 may be similarly explained. At $t = t_3$ the ground returns to its original position at $x_g = 0$ and remains there, and the system describes free vibrations with starting conditions determined by point $3, 3'$.

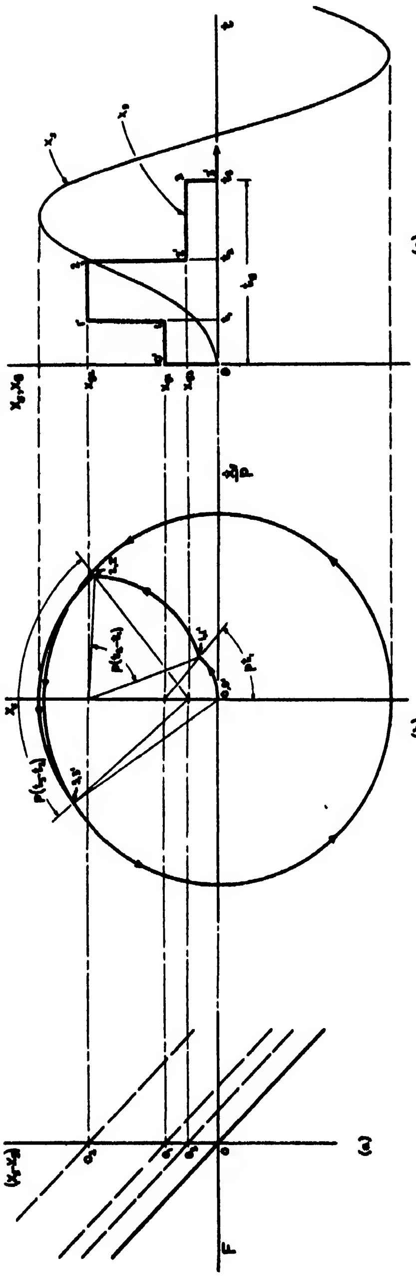

FIG. A-4. Linear system; stepwise ground motion.

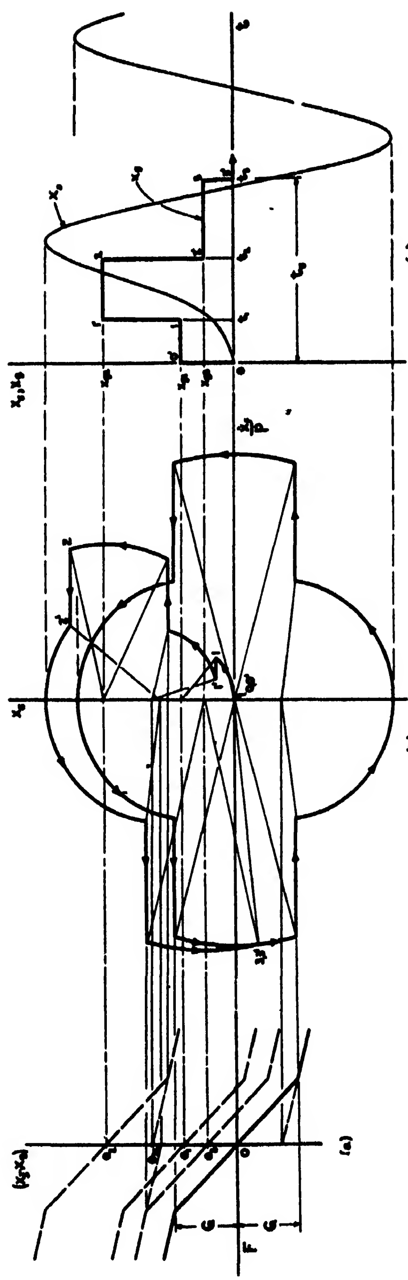

FIG. A-5. Bi-linear system; stepwise ground motion.

While the above example illustrates several important points regarding change in x from a value smaller than G to one larger than G, and *vice versa*, it is somewhat distorted by the fact that only three steps have been taken to approximate the ground pulse. Ordinarily more steps would be used.

The distortions during the ground motion are readily obtained from the $\dot{x}_s/p$, x_s diagram, or the x_s,t curve, and the x_g,t curve. For this purpose it is usually more accurate to take values of x_g from the actual curve of ground motion rather than from the stepped approximation.

Accuracy

In order to obtain a given accuracy the number of steps employed in the approximation of the ground pulse varies somewhat depending on the form of the pulse, the ratio t_g/T and the degree of bi-linearity of the system. Some idea of the accuracy obtained with a linear system is indicated below:

Cosine pulse with t_g/T corresponding to X_m:
16 equal steps 99 per cent, 8 equal steps 97 per cent, and 6 equal steps 97 per cent.

"Skewed" cosine pulse $\sigma = \frac{1}{4}$ and with t_g/T corresponding to X_m:
24 equal steps 99 per cent, 12 unequal steps 98 per cent, 8 unequal steps 96 per cent, and 6 unequal steps 94 per cent.

General Application

The method is a powerful one. In application to linear or to stepped non-linear systems its theoretical accuracy is limited only by the number of steps one is willing to use in approximating the forcing function. In application to a system with continuous non-linearity the continuous restoring force-distortion function must also be replaced by some stepped approximation (6).

The restoring force-distortion function may be asymmetrical instead of symmetrical and may include portions with negative slope (8). Damping may be treated by continuously modifying the length of the rotating vector according to the law of damping (6). There is no restriction on the starting conditions.

APPENDIX B, EXPERIMENTAL APPARATUS

The experimental apparatus consisted of a translatory mechanical system subjected to transient ground motions by means of a cam-driven mechanical linkage as shown in Fig. B-1. The cam C, shaped to generate the ground pulse, is driven at constant velocity, variable over a wide range. The pulse is imparted to the ground structure GS through the cam follower F and linkage A, L. A restoring spring RS

maintains continuous contact between the follower and the cam. The natural frequency of the linkage, ground structure and restoring spring system is high relative to the operating frequencies. The wire W is practically inextensible. The mass of the upper spring system, S_2, W,

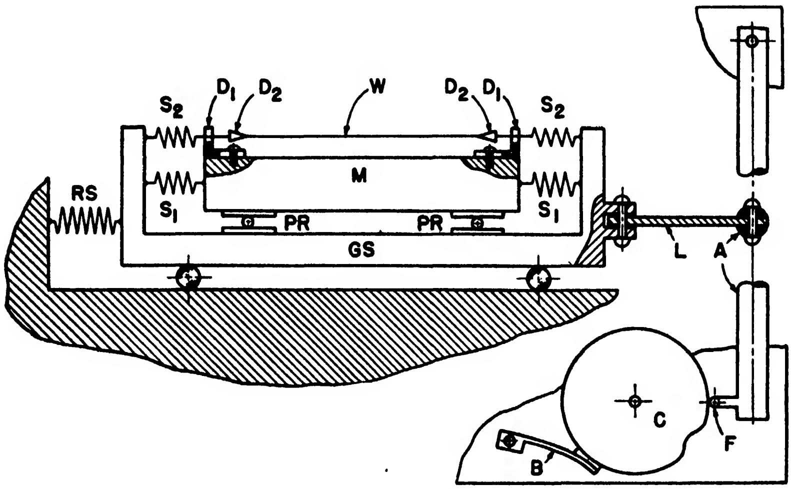

Fig. B-1. Schematic diagram of experimental apparatus.

C	cam	A, L	linkage
F	cam follower	GS	ground structure
B	brake	RS	restoring spring

M mass
S_1 springs of combined constant k_1
S_2 springs of combined constant k_2'
W wire, connecting springs S_2
D_1 adjustable stops mounted on M
D_2 stops attached to W
PR glass plate—steel roller supports

D_2, is very small in comparison with the mass M; therefore, kinetic energy transfers to the upper system are negligible. Friction was minimized by the use of the glass plate-steel roller supports PR under the mass. The displacements x and x_g were recorded continuously by a direct-inking, mechanical device.

A MODERN APPROACH TO "DIMENSIONS"

BY

PARRY MOON[1] AND DOMINA EBERLE SPENCER[2]

A characteristic of nineteenth-century physics was the belief that the entire subject could be based on the laws of classical mechanics. These principles had been found applicable over a tremendous range, from astronomical values of $10^{12}\ m$ to molecular dimensions of $10^{-10}\ m$. The apparently foreign subject of heat had been made a branch of mechanics by means of kinetic theory. It was felt that further work would undoubtedly make electromagnetism another branch of mechanics.

Under these circumstances, it was natural that the concepts of length, mass, and time, which had been chosen as fundamental in mechanics, should be applied to all branches of physics. But the twentieth century brought quantum mechanics and relativity, which practically destroyed the mechanistic idea of the universe. As a result, the feeling that everything had to be expressed in terms of the primary "dimensions" l, m, t was abandoned by most physicists. Bridgman (1)[3] emphasized the fact that there is nothing sacrosanct about l, m, t and that "dimensional" analysis is merely a man-made tool that may be manipulated at will. This principle of free choice of the primary concepts has been widely accepted, though one still finds references to "true" or "absolute dimensions" and statements that the "real dimensions" of ϵ and μ are not yet known.

After having accepted the principle of relativity of "dimensions," one is faced by the question of what choice of primary concepts will be of maximum utility. This is the subject of the present paper.

1. FUNDAMENTALS

As pointed out previously (2), the geometric approach to "dimension" theory has advantages. A complete physical quantity[4] possesses both magnitude and "quality" and is thus similar to a vector. The *magnitude* of the physical quantity is represented by the length of the vector, while the *kind* of physical quantity is represented by the direction of the vector.

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[3] The boldface numbers in parentheses refer to the references appended to this paper.

[4] A scalar quantity or one component of a vector in euclidean 3-space. Note that the "vector" referred to in the above sentence of the text is an *idon* vector.

We write one of these "dimensional" vectors as a symbol inclosed in curly brackets, as $\{A\}$. The magnitude is denoted by A and the direction is specified by the *designation symbol* $[a^1, a^2, \cdots a^k]$. Thus,

$$\{A\} = A[a^1, a^2, \cdots a^k] = Aa^i, \qquad (i = 1, 2, \cdots k).$$

Geometrically, $\{A\}$ is a vector in a k-dimensional affine space, A is a scalar, and a^i is a *base vector* which fixes the direction of $\{A\}$. To distinguish $\{A\}$ from vectors in the euclidean space in which the physical phenomena occur, $\{A\}$ is called an *idon*. The fictitious k-space of the idon vectors is called *idon space*. The *coordinates* $a^1, a^2, \cdots a^k$ of the base vector are equal to the exponents of the usual "dimensional" formulas.

Allowable operations on idons are as follows (2):

(*1*) *Addition*. The addition of two idons is possible only if they are collinear. This corresponds to the well-known principle of "dimensional" homogeneity. In this case, the sum of

$$\{A\} = Aa^i \qquad \text{and} \qquad \{B\} = Bb^i$$

is defined as

$$\{A\} + \{B\} = (A + B)a^i = (A + B)b^i. \tag{1}$$

(*2*) *Multiplication by a scalar*. Let k be an *idonic scalar* ("dimensionless" quantity). Then the product of it and an idon is defined as

$$k\{A\} = kAa^i = Aka^i. \tag{2}$$

(*3*) *Multiplication*. Multiplication of two idons is defined by the equation,

$$\{A\}\{B\} = AB[a^i + b^i]. \tag{3}$$

Note that this product is similar to the familiar product of two complex quantities in polar form, where the lengths are multiplied and the angles added. Here the corresponding coordinates of the base vectors are added in taking the product.

Repeated multiplication leads to the equation,

$$\{A\}^\gamma = (A)^\gamma[\gamma a^i] \tag{4}$$

where γ may be extended to non-integer values if desired.

(*4*) *Division*. Division of idons is defined as

$$\frac{\{A\}}{\{B\}} = \frac{A}{B}[a^i - b^i]. \tag{5}$$

An example is shown in Fig. 1. The primary concepts are chosen as length, mass, and time so $k = 3$ and the coordinate axes are l, m, t. Any length is written

$$\{l\} = l[1, 0, 0],$$

any mass is

$$\{m\} = m[0, 1, 0],$$

and any time is

$$\{t\} = t[0, 0, 1].$$

Any equation of physics can be rewritten in idon form by merely replacing the usual quantities by idons. For instance, velocity is defined as

$$v = dl/dt.$$

In idon form,

$$\{v\} = \frac{d\{l\}}{d\{t\}}$$

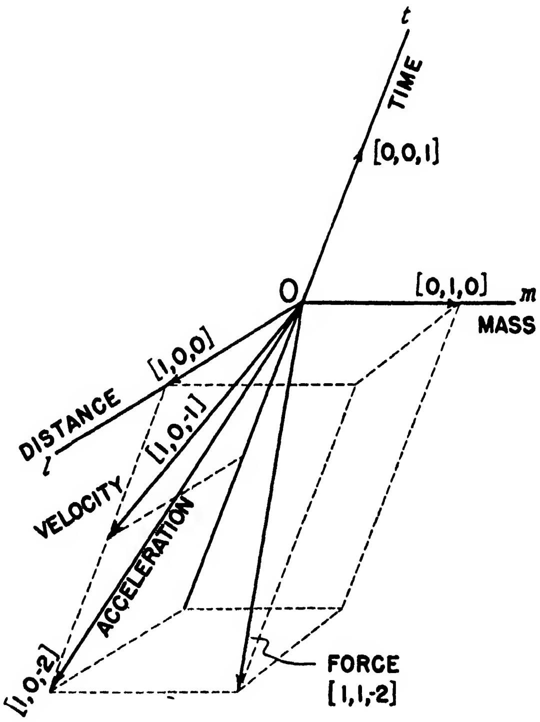

FIG. 1. Division of l and t idons result in idons for v and a.

or, since the base vectors are invariant with respect to the differentiation,

$$v[a^1, a^2, a^3] = \frac{dl}{dt}\frac{[1, 0, 0]}{[0, 0, 1]} = \frac{dl}{dt}[1, 0, -1].$$

For equality, both magnitude and direction must be the same, giving the two equations,

$$v = dl/dt \qquad \text{and} \qquad [a^1, a^2, a^3] = [1, 0, -1].$$

Similarly, acceleration has the base vector $[1, 0, -2]$. Any accelera-

tion is therefore represented by a vector in the direction of the acceleration base vector (Fig. 1).

If we define *force* as

$$F = ma,$$

then

$$\{F\} = \{m\}\{a\} = ma([0, 1, 0] + [1, 0, -2])$$
$$= ma[1, 1, -2].$$

Thus the base vector associated with the concept of force is $[1, 1, -2]$, if l, m, t are chosen as primary concepts.

2. AMBIGUITY

Since the number and nature of the primary concepts are arbitrary, their choice can be put on a purely pragmatic basis. That is, a particular number of primary concepts and a particular choice of concepts may turn out to be most convenient for a given problem, and this choice should be employed rather than an arrangement that is less useful. To answer the question of what primary concepts are best, therefore, one must consider the purpose of "dimensions."

In a previous paper (3), six uses of idon theory were listed. The fundamental application is to the *designation of concepts*. The *designation symbol* $[a^1, a^2, \cdots a^k]$ or *base vector* shows at a glance what the concept is and what relation it has to other concepts. If the method is to be effective, a 1:1 relation should exist between base vectors and concepts. But this ideal condition does not obtain, in general, when l, m, t are chosen as primary concepts. Thus a looseness and an ambiguity are introduced that detract from the value of all six applications of idon theory.

That the use of l, m, t as primary concepts leads to ambiguity is well known. One instance is given by *energy* and *torque*, both of which have the base vector $[2, 1, -2]$. Another example is that of *luminous intensity* and *luminous flux*, which are regarded by all photometrists as distinct concepts but which have the same base vector.

In electricity and magnetism, the same concept has different designation symbols depending on whether the "electrostatic" or the "electromagnetic" cgs. system is employed. Moreover, some of the base vectors assigned to electric and magnetic concepts are already assigned to familiar mechanical concepts. In the "electrostatic" system, for instance, *capacitance* has the same base vector as *length*, *resistivity* has the same base vector as *time*, *resistance* is equivalent "dimensionally" to reciprocal velocity, and *permeability* is equivalent to reciprocal velocity squared. In the "electromagnetic" system, *inductance* and *length* have the same base vector, as do *resistance* and *velocity*.

To talk of inductance and capacitance in *centimeters*, resistivity in *seconds*, and resistance in *centimeters per second* is not wrong but it is

confusing, and only our long familiarity with such "dimensional" monstrosities makes us willing to tolerate them. Physicists in general have learned to take these discrepancies in their stride and to consider them as normal. For instance, Birge (4) says, ". . . one cannot, from the adopted dimensions of a quantity, draw any conclusions as to its physical nature; thus two quantities of quite different character, such as moment of force and energy, have, in the cgs. system, the same dimensions."

The point is that this "dimensional" ambiguity is not a fact of nature. It is merely an imperfection in our man-made scheme of assigning primary concepts. If idon theory is to be of maximum serviceability, it must be revised so that each concept has its unique base vector.

3. THE l, m, t SYSTEM

Table I lists several possible designation systems for the concepts of physics and engineering. The second column indicates the result of choosing the traditional l, m, and t as primary concepts. For mechanics, the designation symbols are well known and are fairly satisfactory. The most notable ambiguity occurs with *energy* and *torque*, as mentioned in the previous section. Despite the identity in "dimensions," physicists always regard energy and torque as distinct concepts. Another example is the confusion among *normal stress*, *shearing stress*, *pressure*, and *elastic moduli*, all having the base vector $[-1, 1, -2]$. These concepts are often considered identical, from the mistaken notion that "dimensions" are associated with units instead of concepts and that identical *units* imply identical *concepts* (3). From an operational standpoint, however, the above concepts are as distinct as energy and torque. It would seem preferable, therefore, to associate normal stress and shearing stress with different base vectors.

Turning now to the subject of *heat*, we find that l, m, t can still be employed, but an assumption must be made about the "dimensionality" of some constant. One suggestion (5) has been to base the analysis on the equation,

$$pv = RT$$

and to assume that R has the same designation symbol as mass. This assumption leads to the interesting result that *entropy* and *mass* are "dimensionally" identical and that *temperature* has the same base vector as velocity squared. These ambiguities are listed in Table II.

There is, of course, no necessity that this particular assumption be made. Condon (6) takes the Wien displacement relation, λT = const. and assumes that the constant is an idonic scalar. The results of this proposal are marked (*b*) in the second column of Table I. Ambiguities are found between *temperature* and *reciprocal length*, *thermal power* and

TABLE I.—*Designation Symbols for Physical Concepts.*

Name	$[l, m, t]$		$[l, m, t, T, Q, F]$	$[l, t, P, T, Q, F]$
Geometry				
Distance	[1, 0, 0]		[1, 0, 0, 0, 0, 0]	[1, 0, 0, 0, 0, 0]
Area	[2, 0, 0]		[2, 0, 0, 0, 0, 0]	[2, 0, 0, 0, 0, 0]
Volume	[2, 0, 0]		[3, 0, 0, 0, 0, 0]	[3, 0, 0, 0, 0, 0]
Angle	[0]		[0]	[0]
Solid angle	[0]		[0]	[0]
Kinematics				
Time	[0, 0, 1]		[0, 0, 1, 0, 0, 0]	[0, 1, 0, 0, 0, 0]
Velocity	[1, 0, −1]		[1, 0, −1, 0, 0, 0]	[1, −1, 0, 0, 0, 0]
Acceleration	[1, 0, −2]		[1, 0, −2, 0, 0, 0]	[1, −2, 0, 0, 0, 0]
Angular velocity	[0, 0, −1]		[0, 0, −1, 0, 0, 0]	[0, −1, 0, 0, 0, 0]
Angular acceleration	[0, 0, −2]		[0, 0, −2, 0, 0, 0]	[0, −2, 0, 0, 0, 0]
Frequency	[0, 0, −1]		[0, 0, −1, 0, 0, 0]	[0, −1, 0, 0, 0, 0]
Dynamics				
Force	[1, 1, −2]		[1, 1, −2, 0, 0, 0]	[−1, 1, 1, 0, 0, 0]
Momentum	[1, 1, −1]		[1, 1, −1, 0, 0, 0]	[−1, 2, 1, 0, 0, 0]
Impulse	[1, 1, −1]		[1, 1, −1, 0, 0, 0]	[−1, 2, 1, 0, 0, 0]
Torque	[2, 1, −2]		[2, 1, −2, 0, 0, 0]	[0, 1, 1, 0, 0, 0]
Energy	[2, 1, −2]		[2, 1, −2, 0, 0, 0]	[0, 1, 1, 0, 0, 0]
Power	[2, 1, −3]		[2, 1, −3, 0, 0, 0]	[0, 0, 1, 0, 0, 0]
Mass	[0, 1, 0]		[0, 1, 0, 0, 0, 0]	[−2, 3, 1, 0, 0, 0]
Moment of inertia	[2, 1, 0]		[2, 1, 0, 0, 0, 0]	[0, 3, 1, 0, 0, 0]
Dissipation coefficient	[0, 1, −1]		[0, 1, −1, 0, 0, 0]	[−2, 2, 1, 0, 0, 0]
Stiffness	[0, 1, −2]		[0, 1, −2, 0, 0, 0]	[−2, 1, 1, 0, 0, 0]
Efficiency	[0]		[0]	[0]
Elasticity				
Stress, normal	[−1, 1, −2]		[−1, 1, −2, 0, 0, 0]	[−3, 1, 1, 0, 0, 0]
shear	[−1, 1, −2]		[−1, 1, −2, 0, 0, 0]	[−3, 1, 1, 0, 0, 0]
Strain, normal	[0]		[0]	[0]
shear	[0]		[0]	[0]
Elastic moduli	[−1, 1, −2]		[−1, 1, −2, 0, 0, 0]	[−3, 1, 1, 0, 0, 0]
Hydrodynamics				
Pressure	[−1, 1, −2]		[−1, 1, −2, 0, 0, 0]	[−3, 1, 1, 0, 0, 0]
Velocity potential	[2, 0, −1]		[2, 0, −1, 0, 0, 0]	[2, −1, 0, 0, 0, 0]
Density	[−3, 1, 0]		[−3, 1, 0, 0, 0, 0]	[−5, 3, 1, 0, 0, 0]
Viscosity	[−1, 1, −1]		[−1, 1, −1, 0, 0, 0]	[−3, 2, 1, 0, 0, 0]
Surface tension	[0, 1, −2]		[0, 1, −2, 0, 0, 0]	[−2, 1, 1, 0, 0, 0]
Heat	*a*	*b*		
Temperature	[2, 0, −2]	[−1, 0, 0]	[0, 0, 0, 1, 0, 0]	[0, 0, 0, 1, 0, 0]
Thermal energy (heat)	[2, 1, −2]	[2, 1, −2]	[2, 1, −2, 0, 0, 0]	[0, 1, 1, 0, 0, 0]
Thermal power	[2, 1, −3]	[2, 1, −3]	[2, 1, −3, 0, 0, 0]	[0, 0, 1, 0, 0, 0]
Entropy	[0, 1, 0]	[3, 1, −2]	[2, 1, −2, −1, 0, 0]	[0, 1, 1, −1, 0, 0]

TABLE I.—*Continued.*

Name	[*l, m, t*]		[*l, m, t, T, Q, F*]	[*l, t, P, T, Q, F*]
Power per unit volume	[−1, 1, −3]	[−1, 1, −3]	[−1, 1, −3, 0, 0, 0]	[−3, 0, 1, 0, 0, 0]
Thermal conductivity	[−1, 1, −1]	[+2, +1, −3]	[1, 1, −3, −1, 0, 0]	[−1, 0, 1, −1, 0, 0]
Thermitivity (spec. ht.)	[0]	[3, 0, −2]	[2, 0, −2, −1, 0, 0]	[2, −2, 0, −1, 0, 0]
Diffusivity	[2, 0, −1]	[2, 0, −1]	[2, 0, −1, 0, 0, 0]	[2, −1, 0, 0, 0, 0]
Thalpance ("emissivity")	[−2, 1, −1]	[1, 1, −3]	[0, 1, −3, −1, 0, 0]	[−2, 0, 1, −1, 0, 0]
Coefficient of linear expansion	[−2, 0, 2]	[+1, 0, 0]	[0, 0, 0, −1, 0, 0]	[0, 0, 0, −1, 0, 0]
Thermoelectric coefficient	$[-\frac{3}{2}, \frac{1}{2}, 1]$[c]	$[\frac{3}{2}, \frac{1}{2}, -1]$[c]	[2, 1, −2, −1, −1, 0]	[0, 1, 1, −1, −1, 0]
Temp. coefficient of resistance	[−2, 0, 2]	[+1, 0, 0]	[0, 0, 0, −1, 0, 0]	[0, 0, 0, −1, 0, 0]
Electrostatics				
	d	e		
Charge	$[\frac{3}{2}, \frac{1}{2}, -1]$	$[\frac{1}{2}, \frac{1}{2}, 0]$	[0, 0, 0, 0, 1, 0]	[0, 0, 0, 0, 1, 0]
Charge per unit length	$[\frac{1}{2}, \frac{1}{2}, -1]$	$[-\frac{1}{2}, \frac{1}{2}, 0]$	[−1, 0, 0, 0, 1, 0]	[−1, 0, 0, 0, 1, 0]
Charge per unit area	$[-\frac{1}{2}, \frac{1}{2}, -1]$	$[-\frac{3}{2}, \frac{1}{2}, 0]$	[−2, 0, 0, 0, 1, 0]	[−2, 0, 0, 0, 1, 0]
Charge per unit volume	$[-\frac{3}{2}, \frac{1}{2}, -1]$	$[-\frac{5}{2}, \frac{1}{2}, 0]$	[−3, 0, 0, 0, 1, 0]	[−3, 0, 0, 0, 1, 0]
Electric field strength	$[-\frac{1}{2}, \frac{1}{2}, -1]$	$[\frac{1}{2}, \frac{1}{2}, -2]$	[1, 1, −2, 0, −1, 0]	[−1, 1, 1, 0, −1, 0]
Electric potential (and pot. difference)	$[\frac{1}{2}, \frac{1}{2}, -1]$	$[\frac{3}{2}, \frac{1}{2}, -2]$	[2, 1, −2, 0, −1, 0]	[0, 1, 1, 0, −1, 0]
Electric flux density	$[-\frac{1}{2}, \frac{1}{2}, -1]$	$[-\frac{3}{2}, \frac{1}{2}, 0]$	[−2, 0, 0, 0, 1, 0]	[−2, 0, 0, 0, 1, 0]
Electric flux	$[\frac{3}{2}, \frac{1}{2}, -1]$	$[\frac{1}{2}, \frac{1}{2}, 0]$	[0, 0, 0, 0, 1, 0]	[0, 0, 0, 0, 1, 0]
Permittivity	[0]	[−2, 0, 2]	[−3, −1, 2, 0, 2, 0]	[−1, −1, −1, 0, 2, 0]
Capacitance	[1, 0, 0]	[−1, 0, 2]	[−2, −1, 2, 0, 2, 0]	[0, −1, −1, 0, 2, 0]
Electric Current				
Current	$[\frac{3}{2}, \frac{1}{2}, -2]$	$[\frac{1}{2}, \frac{1}{2}, -1]$	[0, 0, −1, 0, 1, 0]	[0, −1, 0, 0, 1, 0]
Current density	$[-\frac{1}{2}, \frac{1}{2}, -2]$	$[-\frac{3}{2}, \frac{1}{2}, -1]$	[−2, 0, −1, 0, 1, 0]	[−2, −1, 0, 0, 1, 0]
Resistivity	[0, 0, 1]	[2, 0, −1]	[3, 1, −1, 0, −2, 0]	[1, 2, 1, 0, −2, 0]
Resistance	[−1, 0, 1]	[1, 0, −1]	[2, 1, −1, 0, −2, 0]	[0, 2, 1, 0, −2, 0]
Magnetism				
Magnetic flux density	$[-\frac{3}{2}, \frac{1}{2}, 0]$	$[-\frac{1}{2}, \frac{1}{2}, -1]$	[0, 1, −1, 0, −1, 0]	[−2, 2, 1, 0, −1, 0]
Magnetic flux	$[\frac{1}{2}, \frac{1}{2}, 0]$	$[\frac{3}{2}, \frac{1}{2}, -1]$	[2, 1, −1, 0, −1, 0]	[0, 2, 1, 0, −1, 0]
Magnetic field strength	$[\frac{1}{2}, \frac{1}{2}, -2]$	$[-\frac{1}{2}, \frac{1}{2}, -1]$	[−1, 0, −1, 0, 1, 0]	[−1, −1, 0, 0, 1, 0]

TABLE I.—*Continued.*

Name	[l, m, t]		[l, m, t, T, Q, F]	[l, t, P, T, Q, F]
Permeability	[−2, 0, 2]	[0]	[1, 1, 0, 0, −2, 0]	[−1, 3, 1, 0, −2, 0]
Magnetomotive force	[$\frac{3}{2}$, $\frac{1}{2}$, −2]	[$\frac{1}{2}$, $\frac{1}{2}$, −1]	[0, 0, −1, 0, 1, 0]	[0, −1, 0, 0, 1, 0]
Vector potential	[−$\frac{1}{2}$, $\frac{1}{2}$, 0]	[$\frac{1}{2}$, $\frac{1}{2}$, −1]	[1, 1, −1, 0, −1, 0]	[−1, 2, 1, 0, −1, 0]
Inductance	[−1, 0, 2]	[1, 0, 0]	[2, 1, 0, 0, −2, 0]	[0, 3, 1, 0, −2, 0]
Poynting vector	[0, 1, −3]	[0, 1, −3]	[0, 1, −3, 0, 0, 0]	[−2, 0, 1, 0, 0, 0]
Radiometry				
Radiant pharos (radiant power)	[2, 1, −3]		[2, 1, −3, 0, 0, 0]	[0, 0, 1, 0, 0, 0]
Radiant pharosage	[0, 1, −3]		[0, 1, −3, 0, 0, 0]	[−2, 0, 1, 0, 0, 0]
Radiant pharosum	[−1, 1, −3]		[−1, 1, −3, 0, 0, 0]	[−3, 0, 1, 0, 0, 0]
Radiant phos (energy)	[2, 1, −2]		[2, 1, −2, 0, 0, 0]	[0, 1, 1, 0, 0, 0]
Radiant phosage	[0, 1, −2]		[0, 1, −2, 0, 0, 0]	[−2, 1, 1, 0, 0, 0]
Radiant phosum	[−1, 1, −2]		[−1, 1, −2, 0, 0, 0]	[−3, 1, 1, 0, 0, 0]
Radiant helios	[0, 1, −3]		[0, 1, −3, 0, 0, 0]	[−2, 0, 1, 0, 0, 0]
Radiant heliosent	[−1, 1, −3]		[−1, +1, −3, 0, 0, 0]	[−3, 0, 1, 0, 0, 0]
Phengosage	[−1, 1, −3]		[−1, 1, −3, 0, 0, 0]	[−3, 0, 1, 0, 0, 0]
Photometry	f			
Luminous pharos (luminous flux)	[2, 1, −3]		[0, 0, 0, 0, 0, 1]	[0, 0, 0, 0, 0, 1]
Luminous pharosage	[0, 1, −3]		[−2, 0, 0, 0, 0, 1]	[−2, 0, 0, 0, 0, 1]
Luminous pharosum	[−1, 1, −3]		[−3, 0, 0, 0, 0, 1]	[−3, 0, 0, 0, 0, 1]
Luminous phos ("light")	[2, 1, −2]		[0, 0, 1, 0, 0, 1]	[0, 1, 0, 0, 0, 1]
Luminous phosage	[0, 1, −2]		[−2, 0, 1, 0, 0, 1]	[−2, 1, 0, 0, 0, 1]
Luminous phosum	[−1, 1, −2]		[−3, 0, 1, 0, 0, 1]	[−3, 1, 0, 0, 0, 1]
Luminous helios	[0, 1, −3]		[−2, 0, 0, 0, 0, 1]	[−2, 0, 0, 0, 0, 1]
Luminous heliosent	[−1, 1, −3]		[−3, 0, 0, 0, 0, 1]	[−3, 0, 0, 0, 0, 1]
Lamprosity ("visibility")	[0]		[−2, −1, 3, 0, 0, 1]	[0, 0, −1, 0, 0, 1]

[a] On the assumption that {R} has base vector [0, 1, 0] in $p\mathcal{V} = RT$.
[b] On the assumption that c is an idonic scalar in $\{\lambda\}\{T\} = c$.
[c] "Electrostatic" system.
[d] "Electrostatic" system. Assumption that ϵ_0 is an idonic scalar.
[e] "Electromagnetic" system. Assumption that μ_0 is an idonic scalar.
[f] Assuming that lamprosity ("visibility") is an idonic scalar. The word endings have the following meanings: ENT = per unit length, −AGE = per unit area, −UM = per unit volume.

thermal conductivity. Still other assumptions are possible, leading to other sets of designation symbols.

In electromagnetism, conditions are even worse than in heat. The classical arrangement employs "electrostatic" and "electromagnetic" systems, and the same concept has different designation symbols in each system. The "electrostatic" system is based on the arbitrary assumption that the permittivity of free space is an idonic scalar. The "electromagnetic" system, on the other hand, employs the hypothesis that the permeability of free space is an idonic scalar.

Evidently, both these systems are arbitrary. In adding electromagnetic theory to mechanics, one must either make an arbitrary assumption about the "dimensionality" of a constant or select a fourth primary concept. The old idea that mechanics dominates all science shows itself in the choice of the former alternative, while the more modern approach consists in the selection of a new primary concept such as an *electric charge*.

As shown in Table II, the classical "electrostatic" system does not distinguish idonically between *capacitance* and *length*, or between *resistivity* and *time*, though no one believes these concepts to be anything but distinct. In the "electromagnetic" system, one finds such peculiarities as *inductance* and *length* with the same base vector. Another strange property is that in the "electrostatic" system one can distinguish idonically between B and H but not between D and E, while the converse is true in the "electromagnetic" system.

In the radiometric and photometric branches of physics, one finds ambiguities. Ordinarily the *lamprosity* (relative "visibility" of radiant energy) is assumed to be an idonic scalar, which gives to the photometric concepts the same designation symbols as those assigned to the corresponding radiometric concepts. Table II indicates also that *heliosent* and *phengosage* are idonically the same, though the concepts are completely different. Additional confusion is caused by the fact that *pharosage*, *helios*, and the *Poynting vector* are "dimensionally" indistinguishable, all having the base vector [0, 1, −3]. Table II is based on the modern treatment of photometrics which abandons the antiquated concept of *intensity* (candlepower). If that concept is retained, there is an additional confusion between *luminous pharos* (expressed in lumens) and *luminous intensity* (expressed in candles), and there is a similar confusion between the analogous radiometric concepts.

4. ADDITIONAL PRIMARY CONCEPTS

In the preceding section, l, m, t were considered as primary concepts. A more satisfactory system is obtained by introducing an additional primary concept for each branch of physics beyond mechanics. To eliminate the ambiguities in the subject of heat, we introduce temperature T as a primary concept. Charge Q as a primary concept helps to

TABLE II.—*Ambiguities, lmt System.*

Concepts*	Designation Symbol
Mechanics	
Energy, torque	[2, 1, −2]
Momentum, impulse	[1, 1, −1]
Pressure, normal stress, shear stress, elastic moduli	[−1, 1, −2]
Stiffness, surface tension	[0, 1, −2]
Frequency, angular velocity	[0, 0, −1]
Heat (*a*)	
Entropy, mass	[0, 1, 0]
Temperature, velocity squared	[2, 0, −2]
Thermal conductivity, viscosity	[−1, 1, −1]
Diffusivity, velocity potential	[2, 0, −1]
Thermitivity, permittivity (*d*), lamprosity, angle, solid angle, efficiency, normal strain, shear strain, permeability (*e*)	[0, 0, 0]
Heat (*b*)	
Temperature, reciprocal length	[−1, 0,0]
Diffusivity, velocity potential	[2, 0, −1]
Coefficient of linear expansion, coefficient of resistance, length	[1, 0, 0]
Thermal conductivity, thermal power	[2, 1, −3]
Electromagnetism (*d*)	
Charge, electric flux	[$\frac{3}{2}$, $\frac{1}{2}$, −1]
Charge per unit length, electric potential	[$\frac{1}{2}$, $\frac{1}{2}$, −1]
Charge per unit area, electric field strength, electric flux density	[−$\frac{1}{2}$, $\frac{1}{2}$, −1]
Capacitance, length	[1, 0, 0]
Resistivity, time	[0, 0, 1]
Current, mmf.	[$\frac{3}{2}$, $\frac{1}{2}$, −1]
Inductance, reciprocal acceleration	[−1, 0, 2]
Permeability, reciprocal temperature (*a*)	[−2, 0, 2]
Resistance, reciprocal velocity	[−1, 0, 1]
Magnetism (*e*)	
Inductance, length	[1, 0, 0]
Resistivity, diffusivity, velocity potential	[2, 0, −1]
Resistance, velocity	[1, 0, −1]
Charge, electric flux	[$\frac{1}{2}$, $\frac{1}{2}$, 0]
Charge per unit area, electric flux density	[−$\frac{3}{2}$, $\frac{1}{2}$, 0]
Magnetic flux density, mag. field strength	[−$\frac{1}{2}$, $\frac{1}{2}$, −1]
Current, mmf., vector potential	[$\frac{1}{2}$, $\frac{1}{2}$, −1]
Capacitance, reciprocal acceleration	[−1, 0, 2]
Magnetic flux, thermoelectric coefficient	[$\frac{3}{2}$, $\frac{1}{2}$, −1]
Radiometry and Photometry	
Radiant pharos, luminous pharos, thermal conductivity (*b*)	[2, 1, −3]
Radiant pharosage, radiant helios, luminous pharosage, luminous helios, Poynting vector	[0, 1, −3]
Radiant phos, luminous phos, torque	[2, 1, −2]
Radiant phosum, luminous phosum, pressure	[−1, 1, −2]
Radiant phosage, luminous phosage, surface tension	[0, 1, −2]
Radiant heliosent, radiant pharosum, luminous heliosent, luminous pharosum, phengosage	[−1, 1, −3]

* Italic letters in parentheses refer to the footnotes to Table I.

eliminate ambiguity in electricity and magnetism, while the treatment of luminous pharos F as a basic quantity makes possible idonic discrimination between radiometric and photometric concepts.

Table III indicates the discrepancies that remain in the *lmtTQF*

TABLE III.—*Ambiguities, lmtTQF and ltPTQF System.*

Concepts	Designation Symbols	
	lmtTQF	*ltPTQF*
Mechanics		
Energy, torque	[2, 1, −2, 0, 0, 0]	[0, 1, 1, 0, 0, 0]
Momentum, impulse	[1, 1, −1, 0, 0, 0]	[−1, 2, 1, 0, 0, 0]
Normal stress, shear stress, Young's modulus, shear modulus, bulk modulus, pressure	[−1, 1, −2, 0, 0, 0]	[−3, 1, 1, 0, 0, 0]
Angle, solid angle, efficiency, shear strain, normal strain	[0]	[0]
Stiffness, surface tension	[0, 1, −2, 0, 0, 0]	[−2, 1, 1, 0, 0, 0]
Frequency, angular velocity	[0, 0, −1, 0, 0, 0]	[0, −1, 0, 0, 0, 0]
Heat		
Diffusivity, velocity potential	[2, 0, −1, 0, 0, 0]	[2, −1, 0, 0, 0, 0]
Coefficient of linear expansion, temperature coefficient of resistance	[0, 0, 0, −1, 0, 0]	[0, 0, 0, −1, 0, 0]
Electromagnetism		
Charge, electric flux	[0, 0, 0, 0, 1, 0]	[0, 0, 0, 0, 1, 0]
Charge per unit area, electric flux density	[−2, 0, 0, 0, 1, 0]	[−2, 0, 0, 0, 1, 0]
Current, mmf.	[0, 0, −1, 0, 1, 0]	[0, −1, 0, 0, 1, 0]
Radiometry and Photometry		
Radiant pharosage, radiant helios, Poynting vector	[0, 1, −3, 0, 0, 0]	[−2, 0, 1, 0, 0, 0]
Radiant heliosent, radiant pharosum, phengosage	[−1, 1, −3, 0, 0, 0]	[−3, 0, 1, 0, 0, 0]
Luminous pharosage, luminous helios	[−2, 0, 0, 0, 0, 1]	[−2, 0, 0, 0, 0, 1]
Luminous heliosent, luminous pharosum	[−3, 0, 0, 0, 0, 1]	[−3, 0, 0, 0, 0, 1]

system. The ambiguities in heat have been almost eliminated by the introduction of T as a basic quantity. But discrepancies still remain in mechanics, electromagnetism, radiometry, and photometry.

There is also the possibility of replacing l, m, or t by other concepts considered as primary. An example is included in Table I, where power P is employed as a basic quantity instead of mass. In mechanics, this choice has no particular advantage, some of the idonic equations being more complicated and others being simpler than in the *lmtTQF* system. In heat, there is a slight advantage in the *ltPTQF* system; but the principal value occurs in radiometry, where the use of P emphasizes the parallelism between the radiometric and photometric concepts. As shown in Table III, the discrepancies are exactly the same whether P or m is used as a primary concept.

5. DIRECTED LENGTHS

Tables I to III show that the ambiguities in the *lmt* system can be greatly reduced by adding a new primary concept for each branch of physics beyond mechanics. But ambiguities still remain, even in mechanics. They occur when the concept requires the measurement of length in specific directions. In the concept of energy ("work"), for instance, length is measured in the direction of the force; while with torque, length is measured in a direction perpendicular to the force. This is the characteristic that distinguishes the two concepts, and it is a characteristic that is ignored by the systems of Table I. Consider now the possibility of distinguishing between distances in different directions.

This subject was studied by Williams (7) in 1892. In place of l, he used three fundamental lengths X, Y, Z. A set of orthogonal axes is set up for each problem and is oriented arbitrarily. The same unit of length is usually employed for X, Y, and Z, but the three concepts are considered as distinct. In this way, energy has the designation symbol X^2mt^{-2} while torque is $XYmt^{-2}$, normal stress is $XY^{-1}Z^{-1}mt^{-2}$ while shear stress is $Z^{-1}mt^{-2}$, angle is $X^{-1}Y$ and solid angle is $X^{-2}YZ$. Thus ambiguities are eliminated (8).

In some cases the additional discriminatory ability offered by the Williams system is an embarrassment rather than a help. In specifying length, for instance, one often has no need of designating a particular direction. Williams is forced to write "X or Y or Z" as designation symbol for length. Similarly, area is specified as YZ or XZ or XY, energy is X^2mt^{-2}, Y^2mt^{-2}, or Z^2mt^{-2}, with similar lack of simplicity for other concepts.

It seems that three fundamental lengths are unnecessary in idonic specification. Accordingly, we have retained only two (9), which we have called l_r and l_t. Furthermore, to obtain maximum simplicity, we have formulated a set of rules that specify whether a given length is to be called l_r or l_t. These rules are as follows:

(1) If a single direction is considered, call it l_r.

(2) If there is an area, then distances measured in the plane that is tangent to the area are called l_t, and distance perpendicular to l_t is called l_r.

(3) If there is an axis or a center, radial distance is called l_r and distance perpendicular to l_r is called l_t.

Of course these rules are arbitrary and may be replaced by any rules that the user prefers. Unless a definite convention of some kind is employed, however, it will be impossible to obtain a simple idonic designation of physical concepts. Table IV lists designation symbols on the basis of the foregoing rules. In most cases the idonic representation results from a routine application of the above ideas, though sometimes a little care must be exercised.

TABLE IV.—*Designation Symbols for Physical Concepts.*
(Scalar formulation, l_r and l_t taken as basic lengths.)

Name	Defining Equation	Idonic Formula	Base Vector $[l_r, l_t, m, t, T, \lambda, Q, F]$
Geometry			
Distance	s	l_r	[1, 0, 0, 0, 0, 0, 0, 0]
Area	$A = s_t^2$	l_t^2	[0, 2, 0, 0, 0, 0, 0, 0]
Volume	$\mathcal{V} = As_r$	$l_r l_t^2$	[1, 2, 0, 0, 0, 0, 0, 0]
Angle	$\theta = s_t/s_r$	$l_t l_r^{-1}$	[−1, 1, 0, 0, 0, 0, 0, 0]
Solid angle	$\Omega = A/r^2$	$l_t^2 l_r^{-2}$	[−2, 2, 0, 0, 0, 0, 0, 0]
Kinematics			
Time	t	t	[0, 0, 0, 1, 0, 0, 0, 0]
Velocity	$v = ds/dt$	$l_r t^{-1}$	[1, 0, 0, −1, 0, 0, 0, 0]
Acceleration	$a = dv/dt$	$l_r t^{-2}$	[1, 0, 0, −2, 0, 0, 0, 0]
Angular velocity	$\omega = d\theta/dt$	$l_t l_r^{-1} t^{-1}$	[−1, 1, 0, −1, 0, 0, 0, 0]
Angular acceleration	$\alpha = d\omega/dt$	$l_t l_r^{-1} t^{-2}$	[−1, 1, 0, −2, 0, 0, 0, 0]
Frequency	$= 1/t$	t^{-1}	[0, 0, 0, −1, 0, 0, 0, 0]
Dynamics			
Force	$F = ma$	$l_r m t^{-2}$*	[1, 0, 1, −2, 0, 0, 0, 0]
Momentum	mv	$l_r m t^{-1}$	[1, 0, 1, −1, 0, 0, 0, 0]
Impulse	$\int F dt$	$l_r m t^{-1}$	[1, 0, 1, −1, 0, 0, 0, 0]
Torque	$\mathcal{T} = Fr$	$l_r l_t m t^{-2}$	[1, 1, 1, −2, 0, 0, 0, 0]
Energy	$U = \int F ds$	$l_r^2 m t^{-2}$**	[2, 0, 1, −2, 0, 0, 0, 0]
Power	$P = dU/dt$	$l_r^2 m t^{-3}$**	[2, 0, 1, −3, 0, 0, 0, 0]
Mass	m	m	[0, 0, 1, 0, 0, 0, 0, 0]
Moment of inertia	$\mathcal{I} = mr^2$	$l_r^2 m$	[2, 0, 1, 0, 0, 0, 0, 0]
Dissipation coefficient	$B = F/v$	$m t^{-1}$	[0, 0, 1, −1, 0, 0, 0, 0]
Stiffness	$K = F/s$	$m t^{-2}$	[0, 0, 1, −2, 0, 0, 0, 0]
Efficiency	$\eta = P_o/P_i$	1	[0]
Elasticity			
Stress, normal	$T_n = F_r/A$	$l_r l_t^{-2} m t^{-2}$	[1, −2, 1, −2, 0, 0, 0, 0]
shear	$T_s = F_t/A$	$l_t^{-1} m t^{-2}$	[0, −1, 0, −2, 0, 0, 0, 0]
Strain, normal	$S_n = \Delta s/s$	1	[0]
shear	$S_s = \Delta s_t/s_r$	$l_r^{-1} l_t$	[−1, 1, 0, 0, 0, 0, 0, 0]
Elastic moduli	T/S	$l_r l_t^{-2} m t^{-2}$	[1, −2, 1, −2, 0, 0, 0, 0]
Hydrodynamics			
Pressure	$p = F_r/A$	$l_r l_t^{-2} m t^{-2}$	[1, −2, 1, −2, 0, 0, 0, 0]
Velocity potential	$\varphi = \int v ds$	$l_r^2 t^{-1}$	[2, 0, 0, −1, 0, 0, 0, 0]
Density	$\delta = m/\mathcal{V}$	$l_r^{-1} l_t^{-2} m$	[−1, −2, 1, 0, 0, 0, 0, 0]
Viscosity	$\eta = \dfrac{T_s}{\Delta v_t/s_r}$	$l_r l_t^{-2} m t^{-1}$	[1, −2, 1, −1, 0, 0, 0, 0]
Surface tension	$\mathcal{J} = F_r/s_t$	$l_r l_t^{-1} m t^{-2}$	[1, −1, 1, −2, 0, 0, 0, 0]
Heat			
Temperature	T	T	[0, 0, 0, 0, 1, 0, 0, 0]
Thermal energy (heat)	U	$l_r^2 m t^{-2}$	[2, 0, 1, −2, 0, 0, 0, 0]
Thermal power	P	$l_r^2 m t^{-3}$	[2, 0, 1, −3, 0, 0, 0, 0]

* Or $l_t m t^{-2}$, depending on direction of force.
** Or $l_t^2 m t^{-2}$ and $l_t^2 m t^{-3}$, depending on direction of force.

TABLE IV.—*Continued.*

Name	Defining Equation	Idonic Formula	Base Vector $[l_r, l_t, m, t, T, \lambda, Q, F]$
Entropy	$S = \int \frac{du}{T}$	$l_r^2 m t^{-2} T^{-1}$	[2, 0, 1, −2, −1, 0, 0, 0]
Power per unit volume	$\mathfrak{p} = P/\mathcal{V}$	$l_r l_t^{-2} m t^{-3}$	[1, −2, 1, −3, 0, 0, 0, 0]
Thermal conductivity	$k = \frac{P s_r}{A \Delta T}$	$l_r^3 l_t^{-2} m t^{-3} T^{-1}$	[3, −2, 1, −3, −1, 0, 0, 0]
Thermitivity (sp. heat)	$c = \frac{1}{m}\frac{\Delta U}{\Delta T}$	$l_r^2 t^{-2} T^{-1}$	[2, 0, 0, −2, −1, 0, 0, 0]
Diffusivity	$h^2 = k/\delta c$	$l_r^2 t^{-1}$	[2, 0, 0, −1, 0, 0, 0, 0]
Thalpance ("emissivity")	$\vartheta = \frac{P}{A \Delta T}$	$l_r^2 l_t^{-2} m t^{-3} T^{-1}$	[2, −2, 1, −3, −1, 0, 0, 0]
Coefficient of linear expansion	$\alpha = \Delta l / l \Delta T$	T^{-1}	[0, 0, 0, 0, −1, 0, 0, 0]
Thermoelectric coefficient	$= \Delta V / \Delta T$	$l_t^2 m t^{-2} T^{-1} Q^{-1}$	[0, 2, 1, −2, −1, 0, −1, 0]
Temperature coefficient of resistance	$\alpha = \Delta R / (R \Delta T)$	T^{-1}	[0, 0, 0, 0, −1, 0, 0, 0]
Electrostatics			
Charge	Q	Q	[0, 0, 0, 0, 0, 0, 1, 0]
Charge per unit length	$\lambda = Q/s_t$	$Q l_t^{-1}$	[0, −1, 0, 0, 0, 0, 1, 0]
Charge per unit area	$\sigma = Q/A$	$Q l_t^{-2}$	[0, −2, 0, 0, 0, 0, 1, 0]
Charge per unit volume	$\rho = Q/\mathcal{V}$	$Q l_r^{-1} l_t^{-2}$	[−1, −2, 0, 0, 0, 0, 1, 0]
Electric field strength	$E = F/Q$	$l_t m t^{-2} Q^{-1}$	[0, 1, 1, −2, 0, 0, 1, 0]
Electric potential (and potential difference)	$V = \int E ds$	$l_t^2 m t^{-2} Q^{-1}$	[0, 2, 1, −2, 0, 0, 1, 0]
Electric flux density	$D = Q/A_{rt}$	$Q l_t^{-1} l_r^{-1}$	[−1, −1, 0, 0, 0, 0, 1, 0]
Electric flux	$\Psi = \int D dt$	Q	[0, 0, 0, 0, 0, 0, 1, 0]
Permittivity	$\epsilon = D/E$	$l_r^{-1} l_t^{-2} m^{-1} t^2 Q^2$	[−1, −2, 1, 2, 0, 0, 2, 0]
Capacitance	$C = Q/V$	$l_t^{-2} m^{-1} t^2 Q^2$	[0, −2, −1, 2, 0, 0, 2, 0]
Electric Current			
Current	$I = dQ/dt$	$Q t^{-1}$	[0, 0, 0, −1, 0, 0, 1, 0]
Current density	$J = dI/dA$	$l_r^{-1} l_t^{-1} Q t^{-1}$	[−1, −1, 0, −1, 0, 0, 1, 0]
Resistivity	$\mathfrak{R} = E/J$	$l_r l_t^2 m t^{-1} Q^{-2}$	[1, 2, 1, −1, 0, 0, −2, 0]
Resistance	$R = V/I$	$l_t^2 m t^{-1} Q^{-2}$	[0, 2, 1, −1, 0, 0, −2, 0]
Magnetism			
Magnetic flux density	$B = F_t / I l_t$	$m t^{-1} Q^{-1}$	[0, 0, 1, −1, 0, 0, −1, 0]
Magnetic flux	$\Phi = \int B da$	$l_t^2 m t^{-1} Q^{-1}$	[0, 2, 1, −1, 0, 0, −1, 0]
Magnetic field strength	$H = NI/l_r$	$l_r^{-1} t^{-1} Q$	[−1, 0, 0, −1, 0, 0, 1, 0]
Permeability	$\mu = B/H$	$l_r m Q^{-2}$	[1, 0, 1, 0, 0, 0, −2, 0]
Magnetomotive force	$\mathfrak{F} = NI$	$Q t^{-1}$	[0, 0, 0, −1, 0, 0, 1, 0]
Inductance	$L = N\Phi/I$	$l_t^2 m Q^{-2}$	[0, 2, 1, 0, 0, 0, −2, 0]
Poynting vector	$\mathbf{D}_r = \mathbf{E} \times \mathbf{H}$	$l_r^{-1} l_t m t^{-3}$	[−1, 1, 1, −3, 0, 0, 0, 0]
Radiometry			
Radiant pharos (radiant power)	$F_r = P$	$l_r^2 m t^{-3}$	[2, 0, 1, −3, 0, 0, 0, 0]
Radiant pharosage	$D_r = F_r/A$	$l_r^2 l_t^{-2} m t^{-3}$	[2, −2, 1, −3, 0, 0, 0, 0]
Radiant pharosum	$p_r = F_r/\mathcal{V}$	$l_r l_t^{-2} m t^{-3}$	[1, −2, 1, −3, 0, 0, 0, 0]
Radiant phos (energy)	$Q_r = U$	$l_r^2 m t^{-2}$	[2, 0, 1, −2, 0, 0, 0, 0]

TABLE IV.—*Continued.*

Name	Defining Equation	Idonic Formula	Base Vector $[l_r, l_t, m, t, T, \lambda, Q, F]$
Radiant phosage	$W_r = Q_r/A$	$l_r^2 l_t^{-2} m t^{-2}$	[2, −2, 1, −2, 0, 0, 0, 0]
Radiant phosum	$u_r = Q_r/\mathcal{V}$	$l_r l_t^{-2} m t^{-2}$	[1, −2, 1, −2, 0, 0, 0, 0]
Radiant helios	$H_r = \pi \dfrac{dD_r}{d\Omega}$	$l_r^4 l_t^{-4} m t^{-3}$	[4, −4, 1, −3, 0, 0, 0, 0]
Radiant heliosent	$G_r = H_r/s$	$l_r^3 l_t^{-4} m t^{-3}$	[3, −4, 1, −3, 0, 0, 0, 0]
Phengosage	$J(\lambda) = dD_r/d\lambda$	$l_r^2 l_t^{-2} m t^{-3} \lambda^{-1}$	[2, −2, 1, −3, 0, −1, 0, 0]
Photometry			
Luminous pharos (luminous flux)	F_l	F	[0, 0, 0, 0, 0, 0, 0, 1]
Luminous pharosage	$D_l = F_l/A$	$F l_t^{-2}$	[0, −2, 0, 0, 0, 0, 0, 1]
Luminous pharosum	$p_l = F_l/\mathcal{V}$	$F l_r^{-1} l_t^{-2}$	[−1, −2, 0, 0, 0, 0, 0, 1]
Luminous phos ("light")	$Q_l = F_l t$	Ft	[0, 0, 0, 1, 0, 0, 0, 1]
Luminous phosage	$W_l = Q_l/A$	Ftl_t^{-2}	[0, −2, 0, 1, 0, 0, 0, 1]
Luminous phosum	$u_l = Q_l/\mathcal{V}$	$Ftl_r^{-1} l_t^{-2}$	[−1, −2, 0, 1, 0, 0, 0, 1]
Luminous helios	$H_l = \pi \dfrac{dD_l}{d\Omega}$	$F l_r^2 l_t^{-4}$	[2, −4, 0, 0, 0, 0, 0, 1]
Luminous heliosent	$G_l = H_l/s$	$F l_r l_t^{-4}$	[1, −4, 0, 0, 0, 0, 0, 1]
Lamprosity ("visibility")	$\mathcal{G} = D_l/D_r$	$F l_r^{-2} m^{-1} t^3$	[0, −2, −1, 3, 0, 0, 0, 1]

For example, *force* is generally considered to be in the l_r-direction in accordance with Rule 1, the idonic formula being written $l_r m t^{-2}$. But if the force acts about an axis, it must be $l_t m t^{-2}$ in accordance with Rule 3. Thus *torque* is expressed idonically as $l_r l_t m t^{-2}$, while energy or "work" is $l_r^2 m t^{-2}$. A similar distinction is made between *normal stress* and *shear stress*. Normal stress is taken in the l_r-direction in accordance with Rules 1 and 2, but a shear force is tangent to an area and is therefore called $l_t m t^{-2}$ by Rule 2.

The most troublesome part of classical physics is electromagnetism, where the involved spacial relationships require thoughtful consideration if a consistent idonic specification is to be obtained. If electrostatics only is under consideration, the **E**-vector may be taken in the l_r-direction (Rule 1). But if all of electromagnetism is to form a consistent whole, the scheme of Fig. 2 seem preferable. Here the **B**-vector is taken in the l_r-direction, which gives l_t^2 as the area needed in obtaining total magnetic flux. Then **E** and **J** are along one l_t-axis, while the force **F** on a current-carrying conductor is along the other l_t-axis. This arbitrary arrangement gives the particular designation symbols given in Table IV for electromagnetism. Needless to say, the principle of idonic relativity allows many other specifications of equal validity.

As an indication that the designation symbols apply to other equations beside the particular ones employed in defining the concepts, we consider three other equations of electromagnetism. The first equation is concerned with energy stored in an electric field:

$$u = \tfrac{1}{2}\epsilon E^2. \tag{6}$$

The energy per unit volume u is related to the force in the **E**-direction (l_t-direction) and is therefore

$$\frac{ml_t^2t^{-2}}{l_rl_t^2} = \mathbf{ml_r^{-1}t^{-2}}.$$

Now consider the right side of Eq. 6. From Table IV,

$$(l_r^{-1}l_t^{-2}m^{-1}t^2Q^2)(l_t^2m^2t^{-2}Q^{-2}) = \mathbf{ml_r^{-1}t^{-2}}.$$

Thus the designation symbols of Table IV satisfy Eq. (6).

As a second illustration, take the equation

$$\epsilon\mu = 1/c^2 \qquad (7)$$

containing the velocity c of an electromagnetic wave. Since the wave is propagated in the l_t-direction (Fig. 2),

$$\frac{1}{c^2} = \mathbf{l_t^{-2}t^2}.$$

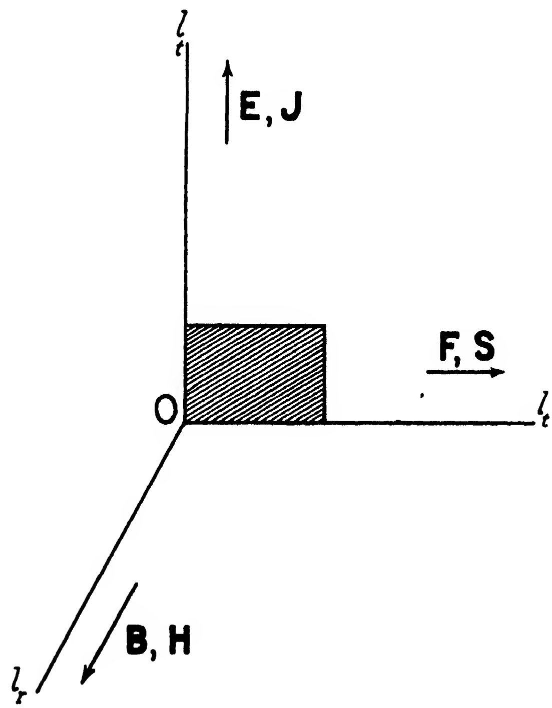

FIG. 2. Direction vectors for electromagnetic vectors.

The left side of Eq. 2 gives

$$(l_r^{-1}l_t^{-2}m^{-1}t^2Q^2)(l_rmQ^{-2}) = \mathbf{l_t^{-2}t^2}.$$

The third example deals with the Poynting vector **S**:

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}. \qquad (8)$$

Since **S** is the power per unit area propagated horizontally (Fig. 2) in the l_t-direction, its idonic specification is

$$\frac{ml_t^2t^{-3}}{l_rl_t} = \mathbf{l_r^{-1}l_tmt^{-3}}.$$

For the right side of Eq. 8, we have

$$(l_t m t^{-2} Q^{-1})(l_r^{-1} t^{-1} Q) = \mathbf{m l_r^{-1} l_t t^{-3}}$$

which again checks.

TABLE V.—*Ambiguities in Designations of Table IV.*
(l_r and l_t taken as basic lengths.)

Concepts	Base Vector $[l_r, l_t, m, t, T, \lambda, Q, F]$
Momentum, impulse	[1, 0, 1, −1, 0, 0, 0, 0]
Angle, shear strain	[−1, 1, 0, 0, 0, 0, 0, 0]
Normal stress, pressure, elastic moduli, radiant phosum	[1, −2, 1, −2, 0, 0, 0, 0]
Coefficient of linear expansion, temperature coefficient of resistance	[0, 0, 0, 0, −1, 0, 0, 0]
Velocity potential, diffusivity	[2, 0, 0, −1, 0, 0, 0, 0]
Charge, electric flux	[0, 0, 0, 0, 0, 0, 1, 0]
Current, mmf.	[0, 0, 0, −1, 0, 0, 1, 0]

TABLE VI.—*Kinematics.*

Concept	Symbol	Base Vector $[l_r, l_t, t]$
Distance	s	[1, 0, 0]
Area	A	[0, 2, 0]
Volume	$\mathcal{V}$	[1, 2, 0]
Angle	θ	[−1, 1, 0]
Solid angle	Ω	[−2, 2, 0]
Time	t	[0, 0, 1]
Velocity	v	[1, 0, −1]
Acceleration	a	[1, 0, −2]
Angular velocity	ω	[−1, 1, −1]
Angular acceleration	α	[−1, 1, −2]
Frequency	f	[0, 0, −1]

TABLE VII.—*Dynamics.*

Concept	Symbol	Base Vector $[l_r, l_t, m, t]$
Force	F	[1, 0, 1, −2]
Momentum	mv	[1, 0, 1, −1]
Impulse	$\int F dt$	[1, 0, 1, −1]
Torque	$\mathcal{T}$	[1, 1, 1, −2]
Energy	U	[2, 0, 1, −2]
Power	P	[2, 0, 1, −3]
Mass	m	[0, 0, 1, 0]
Moment of inertia	$\mathcal{I}$	[2, 0, 1, 0]
Dissipation coefficient	B	[0, 0, 1, −1]
Stiffness	K	[0, 0, 1, −2]
Efficiency	η	[0]

That l_r and l_t provide a good system (9) for designating the concepts of physics is shown by Table V. Most of the ambiguities of the other systems (Tables II and III) have been eliminated by the use of two length concepts. Most of the remaining duplications are evidently redundancies in the formulation of physical concepts rather than errors in the idonic method. For example, shear strain is actually an angle, and this fact is evidenced by the identity of designation symbols for the two concepts. Similarly, there is no way to distinguish idonically between *electric charge* and *electric flux* or between *current* and *magnetomotive force*. On the other hand, there are a few cases, such as *pressure* and *radiant energy per unit volume* or *velocity potential* and *diffusivity*, where the concepts are physically distinct but the designation symbols are identical. One may regard these few cases as defects of the l_rl_t-system, though it is doubtful if they could cause any inconvenience.

Tables I and IV indicate that the designation symbols are somewhat cumbersome when based on the primary concepts needed for all of

TABLE VIII.—*Heat.*

Concept	Symbol	Base Vector $[l_r, l_t, t, T, P]$
Temperature	T	[0, 0, 0, 1, 0]
Thermal energy (heat)	U	[0, 0, 1, 0, 1]
Thermal power	P	[0, 0, 0, 0, 1]
Entropy	$\mathcal{S}$	[0, 0, 1, −1, 1]
Power per unit volume	$\mathfrak{p}$	[−1, −2, 0, 0, 1]
Thermal conductivity	k	[1, −2, 0, −1, 1]
Thermitivity	c	[2, 0, −2, −1, 0]
Diffusivity	h^2	[2, 0, −1, 0, 0]
Thalpance	ϑ	[0, −2, 0, −1, 1]
Coefficient of linear expansion	α	[0, 0, 0, −1, 0]

physics. It should be realized, however, that in any given problem the complete set of concepts is ordinarily not required. In kinematics, for instance, only l_r, l_t, and t are required and idon space is three-dimensional. The designation symbols are listed in Table VI. For dynamics, an additional primary concept (such as *mass*) is introduced; but the base vectors are still fairly simple, as shown in Table VII.

When a problem deals exclusively with heat, there is no need of expressing the designation symbols in terms of mechanical quantities. As shown by Table VIII, the base vectors are expressed very simply in terms of l_r, l_t, t, T, and P. Table IX lists the concepts of electromagnetism, expressed in terms of l_r, l_t, t, I, and P. For electromagnetic problems, this set of primary concepts is often found to be most convenient, though other sets (such as l_r, l_t, t, Q, U) are also good. In radiometry, the use of P rather than m as a primary concept exhibits

TABLE IX.—*Electricity and Magnetism.*

Concept	Symbol	Base Vector $[l_r, l_t, t, I, P]$
Electrostatics		
Charge	Q	[0, 0, 1, 1, 0]
Charge per unit length	λ	[0, −1, 1, 1, 0]
Charge per unit area	σ	[0, −2, 1, 1, 0]
Charge per unit volume	ρ	[−1, −2, 1, 1, 0]
Electric field strength	E	[0, −1, 0, −1, 1]
Electric potential	V	[0, 0, 0, −1, 1]
Electric flux density	D	[−1, −1, 1, 1, 0]
Permittivity	ϵ	[−1, 0, 1, 2, −1]
Capacitance	C	[0, 0, 1, 2, −1]
Electric Current		
Current	I	[0, 0, 0, 1, 0]
Current density	J	[−1, −1, 0, 1, 0]
Resistivity	$\mathcal{R}$	[1, 0, 0, −2, 1]
Resistance	R	[0, 0, 0, −2, 1]
Magnetism		
Magnetic flux density	B	[0, −2, 1, −1, 1]
Magnetic flux	Φ	[0, 0, 1, −1, 1]
Magnetic field strength	H	[−1, 0, 0, 1, 0]
Permeability	μ	[1, −2, 1, −2, 1]
Mmf.	$\mathfrak{F}$	[0, 0, 0, 1, 0]
Inductance	L	[0, 0, 1, −2, 1]
Poynting vector	D_r	[−1, −1, 0, 0, 1]

TABLE X.—*Radiometry and Photometry.*

Concept	Symbol	Base Vector $[l_r, l_t, t, \lambda, P, F]$
Radiometry		
Radiant pharos (radiant power)	F_r	[0, 0, 0, 0, 1, 0]
Radiant pharosage	D_r	[0, −2, 0, 0, 1, 0]
Radiant pharosum	p_r	[−1, −2, 0, 0, 1, 0]
Radiant phos (radiant energy)	Q_r	[0, 0, 1, 0, 1, 0]
Radiant phosage	W_r	[0, −2, 1, 0, 1, 0]
Radiant phosum	u_r	[−1, −2, 1, 0, 1, 0]
Radiant helios	H_r	[2, −4, 0, 0, 1, 0]
Radiant heliosent	G_r	[1, −4, 0, 0, 1, 0]
Phengosage	$J(\lambda)$	[0, −2, 0, −1, 1, 0]
Photometry		
Luminous pharos (luminous flux)	F_l	[0, 0, 0, 0, 0, 1]
Luminous pharosage	D_l	[0, −2, 0, 0, 0, 1]
Luminous pharosum	p_l	[−1, −2, 0, 0, 0, 1]
Luminous phos ("light")	Q_l	[0, 0, 1, 0, 0, 1]
Luminous phosage	W_l	[0, −2, 1, 0, 0, 1]
Luminous phosum	u_l	[−1, −2, 1, 0, 0, 1]
Luminous helios	H_l	[2, −4, 0, 0, 0, 1]
Luminous heliosent	G_l	[1, −4, 0, 0, 0, 1]
Lamprosity ("visibility")	ϑ	[0, 0, 0, 0, −1, 1]

the parallelism between the radiometric and photometric concepts (Table X). Tables VI to X are examples which show that designation symbols are not unduly complicated in the $l_r l_t$-system and suggest some of the variety that is allowable under the principle of idonic relativity.

6. OTHER USES OF DESIGNATION SYMBOLS

Undoubtedly the principal value of designation symbols ("dimensions") is that they give a shorthand specification of concepts and show at a glance the relations among concepts. There are other uses, however, five of which were listed previously (3). Change of *units* is easily effected in the $l_r l_t$-system by condensing the two l's into one:

$$l_r = l_t = l$$

and proceeding as in the *lmt* system. This is based on the usual assumption that the same unit of length is employed throughout. Note, however, that the $l_r l_t$-system allows the calculation of change of units even when lengths are measured in different units in different directions (volumes in *acre feet*, for instance, or angles in *degrees*). Other uses of designation symbols are handled in the new system just as in the *lmt*-system. To demonstrate this fact, we give two illustrations.

The first example deals with the idonic checking of equations. Physicists and engineers frequently substitute idonic expressions on both sides of an equation to determine if it is "dimensionally" homogeneous. Homogeneity is a necessary but not a sufficient condition for the validity of the equation. If such a check is obtained with *lmt*, there is still considerable chance that the assumed equation is wrong. When additional fundamental concepts are introduced into the idonic system, the margin of error is reduced, though the method never insures validity.

Suppose that a physicist, accustomed to use Gaussian units and to omit ϵ_0 and μ_0, is trying to remember how to write the expression for the capacitance of a parallel-plate capacitor. He knows that C is directly proportional to the area A of the plates, inversely proportional to their separation l, and depends on the permittivity ϵ of the medium. How does ϵ enter the equation? From Table IX and Fig. 2, he obtains the following base bectors:

For C,	$[0, 0, 1, 2, -1]$
For A,	$[1, 1, 0, 0, 0]$
For l,	$[0, 1, 0, 0, 0]$
For ϵ,	$[-1, 0, 1, 2, -1]$.

Thus,

$$[0, 0, 1, 2, -1] = [1, 1, 0, 0, 0] - [0, 1, 0, 0, 0] \pm [-1, 0, 1, 2, -1].$$

Evidently, homogeneity is obtained with the $+$ sign, so the equation

must be

$$C = \frac{\epsilon A}{l} \text{ (possible idonic scalar).} \tag{9}$$

Try the same problem with the customary *lmt* system. From Table I,

$$[1, 0, 0] = [2, 0, 0] - [1, 0, 0] \pm [0],$$

which tells absolutely nothing about the position of ϵ in the capacitance equation.

As a second illustration, consider the derivation of Child's relation on the idonic basis. Assume two large parallel electrodes, one of which emits electrons. Evidently, the pertinent concepts are the current density J (equilibrium conditions), the potential difference V between electrodes, charge Q of an electron, mass m of an electron, distance l between electrodes, and permittivity ϵ. The associated base vectors are obtained from Table IX, giving

$$\begin{aligned}
\{J\} &= J[-1, -1, 0, 1, 0] \\
\{V\} &= V[0, 0, 0, -1, 1] \\
\{Q\} &= Q[0, 0, 1, 1, 0] \\
\{m\} &= m[0, -2, 3, 0, 1] \\
\{l\} &= l[0, 1, 0, 0, 0] \\
\{\epsilon\} &= \epsilon[-1, 0, 1, 2, -1].
\end{aligned}$$

Let

$$\{J\} = \{V\}^{\alpha}\{Q\}^{\beta}\{m\}^{\gamma}\{l\}^{\delta}\{\epsilon\}^{\mu}\cdot\text{const.},$$

where α, β, γ, δ, μ are constants that are to be determined. Substitution of the foregoing idonic equations gives $\alpha = \frac{3}{2}$, $\beta = \frac{1}{2}$, $\gamma = -\frac{1}{2}$, $\delta = -2$, $\mu = 1$. Thus the equation for current density limited by space charge must be

$$J = \text{Const.}\ \sqrt{Q/m}\,\frac{\epsilon V^{\frac{3}{2}}}{l^2} \tag{10}$$

which is the usual form of Child's equation.

A less satisfactory result is obtained if a single fundamental length is employed instead of l_r and l_t. From Table I for base vectors $[l, m, t, Q]$,

$$\begin{aligned}
\{J\} &= J[-2, 0, -1, 1] \\
\{V\} &= V[2, 1, -2, -1] \\
\{Q\} &= Q[0, 0, 0, 1] \\
\{m\} &= m[0, 1, 0, 0] \\
\{l\} &= l[1, 0, 0, 0] \\
\{\epsilon\} &= \epsilon[-3, -1, 2, 2].
\end{aligned}$$

Using the same process as before, one finds that the equation must have

the form,

$$J = \text{Const.}\sqrt{\frac{V}{m}\frac{Q^{\frac{3}{2}}}{l^3}}\cdot F\left(\frac{Vl\epsilon}{Q}\right). \tag{11}$$

By proper choice of the unknown function F, we can again obtain Child's equation. But there is nothing in this second solution to indicate the form of F, and thus the result is much less satisfastory than that obtained with l_r and l_t.

7. A VECTOR SYSTEM

The previous sections have shown that there is a distinct advantage in the use of two fundamental lengths l_r and l_t in place of the usual l-"dimension." We now consider an alternative system in which the distinction between vectors and scalars is maintained in the idonic specification of physical concepts. It will be recalled that in the previous systems, the vector or scalar nature of the physical quantities is ignored and vector and scalar products are considered to be identical. In some cases, however, the additional information, obtained by discriminating between vectors and scalars in physical space, may be helpful.

The fundamental lengths l_r and l_t are again employed, but with an additional length l which is used where direction is not significant. The three rules of Section 5 are retained and a fourth rule is added:

(4) The defining equations of physics, used in determining the idonic specification, are written in vector form. The distinction between vector and scalar products is maintained, in accordance with the following equations:

$$\begin{aligned} l_t \times l_t &= l_r l \\ l_r \times l_t &= l_t \times l_r = l_t l, \\ l_t \cdot l_t &= l^2. \end{aligned}$$

These product rules are like those of ordinary vector algebra. The only difference is that the distinction between positive and negative signs in vector products is dropped, and hence the vector product becomes commutative.

Table XI lists the principal concepts of physics and their idonic specification in the vector system. *Distance* in general is considered as a vector, and (in accordance with Rule 1) it is in the l_r-direction. In accordance with the usual convention of vector analysis, *area* is taken as a vector that is perpendicular to the element of surface under consideration. *Volume* may be defined as $l_r l \cdot l_r$, which according to Rule 4 is equal to the scalar quantity l^3. *Torque* is

$$\mathbf{T} = \mathbf{s}_r \times \mathbf{F}_t$$

so its designation symbol is

$$l_r \times l_t m t^{-2} = l_t l m t^{-2},$$

which shows that torque is a vector quantity in the direction l_t.

TABLE XI.—*A Vectorial Derivation of Designation Symbols.*
(l_r, l_t, l taken as basic lengths.)

Concept	Symbol	Defining Equation	Idonic Formula	Base Vector $[l_r, l_t, l, m, t, T, \lambda, Q, F]$
Geometry				
Distance	$\mathbf{s}$	$\mathbf{s}$	l_r	[1, 0, 0, 0, 0, 0, 0, 0, 0]
Area	$\mathbf{A}$	$\mathbf{A} = \mathbf{x} \times \mathbf{y}$	$l_r l$	[1, 0, 1, 0, 0, 0, 0, 0, 0]
Volume	$\mathcal{V}$	$\mathcal{V} = \mathbf{A} \cdot \mathbf{s}$	l^3	[0, 0, 3, 0, 0, 0, 0, 0, 0]
Angle	θ	$\|\theta\| = \|s_t\|/\|s_r\|$	$l_t l_r^{-1}$	[−1, 1, 0, 0, 0, 0, 0, 0, 0]
Solid angle	$\mathbf{\Omega}$	$\mathbf{\Omega} = \mathbf{A}/(\mathbf{r} \cdot \mathbf{r})$	$l_r l^{-1}$	[1, 0, −1, 0, 0, 0, 0, 0, 0]
Kinematics				
Time	t	t	t	[0, 0, 0, 0, 1, 0, 0, 0, 0]
Velocity	$\mathbf{v}$	$\mathbf{v} = d\mathbf{s}/dt$	$l_r t^{-1}$	[1, 0, 0, 0, −1, 0, 0, 0, 0]
Acceleration	$\mathbf{a}$	$\mathbf{a} = d\mathbf{v}/dt$	$l_r t^{-2}$	[1, 0, 0, 0, −2, 0, 0, 0, 0]
Angular velocity	ω	$\omega = d\|\theta\|/dt$	$l_t l_r^{-1} t^{-1}$	[−1, 1, 0, 0, −1, 0, 0, 0, 0]
Angular acceleration	α	$\|\alpha\| = \|d\omega/dt\|$	$l_t l_r^{-1} t^{-2}$	[−1, 1, 0, 0, −2, 0, 0, 0, 0]
Frequency	f	$f = 1/t$	t^{-1}	[0, 0, 0, 0, −1,0, 0, 0, 0]
Dynamics				
Force	$\mathbf{F}$	$\mathbf{F} = m\mathbf{a}$	$l_r m t^{-2}$*	[1, 0, 0, 1, −2, 0, 0, 0, 0]
Momentum		$m\mathbf{v}$	$l_r m t^{-1}$	[1, 0, 0, 1, −1, 0, 0, 0, 0]
Impulse		$\int \mathbf{F} dt$	$l_r m t^{-1}$	[1, 0, 0, 1, −1, 0, 0, 0, 0]
Torque	$\mathfrak{T}$	$\mathfrak{T} = \mathbf{r} \times \mathbf{F}_t$	$l_t l m t^{-2}$	[0, 1, 1, 1, −2, 0, 0, 0, 0]
Energy	U	$U = \int \mathbf{F} \cdot d\mathbf{s}$	$l^2 m t^{-2}$	[0, 0, 2, 1, −2, 0, 0, 0, 0]
Power	P	$P = dU/dt$	$l^2 m t^{-3}$	[0, 0, 2, 1, −3, 0, 0, 0, 0]
Mass	m		m	[0, 0, 0, 1, 0, 0, 0, 0, 0]
Moment of inertia	$\mathcal{J}$	$\mathcal{J} = \int \mathbf{r} \cdot \mathbf{r} dm$	$l^2 m$	[0, 0, 2, 1, 0, 0, 0, 0, 0]
Dissipation coefficient	B	$\mathbf{F} = B\mathbf{v}$	$m t^{-1}$	[0, 0, 0, 1, −1, 0, 0, 0, 0]
Stiffness	K	$\mathbf{F} = K\mathbf{s}$	$m t^{-2}$	[0, 0, 0, 1, −2, 0, 0, 0, 0]
Efficiency*	η	$\eta = P_o/P_i$	1	[0]
Elasticity				
Stress, normal	T_n	$\|T_n\| = \|F_r\|/\|A_r\|$	$l^{-1} m t^{-2}$	[0, 0, −1, 1, −2, 0, 0, 0, 0]
shear	T_s	$\|T_s\| = \|F_t\|/\|A_r\|$	$l_t l_r^{-1} l^{-1} m t^{-2}$	[−1, 1, −1, 1, −2, 0, 0, 0, 0]
Strain, normal	S_n	$\|S_n\| = \|\Delta s_r\|/\|s_r\|$	1	[0]
shear	S_t	$\|S_t\| = \|\Delta s_t\|/\|s_r\|$	$l_t l_r^{-1}$	[−1, 1, 0, 0, 0, 0, 0, 0, 0]
Elastic moduli	c	$\|c\| = \|T\|/\|S\|$	$l^{-1} m t^{-2}$	[0, 0, −1, 1, −2, 0, 0, 0, 0]
Hydrodynamics				
Pressure	p	$p = \|F_r\|/\|A_r\|$	$l^{-1} m t^{-2}$	[0, 0, −1, 1, −2, 0, 0, 0, 0]
Velocity potential	φ	$\varphi = \int \mathbf{v} \cdot d\mathbf{s}$	$l^2 t^{-1}$	[0, 0, 2, 0, −1, 0, 0, 0, 0]
Density	δ	$\delta = m/\mathcal{V}$	$m l^{-3}$	[0, 0, −3, 1, 0, 0, 0, 0, 0]
Viscosity	η	$\|T_s\| = \eta\|\Delta v_t\|/\|s_r\|$	$l^{-1} m t^{-1}$	[0, 0, −1, 1, −1, 0, 0, 0, 0]
Surface tension	$\mathcal{I}$	$\|F_r\| = \mathcal{I}\|s_t\|$	$l_r l_t^{-1} m t^{-2}$	[1, −1, 0, 1, −2, 0, 0, 0, 0]
Heat				
Temperature	T		T	[0, 0, 0, 0, 0, 1, 0, 0, 0]
Thermal energy (heat)	U		$l^2 m t^{-2}$	[0, 0, 2, 1, −2, 0, 0, 0, 0]
Thermal power	P		$l^2 m t^{-3}$	[0, 0, 2, 1, −3, 0, 0, 0, 0]
Entropy	$\mathcal{S}$	$\mathcal{S} = \int dU/T$	$l^2 m t^{-2} T^{-1}$	[0, 0, 1, 1, −2, −1, 0, 0, 0]
Power per unit volume	$\mathfrak{p}$	$\mathfrak{p} = P/\mathcal{V}$	$l^{-1} m t^{-3}$	[0, 0, −1, 1, −3, 0, 0, 0, 0]

TABLE XI.—*Continued.*

Concept	Symbol	Defining Equation	Idonic Formula	Base Vector $[l_r, l_t, l, m, t, T, \lambda, Q, F]$
Thermal conductivity	k	$kA\Delta T = P\mathbf{s}_r$	$lmt^{-3}T^{-1}$	[0, 0, 1, 1, −3, −1, 0, 0, 0]
Thermitivity (sp. heat)	c	$c = \frac{1}{m}\frac{\Delta U}{\Delta T}$	$l^2t^{-2}T^{-1}$	[0, 0, 2, 0, −2, 1, 0, 0, 0]
Diffusivity	h^2	$h^2 = \frac{k}{\delta c}$	l^2t^{-1}	[0, 0, 2, 0, −1, 0, 0, 0, 0]
Thalpance ("emissivity")	ə	$\lvert ə \rvert = \frac{P}{\lvert A \rvert \Delta T}$	$mt^{-3}T^{-1}$	[0, 0, 0, 1, −3, −1, 0, 0, 0]
Coefficient of linear expansion	α	$\Delta l = \alpha l \Delta T$	T^{-1}	[0, 0, 0, 0, 0, −1, 0, 0, 0]
Thermoelectric coefficient	$\imath$	$\Delta V = \imath \Delta T$	$l^2mt^{-2}T^{-1}Q^{-1}$	[0, 0, 2, 1, −2, −1, 0, −1, 0]
Temperature coefficient of resistance	α	$\Delta R = \alpha R \Delta T$	T^{-1}	[0, 0, 0, 0, 0, −1, 0, 0, 0]
Electrostatics				
Charge	Q		Q	[0, 0, 0, 0, 0, 0, 0, 1, 0]
Charge per unit length	λ	$\lvert\lambda\rvert = Q/\lvert s_r \rvert$	Ql^{-1}	[0, 0, −1, 0, 0, 0, 0, 1, 0]
Charge per unit area	σ	$\lvert\sigma\rvert = Q/\lvert A \rvert$	Ql^{-2}	[0, 0, −2, 0, 0, 0, 0, 1, 0]
Charge per unit volume	ρ	$\rho = Q/\mathcal{V}$	Ql^{-3}	[0, 0, −3, 0, 0, 0, 0, 1, 0]
Electric field strength	$\mathbf{E}$	$\mathbf{E} = \mathbf{F}/Q$	$l_rmt^{-2}Q^{-1}$**	[1, 0, 0, 1, −2, 0, 0, −1, 0]
Electric potential (and potential difference)	V	$V = \int \mathbf{E}\cdot d\mathbf{s}$	$l^2mt^{-2}Q^{-1}$	[0, 0, 2, 1, −2, 0, 0, −1, 0]
Electric flux density	$\mathbf{D}$	$Q = \int \mathbf{D}\cdot d\mathbf{A}$	$l_rl^{-3}Q$***	[1, 0, −3, 0, 0, 0, 0, 1, 0]
Electric flux	Ψ	$\Psi = \int \mathbf{D}\cdot d\mathbf{A}$	Q	[0, 0, 0, 0, 0, 0, 0, 1, 0]
Permittivity	ϵ	$\mathbf{D} = \epsilon \mathbf{E}$	$l^{-3}m^{-1}t^2Q^2$	[0, 0, −3, −1, 2, 0, 0, 2, 0]
Capacitance	C	$C = Q/V$	$l^{-2}m^{-1}t^2Q^2$	[0, 0, −2, −1, 2, 0, 0, 2, 0]
Electric Current				
Current	I	$I = dQ/dt$	Qt^{-1}	[0, 0, 0, 0, −1, 0, 0, 1, 0]
Current density	$\mathbf{J}$	$I = \int \mathbf{J}\cdot d\mathbf{A}$	$l_rl^{-3}t^{-1}Q$	[1, 0, −3, 0, −1, 0, 0, 1, 0]
Resistivity	$\mathcal{R}$	$\mathbf{E} = \mathcal{R}\mathbf{J}$	$l^3mt^{-1}Q^{-2}$	[0, 0, 3, 1, −1, 0, 0, −2, 0]
Resistance	R	$R = V/I$	$l^2mt^{-1}Q^{-2}$	[0, 0, 2, 1, −1, 0, 0, −2, 0]
Magnetism				
Magnetic flux density	$\mathbf{B}$	$\mathbf{F}_t = I(\mathbf{s}_t \times \mathbf{B})$	$l_rl^{-1}mt^{-1}Q^{-1}$	[1, 0, −1, 1, −1, 0, 0, −1, 0]
Magnetic flux	Φ	$\Phi = \int \mathbf{B}\cdot d\mathbf{A}$	$l^2mt^{-1}Q^{-1}$	[0, 0, 1, 1, −1, 0, 0, 1, 0]
Magnetic field strength	$\mathbf{H}$	$I = \int \mathbf{H}\cdot d\mathbf{s}$	$l_rl^{-2}t^{-1}Q$	[1, 0, −2, 0, −1, 0, 0, 1, 0]
Permeability	μ	$\mathbf{B} = \mu\mathbf{H}$	lmQ^{-2}	[0, 0, 1, 1, 0, 0, 0, −2, 0]
Mmf.	$\mathfrak{F}$	$\mathfrak{F} = NI$	Qt^{-1}	[0, 0, 0, 0, −1, 0, 0, 1, 0]
Vector potential	$\mathbf{A}$	$\mathbf{B} = \text{curl}\,\mathbf{A}$	$l_t^{-1}l^2mt^{-1}Q^{-1}$	[0, −1, 2, 1, −1, 0, 0, −1, 0]
Inductance	L	$L = N\Phi/I$	l^2mQ^{-2}	[0, 0, 2, 1, 0, 0, 0, −2, 0]
Poynting vector	$\mathbf{D}_r$	$\mathbf{D}_r = \mathbf{E}_t \times \mathbf{H}_t$	$l_rl^{-1}mt^{-3}$	[1, 0, −1, 1, −3, 0, 0, 0, 0]

TABLE XI.—*Continued.*

Concept	Symbol	Defining Equation	Idonic Formula	Base Vector $[l_r, l_t, l, m, t, T, \lambda, Q, F]$
Radiometry				
Radiant pharos (radiant power)	F_r	$F_r = P$	l^2mt^{-3}	[0, 0, 2, 1, −3, 0, 0, 0, 0]
Radiant pharosage	$\mathbf{D}_r$	$F_r = \int \mathbf{D}_r \cdot d\mathbf{A}$	$l_rl^{-1}mt^{-3}$	[1, 0, −1, 1, −3, 0, 0, 0, 0]
Radiant pharosum	$\mathfrak{p}_r$	$\mathfrak{p}_r = F_r/\mathfrak{V}$	$l^{-1}mt^{-3}$	[0, 0, −1, 1, −3, 0, 0, 0, 0]
Radiant phos (energy)	Q_r	$Q_r = U$	l^2mt^{-2}	[0, 0, 2, 1, −2, 0, 0, 0, 0]
Radiant phosage	$\mathbf{W}_r$	$Q_r = \int \mathbf{W}_r \cdot d\mathbf{A}$	$l_rl^{-1}mt^{-2}$	[−1, 0, 1, 1, −2, 0, 0, 0, 0]
Radiant phosum	u_r	$u_r = Q_r/\mathfrak{V}$	$l^{-1}mt^{-2}$	[0, 0, −1, 1, −2, 0, 0, 0, 0]
Radiant helios	H_r	$d\mathbf{D}_r = \frac{1}{\pi} H_r d\Omega$	mt^{-3}	[0, 0, 0, 1, −3, 0, 0, 0, 0]
Radiant heliosent	$\mathbf{G}_r$	$H_r = \int \mathbf{G}_r \cdot d\mathbf{s}_r$	$l_rl^{-2}mt^{-3}$	[1, 0, −2, 1, −3, 0, 0, 0, 0]
Phengosage	$\mathbf{J}(\lambda)$	$\mathbf{D}_r = \int \mathbf{J}(\lambda)d\lambda$	$l_rl^{-1}mt^{-3}\lambda^{-1}$	[1, 0, −1, 1, −3, 0, −1, 0, 0]
Photometry				
Luminous pharos (luminous flux)	F_l		F	[0, 0, 0, 0, 0, 0, 0, 0, 1]
Luminous pharosage	D_l	$F_l = \int \mathbf{D}_l d\mathbf{A}$	$l_rl^{-3}F$	[1, 0, −3, 0, 0, 0, 0, 0, 1]
Luminous pharosum	P_l	$P_l = F_l/\mathfrak{V}$	$l^{-3}F$	[0, 0, −3, 0, 0, 0, 0, 0, 1]
Luminous phos ("light")	Q_l	$Q_l = \int F_l dt$	tF	[0, 0, 0, 0, 1, 0, 0, 0, 1]
Luminous phosage	$\mathbf{W}_l$	$Q_l = \int \mathbf{W}_l \cdot d\mathbf{A}$	$l_rl^{-3}tF$	[1, 0, −3, 0, 1, 0, 0, 0, 1]
Luminous phosum	u_l	$u_l = Q_l/\mathfrak{V}$	$l^{-3}tF$	[0, 0, −3, 0, 1, 0, 0, 0, 1]
Luminous helios	H_l	$d\mathbf{D}_l = \frac{1}{\pi} H_l d\Omega$	$l^{-2}F$	[0, 0, −2, 0, 0, 0, 0, 0, 1]
Luminous heliosent	$\mathbf{G}_l$	$H_l = \int \mathbf{G}_l \cdot d\mathbf{s}_r$	$l_rl^{-4}F$	[1, 0, −4, 0, 0, 0, 0, 0, 1]
Lamprosity ("visibility")	$\bar{y}$	$\bar{y} = D_l/D_r$	$l^{-2}m^{-1}t^{-3}F$	[0, 0, −2, −1, −3, 0, 0, 0, 1]

* Or l_tmt^{-2}, depending on direction of force.
** Or $l_tmt^{-2}Q^{-1}$, depending on direction of **E**.
*** Or $l_tl^{-3}Q$, depending on direction of **E**.

As another example, consider the idonic specification of magnetic flux density. The defining equation is

$$\mathbf{F}_t = \mathbf{I}(\mathbf{s}_t \times \mathbf{B}),$$

where $\mathbf{s}_t$ is in the direction of the current. The only area that is significant when dealing with magnetic flux density is that perpendicular to **B**, so **B** is in the l_r-direction and **F** and **s** are in the l_t directions (Fig. 2). Substituting the idonic specification in the defining equation, one obtains

$$l_tmt^{-2} = (Qt^{-1})(l_t \times B),$$

which is satisfied by

$$B = l_rl^{-1}mt^{-1}Q^{-1}.$$

A word should be said about angle and solid angle. According to Rule 3, the tangential distance is called l_t and the radial distance l_r, so

the idonic specification of angle is $l_t l_r^{-1}$. Closely associated with this representation of angle is our treatment of stress and strain. An adequate treatment would require tensor analysis, but for many engineering purposes it is customary to consider only magnitudes. Normal stress is then specified idonically by

$$\frac{l_r m t^{-2}}{l_r l} = l^{-1} m t^{-2}.$$

Shear stress, on the other hand, is specified by $l_r m t^{-2}/l_r l = l_t l_r^{-1} l^{-1} m t^{-2}$. In this way, different designation symbols are given for the two stresses, just as different designation symbols are obtained for energy and torque.

Table XI presents the designation symbols for classical physics, in so far as physics can be described by ordinary vector analysis. The treatment is limited to euclidean space and isotropic media. A more general and more precise description of physical phenomena could be given in terms of tensors, and the designation symbols could be revised in accordance with this extension. Such a step, however, will not be taken in this paper.

The vector system eliminates ambiguities to essentially the same extent as the scalar $l_r l_t$-system did (Table V). The new system also provides a check on the vector character of a given equation. In many cases (as in the examples of Section 6) the physical problem does not need vectors. If this be true, the idonic treatment, on scalar $l_r l_t$ basis, is generally preferable. In other cases, the vector system is advantageous. Study of Table XI gives the following information:

(*a*) If a physical quantity is a scalar, neither l_r nor l_t enter the designation symbol.

(*b*) A vector quantity contains either l_r or l_t (but not both) in its idonic specification.

(*c*) The appearance of both l_r and l_t in a designation symbol indicates a quantity that is not adequately represented by ordinary vector analysis. Such a concept could be better represented by a covariant vector or by a bivector. These conclusions apply, of course, only to the particular representation given in Table XI, with the defining equations there listed. It should be emphasized again that "dimensions" are not absolute and that a system of designation symbols can be arranged in an infinite variety of ways to meet the requirements of the particular problems being considered.

8. SUMMARY

The paper describes an investigation of some possible "dimensional" systems for the designation of physical concepts. The usual system, based on the primary concepts of *length*, *mass*, and *time*, results in a large number of ambiguities (Table II) and is therefore unsatisfactory.

The introduction of an additional primary concept for heat and another for electromagnetism is an improvement, but many ambiguities still remain (Table III). The trouble is caused by our failure to distinguish between distances in different directions.

By introducing two fundamental lengths l_r and l_t (radial and tangential) in place of the usual l, we eliminate the former difficulty and obtain distinct designation symbols for the concepts of *energy* and *torque*, for *normal stress* and *shear stress*, for *pharosage* (radiant energy per unit area) and *helios* (brightness).

There are six types of practical application for idon theory ("dimensions"), all of which are handled as well (or better) by the $l_r l_t$-system as by the old method. Examples are given of the use of the new method in checking equations and in deriving new equations by idonic analysis. Tables of designation symbols are given for the principal concepts of physics (Table VI to X). Finally, an alternative system is outlined which employes the fundamental lengths l_r, l_t and l, and which discriminates between vectors and scalars in physical space (Table XI). This vector system is useful in certain problems, though in most cases the simpler $l_r l_t$-system is adequate.

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P. MOON AND D. E. SPENCER, "Internationality in the names of scientific concepts," *Am. J. Phys.*: Vol. 14, pp. 285 and 431 (1946); Vol. 15, p. 84 (1947);
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"Utilizing the mks system," *Am. J. Phys.*, Vol. 16, p. 25 (1948).

Automatically Vacuum-Cleaned Workbench (*Bottling*, No. 102).—A neat device has recently been placed on the market, and is of especial interest to bottlers operating their own repair shop, where buffing, grinding and other dust-creating processes have to be encountered. The bench is of re-inforced steel plate but on its working surface there is an 18-in. square rubber-covered grid communicating with a suction pump, dust filter and shaking gear, all housed in the interior of the bench. Hence the worker, especially if working with flexible tools, is completely protected from dangerous dusts, the surrounding atmosphere is kept clear, and if the metal or other dusts have any commercial value they can be recovered.

Fluid-Motor Valve Operator (*Rubber Age*, Vol. 65, No. 6).—A simplified fluid-motor operator for gate valves has been developed by the Crane Co., Chicago. This new design makes possible extensive use of motor-operated valves, since the actuating unit can be easily adapted to standard valves in stock or to valves already installed. This previously has been a virtual impossibility, for motor-operated valves have customarily been made up on special order to the requirements of the installation. The device can be operated by water, oil or compressed air or gases.

The motor unit, comprising the motor, gear box and yoke, is bolted to the valve bonnet; and the motor operates the valve stem through the gear box. The motor derives its power from five flexible diaphragms, mounted radially about an eccentric on the drive shaft. The operating fluid is admitted in rotational sequence to the five diaphragms, which actuate pistons that transmit the thrust to the eccentric through either of two entrance ports, controlling the direction of operation of the motor for opening and closing the valve.

When the valve gate reaches the end of its travel in either direction, the motor merely stalls, still under pressure. There can be no leakage, and shut-off devices are unnecessary. The valves can be operated by a wide range of liquids or gases at pressures from 40 to 300 psi.

Aerial-pickup System.—A "fishing reel" aerial-pickup system perfected by an Air Force officer in Panama makes it possible for a light plane to pick up messages and mail without descending to dangerously low altitudes. At 200 ft. the plane lets out, by gravity fall of a heavy lead sinker, a line to which is attached a 2-in. hook. This engages the mail bag or message container, suspended between two poles on the ground. When the "catch" is made, a certain amount of line runs out to absorb the shock, then a spring-loaded brake stops the drum. Should the hook engage too heavy an object, the line runs out to a strand of smaller diameter which parts without ill effect on the aircraft.

Radar for the Medical Profession.—Detection of foreign bodies such as gallstones within human tissues, by means of a technique employing the radar principle, may be possible in the near future. The method, so far used only on animals, has been developed at the Naval Medical Research Institute in Bethesda, Md. It involves transmitting ultrasonic energy into body tissues and observing on a cathode-ray oscillograph screen the reflected and transformed waves from the foreign body. Distance of the echo from the initial pulse gives the depth of the foreign object in the tissue.

WORKHARDENING OF METALS—A GENERAL THEORY

BY

ALFRED M. FREUDENTHAL [1]

I. THE PLASTIC DEFORMATION OF METALS

When the yield limit of a metal specimen under load is exceeded, plastic flow under constant or fluctuating stress is more or less rapidly blocked by changes within the structure of the material. These changes are accompanied by a gradually increasing resistance to further plastic deformation, increasing indentation hardness and increasing fracture strength as well as by changes in density, electric conductivity, magnetic properties and resistance to wear. Whereas the elastic shear modulus remains practically unchanged, slight changes may occur in the value of the bulk modulus and Poisson's ratio (1).[2] This process of changing the mechanical properties of metals by plastic deformation is usually termed strain-hardening or workhardening; the latter term is the more adequate since the changes may be related to an input of work. A general theory of the phenomenon can be developed on the basis of this relation.

The rate of change of the mechanical and other properties is related to the rate of irrecoverable change in the internal structure of the metal; it depends upon the undeformed structure of the material and is different for single crystals and for polycrystalline aggregates of various initial structure, that is, of structures of various grain size. It also depends upon the temperature and the speed at which the material is deformed.

Thus changes in the physical properties by workhardening are related to some modification that has occurred within the configuration of the crystal grains. Since mechanical properties of polycrystalline metals also depend very strongly upon the nature of the grain boundaries, changes of those properties are necessarily related to changes in the character and extent of these boundaries. The specific character of the grain boundaries is due to the distorted atomic arrangement produced by the competing claims as to the lattice to be formed exerted by the adjacent crystal grains on the small number of atoms located within the boundary region between the grains. The arrangement of those atoms is therefore as nearly "disordered" as is the arrangement of particles in an amorphous material, with the result that grain boundaries, as well as distorted atomic layers in general, behave in a way similar to amorphous materials; they show a highly temperature- and time-sensitive "viscous" response to loads.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

When a metal crystal is deformed, slip takes place along the eligible atomic planes which are most favorably oriented with respect to the direction of principal shear of the operative stress-system. The spacing of the slip planes and the extent of slip on individual planes have been found to depend upon the applied rate of strain or load (2). If the deformation is produced very slowly, the amount of slip on individual planes is extremely small and the number of planes correspondingly large; under such conditions individual slip is so uniformly distributed and so small that it may become invisible under the microscope. Above a certain critical strain- or loading rate, on the other hand, the spacing of slip planes and the extent of slip along individual planes suddenly increase and slip bands formed of clusters of atomic slip planes become visible under the microscope. The critical strain or loading rate at which the finely distributed slip is transformed into one essentially concentrated within clusters of adjacent slip planes dividing the crystal into lamellas of finite thicknesses, depends upon the rate at which applied strain energy is dissipated within the intercrystalline regions. If the rate of application of strain energy is higher than its rate of dissipation, extensive breakdown of the cohesion will occur sooner or later within the intercrystalline boundaries, accompanied by an abrupt change in the distribution of the slip. It is reasonable to assume that the rather uniform restraint imposed upon the deformation of an individual crystal by slip (provided by the unbroken, viscously deforming grain boundaries) will favor a fine distribution of slip planes and small amounts of slip on individual planes; such distribution will enable the discontinuous deformation by slip to occur in the course of a relatively small viscous deformation (creep) of the restraining intercrystalline medium. The propagation of extensive slip on widely spaced clusters of glide planes on the other hand requires excessive and rapid deformation of the restraining medium leading to its local destruction. Thus, finely distributed slip is probably characteristic of slowly deforming, relatively fine-grained polycrystalline aggregates in which the crystals deform within the slowly yielding viscous boundaries; this cannot be expected to occur in single crystals, the deformation of which is unrestrained, and in which, therefore, slip is sudden and extensive; nor can it be expected in rapidly deformed or coarse-grained polycrystals, since the restraint imposed by the grain boundaries on the slip along widely spaced slip bands is rapidly removed by local destruction of the cohesion in the boundary regions. Glide planes are thus free to propagate through a number of neighboring crystal grains.

II. STRUCTURAL THEORIES OF WORKHARDENING

Slip is stopped by the distortion of the atomic structure around the advancing edge of the slip planes. The amount of distortion created during slip determines the rate of blocking along the slip planes. Over-

all plastic deformation resulting from short slip on many finely distributed slip planes will therefore be less rapidly blocked than plastic deformation resulting from extensive slip along widely spaced clusters of atomic slip planes (slip bands), which involves the creation and distortion in the course of the motion of glide lamellas of finite thickness between distorted slip bands.

However, the development of a blocking mechanism by distortion of the atomic arrangement within the slip bands may be responsible for workhardening on a macroscopic scale only within the range of relatively small plastic deformation. The change in resistance to deformation that can be explained by this mechanism alone is not of the same order of magnitude as the changes observed in most polycrystalline aggregates. The blocking mechanism developing on individual slip planes or slip bands may explain the workhardening observed in single crystals; however, such workhardening represents only part of the total workhardening effect within the polycrystalline aggregate. It is therefore not so much the blocking of slip within the slip planes but the blocking of this slip at the grain boundaries which is overcome by fragmentation, rotation and elastic distortion and bending of fragments in the course of the deformation which produces the observed workhardening effects of the polycrystal. The crystal fragments may be of different size and shape; it appears, however, that they cannot be broken down, at least not permanently, into smaller units than the limiting minimum size that is stable at the considered temperature of the deformation and which, according to almost all experiments performed on metals, is of an order of magnitude of 0.1 to $1\mu^3$ (3).

Crystal fragmentation is the principal structural change accompanying plastic deformation within the range of strains up to 15 or 20 per cent. Changes in the physical properties within this range are thus essentially due to the refinement of the initial crystal grains producing crystallites of more or less uniform limiting size, and to changes in the energy content of the intercrystalline boundaries.

The primary effect of grain refinement is to increase the specific distortional energy that can be reversibly stored up within an individual crystal grain, since, according to the v. Mises' yield condition, the yield limit is proportional to the sum of the squares of the principal shear strains. If the simplifying assumption is introduced that slip within an individual small grain of the aggregate is primarily the result of the energy which locally produces pure shear in the most eligible slip system of the considered crystal, the critical energy level at which slip is initiated along a single one of the family of eligible planes is represented by the shear energy which can be released by slip taking place over one atomic distance a which is the unit step or "quantum" of slip. Slip over one half only of that step requires energy application, the second half of the step towards the new equilibrium position occurs spontane-

ously after the activation energy between the two positions has been overcome. The critical energy level is therefore directly proportional to the square of the ratio $(a/2\lambda)$, where λ denotes the dimension of the crystal (Fig. 1).

The change in the energy content of the boundaries of the broken-up crystal structure is connected with a decrease in density of the order of magnitude of 0.1 to 1 per cent, accompanying the workhardening process of most polycrystalline metals; no measurable decrease in density can usually be found in cold-worked single crystals. In the course of the fragmentation of the crystal structure, a certain amount of disordered material, containing bonds of high energy content, is created within the glide-planes. This is the result of the distortion of the atomic structure within those planes by slip, rotation and bending of crystal fragments during deformation, as well as of the restrictions imposed, upon load release, on the recovery of shape of the elastically distorted crystal

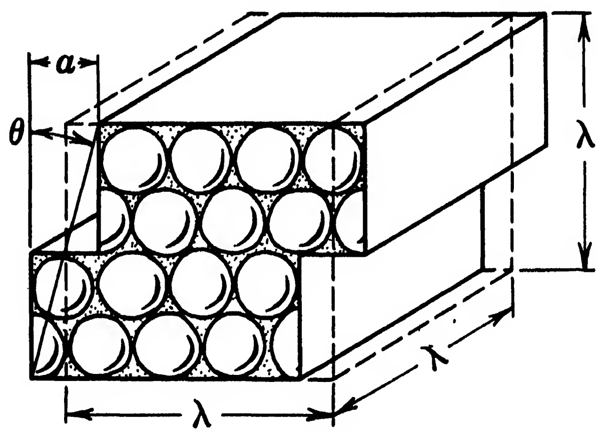

FIG. 1. Slip of a crystallite.

fragments by the preceding irrecoverable deformation of the polycrystalline aggregate.

To stabilize the fragmented and distorted crystalline structure a system of inter-atomic forces of very high potential energy is built up in the course of unloading. Thus a certain percentage of the volume of cold-worked, polycrystalline metals contains particles connected by bonds of very high energy content; the particles within this volume must be markedly further apart than those in the remaining crystalline volume. The potential energy introduced into and latently stored within the polycrystalline structure by the fragmentation of crystals is, therefore, probably contained within the small volume of crystal boundaries and of slip bands of distorted atomic structure.

This conclusion is borne out by the comparison of the changes in the lattice parameter observed in the course of workhardening and the amount of potential energy latently stored up within the fragmented

structure. It has been found that the amounts of strain energy or of volume change necessary to produce an over-all elastic strain consistent with the observed change of the lattice parameter are much smaller than the actually measured latent energy or the observed density change. Therefore, latent energy and local density changes must be concentrated mainly within the intercrystalline boundaries and within the distorted glide planes (4).

When an advanced stage is reached in the process of crystal fragmentation without fracture having occurred, the mechanism of plastic deformation by slip and fragmentation changes and gradually becomes one of formation of texture (orientation) by rotation and by temporary break-up of fragments which are subsequently reformed in the direction of the largest strain velocity. Changes in mechanical and other properties, produced during this stage of the deformation, are an expression of the developing anisotropy of the material; the structural pattern defined by grain size and by the extent of grain boundaries remains practically unaffected, and the hardness increase during this stage is therefore relatively small.

Four different mechanisms may thus be responsible for the increase of resistance to plastic deformation of a polycrystalline metal aggregate stressed above the yield limit. Although it has frequently been attempted to explain the workhardening phenomenon in terms of any one of those mechanisms alone, it is probably the joint or the successive operation of all four mechanisms which is responsible for the observed workhardening effects over the whole range of deformations of metals.

III. A GENERAL LAW OF WORKHARDENING

On the basis of the preceding discussion, workhardening is the result of:

(*a*) The increase in resistance to slip within a single crystal, due to the creation and propagation, under applied shear, of distortions of the atomic lattice, within the slip planes, called "dislocations." (Taylor's Theory.)

(*b*) The increase in resistance to plastic deformation of a polycrystalline aggregate produced by the fragmentation, along slip planes, of crystals and the rotation, elastic distortion and bending of the crystal fragments. The increased resistance is due primarily to the fact that the specific shear energy required to initiate slip in a single crystal is an inverse function of the square of the crystal size. (Bragg's Theory.)

(*c*) The stabilization of the fragmented and distorted crystal structure by a system of micro-residual (textural) stresses set up during unloading after fragmentation. This stress system stabilizes the distorted crystalline structure of the aggregate by introducing into the material a certain amount of "latent" potential energy, which becomes a characteristic feature of the workhardened polycrystalline pattern.

(*d*) Anisotropic change in resistance to plastic deformation produced by practically volume-constant deformation associated with rotation, break-up and immediate reformation of crystal fragments of limiting size in the direction of the maximum strain velocity.

Although these four workhardening mechanisms are assumed to be jointly operative in polycrystalline materials, the degree of their relative importance probably changes with the initial properties of the material, and with the progress of the workhardening process.

The propagation of dislocations [Mechanism (*a*)] is probably responsible for the initial workhardening associated with small plastic deformation, or for the workhardening of polycrystals consisting of a very small number of crystals. Beyond the range of small deformation the workhardening effects resulting from crystal fragmentation and storing up of "latent" energy [Mechanism (*b*) and (*c*)] may be assumed to be mainly responsible for producing the observable relations of stress and strain. Reorientation of fragments which does not produce appreciable hardening is operative in the formation of texture under large uni-directional strains; therefore, in polycrystalline metals the texture-forming mechanism comes into play at a late stage, when crystal fragmentation has been nearly completed, and further deformation can only proceed by directional reorientation of the crystal structure under conditions of considerable thermal instability.

Without attempting to discuss the relative merits of the mechanisms of dislocations and of fragmentation, which are not considered here as alternative but as jointly operative mechanisms, a feature common to both mechanisms will be used to develop a general function of workhardening. This feature is the dependence of the resistance to slip initiation within a crystal or within a polycrystalline aggregate, on an inverse function of a geometrical parameter λ, which is either the range over which a dislocation can migrate (its so-called "mean free path") or the range over which slip can take place in a crystal. The parameter λ can therefore be considered as a measure of the order of magnitude of the average linear dimension of the perfect crystals, or of the crystallites, forming the aggregate.

Considering an elementary crystal cube of dimension λ built up of atoms (Fig. 1) under conditions of pure shear, slip starts by the gliding of one atomic plane over another, with accompanying release of potential energy, when the elastic shear strain g has attained the value $a/2\lambda$; this is equivalent to an elastic displacement of the upper plane against the base plane of one half the atomic distance a. Hence, the elastic energy per unit volume which can be reversibly stored up before slip starts, the "resilience" $\frac{1}{2}Gg^2 = \text{const.}\ (1/\lambda)^2$, is inversely proportional to the square of the parameter λ. It is this limit of elastic energy that provides a measure of the "hardness" H by delimiting the energy level at which plastic deformation starts.

On the other hand, the theory of dislocations as developed by G. I. Taylor (5) leads to the "parabolic" relation of stress and strain.

$$s = \text{const.} \sqrt{g/\lambda} \tag{1}$$

according to which the "critical stress" at constant strain g at which slip proceeds, is inversely proportional to $\sqrt{\lambda}$. In terms of energy the "resilience" H, which is proportional to the square of stress, would be inversely proportional to λ.

If it is assumed that the increase in hardness, defined as increase in the resistance to plastic deformation in terms of energy, is not or not only, due to the "resilience" of individual crystals increasing with decreasing parameter λ, but to the storing-up of "latent" energy within the intercrystalline boundaries and distorted atomic layers of the fragmented polycrystal, the hardness increase dH would be proportional to the increase, in the course of fragmentation, of intercrystalline surfaces within the unit volume of the aggregate. If m crystals of parameter λ_m, making up the unit volume of hardness H_m are broken up into a unit volume of hardness H_n consisting of n crystallites of parameter λ_n, and if the ratio of hardness before and after fragmentation H_m/H_n is assumed to be proportional to the ratio of intercrystalline surfaces A_m and A_n per unit volume before and after fragmentation respectively, this hardness ratio may be expressed by

$$H_n/H_m = \frac{n \cdot \lambda_n^2}{m \cdot \lambda_m^2} = \frac{\lambda_m}{\lambda_n}, \tag{2}$$

since $m \cdot \lambda_m^3 = n \cdot \lambda_n^3 =$ unit volume. The hardness H in terms of energy is therefore inversely proportional to the parameter λ. This is the same relation that has been obtained from the dislocation theory.

Thus, according to any one of the three workhardening mechanisms, the hardness defined in terms of the strain energy required to initiate plastic deformation is inversely proportional to either the square of the parameter λ or to λ itself or to some intermediate power λ^n, where $1 < n < 2$. It is on the basis of this conclusion that a general law of workhardening can be developed. This law describes the change of state in terms of the process of energy transformation accompanying the changes in the structural pattern of the metal. It can therefore be expressed in terms of the basic thermodynamic equations

$$\frac{dW_s}{dt} = \frac{d\Phi_0}{dt} + \frac{dW_D}{dt}, \tag{3}$$

and

$$\frac{dW_D}{dt} = \frac{1}{T}\frac{dS}{dt} \geqq 0, \tag{4}$$

where W_s, Φ_0, W_D, S, and T denote the applied strain energy, the reversibly stored distortional energy, the dissipated energy, the entropy

and the absolute temperature, respectively. The > and = signs in Eq. 4 define irreversible and reversible processes, respectively. For every structural pattern defined by the energy W_D irrecoverably expended in producing it, a limiting amount of distortional energy $\Phi_0 = H$ exists up to which $dW_D/dt = 0$. The function $\Phi_0 = H = f(W_D)$ represents the basic workhardening relation.

Since energy is an additive quantity, the hardness of the polycrystalline aggregate can be assumed to be roughly the sum of the individual resilience values of the constituent crystals; hence

$$\tfrac{1}{2}\sum^{n} G\cdot g_m^2\cdot V_m = \Phi_0(g)\cdot V = \tfrac{1}{2}G\cdot g^2\cdot V, \tag{5}$$

where g denotes the limiting shear-strain and V the volume of the aggregate, while V_m and g_m respectively denote the volume and limiting shear-strain of an individual crystal of size m out of the n sizes of crystals forming the aggregate.

Since the hardness H of the aggregate is thus made up of the sum of the hardnesses H_m of the constituent crystal sizes, the contribution of the crystal size m to the hardness of the aggregate will necessarily be $H_m(V_m/V]$. If the structural pattern producing the hardness H is defined by a certain volume distribution of crystal parameters λ running from λ_l to λ_n, where λ_l denotes the largest size, λ_n the smallest (limiting) size, this pattern is related to the energy W_D irrecoverably expended in producing it from the initial structural pattern of the aggregate, defined by the initial hardness H_0.

The validity of this relation is however limited to conditions for which crystal fragmentation and distortion are the principal operating dissipation mechanisms; this is the case at temperatures at which the thermal stability of the distorted crystal pattern is high and effects of anisotropic reorientation are slight. At temperatures below recrystallization temperature, the energy dissipated by alternative mechanisms will, in general, be negligible. At temperatures near recrystallization temperatures, however, the amount of energy dissipated directly into heat, without producing an identifiable permanent change of structural pattern, that would result in increased hardness, becomes so large that the one-valued relation between hardness H and dissipated energy W_D no longer exists. Similarly, energy is dissipated without producing appreciable change in hardenss, if during deformation directional reorientation is the predominant effect. However, conditions of marked thermal instability of the fragmented structure at room temperature can be expected but at the approach to the upper limit of hardness. Since for most of the structural metals at room temperatures (or even at moderately elevated temperatures) fracture will intervene before the upper limit of workhardening is attained, the one-valued relation between hardness H and dissipated strain energy W_D may be assumed to

hold with fair approximation within the practically important range of deformation.

An increase in hardness dH is brought about by expending the energy dW_D to change to volume distribution of crystal sizes through the fragmentation of a part v_m of the total volume V_m of crystals of size λ_m and hardness H_m, into an equal volume of crystals of size λ_n and hardness H_n. Therefore the rate of workhardening of the aggregate

$$\frac{dH}{dW_D} = \sum_1^m (H_n - H_m) \frac{dv_m}{dW_D} \tag{6}$$

if the considered volume of the aggregate V is unity; the sum is to be taken over all crystal sizes affected, that is over all sizes $m > n$.

Fragmentation of crystals may be either gradual, through consecutive stages, when every crystal size is broken up into a smaller, but not necessarily limiting size, or "catastrophic," when every crystal size is shattered directly into fragments of limiting size, or it may be of an intermediate nature. Since it has been found that the first assumption leads to workhardening functions inconsistent with observation (6), the catastrophic type of fragmentation is assumed to be prevalent.

If q_m denotes the ratio of the volume of one crystallite of parameter λ_m to the volume of the crystal fragment of limiting parameter λ_n, the ratio of hardness of the two crystal sizes

$$\frac{H_n}{H_m} = \left(\frac{\lambda_m}{\lambda_n}\right)^2 = q_{m,n}^{\frac{2}{3}} \tag{7}$$

if the hardness is defined by the "resilience" of crystal grains alone. If, on the other hand, it is assumed that the increase of hardness is not due to the increasing resilience of the refined crystal grains, but results from the "latent" energy stored up in the intercrystalline surfaces, the hardness ratio according to Eq. 2 is

$$\frac{H_n}{H_m} = \frac{\lambda_m}{\lambda_n} = q_{m,n}^{\frac{1}{3}}. \tag{8}$$

Adopting Eq. 7 and introducing it into Eq. 6 the rate of workhardening

$$\frac{dH}{dW_D} = \sum_1^m H_m \cdot (q_{m,n}^{\frac{2}{3}} - 1) \frac{dv_m}{dW_D}. \tag{9}$$

By integration,

$$H = \sum_1^m H_m \cdot (q_{m,n}^{\frac{2}{3}} - 1) \cdot v_m + \text{const.} \tag{10}$$

If v_m denotes the relative volume of grains of parameter λ_m fragmented to date out of the volume V_{0m} of such crystals initially existing in the unit volume V, the rate of fragmentation of grains of any particular

parameter λ_m may reasonably be assumed proportional to the relative volume of such grains $V_m = (V_{m0} - v_m)$, existing at the considered stage of fragmentation defined by W_D. This assumption is generally used in all physical disintegration (decay) processes. Hence

$$\frac{dv_m}{dW_D} = (V_{m0} - v_m)\frac{1}{\alpha_m}, \tag{11}$$

where α_m is a factor of proportionality characteristic for the stability of the grain size; its inverse value is proportional to the rate of disintegration. With the condition that $v_m = 0$ for $W_D = W_{D0m}$, integration of Eq. 11 gives

$$v_m = V_{m0}[1 - e^{-(W_D - W_{D0m})/\alpha_m}], \tag{12}$$

valid for $W_D > W_{D0m}$.

The workhardening curve corresponding to Eq. 10 thus has different ranges governed by different equations, and delimited by the values of W_{D0m}. In the first range, from $W_{D01} = 0$ to W_{D02} the largest size of crystals of parameter λ_1 and hardness H_1 is fragmented into crystallites of limiting size, until the over-all hardness H of the polycrystalline aggregate (which, in the initial stage, that is, for $W_D = 0$ is $H = H_0 = H_1$) reaches the hardness H_2 of the grains of parameter λ_2. From there on the fragmentation of the crystal size of parameter λ_2 sets in, proceeding simultaneously with the continued breaking up of still existing grains of parameter λ_1 and so on. Thus for the first stage $(0 < W_D < W_{D02})$, with $H = H_1$ for $W_D = 0$:

$$H = H_1[1 + (q_{1,n}^{\frac{1}{2}} - 1)\cdot V_{10}(1 - e^{-W_D/\alpha_1})]. \tag{13}$$

For the second stage $(W_{D02} < W_D < W_{D03})$ with $H = H_2$ for $W_D = W_{D02}$:

$$H = H_1\{q_{1,2}^{\frac{1}{2}}[1 + (q_{2,n}^{\frac{1}{2}} - 1)\cdot V_{20}(1 - e^{-(W_D - W_{D02})/\alpha_2}] + (q_{1,n}^{\frac{1}{2}} - 1)\cdot V_{10}(e^{-W_{D02}/\alpha_1} - e^{-W_D/\alpha_1})\}, \tag{14}$$

where $q_{1,2}^{\frac{1}{2}} = H_2/H_1$.

Generally, the hardness $H(W_{Dm})$ within the m-th stage of fragmentation

$$H(W_{Dm}) = H_1\cdot\{q_{1,m}^{\frac{1}{2}} + \sum_{k=1}^{k=m}[q_{1,k}^{\frac{1}{2}}(q_{k,n}^{\frac{1}{2}} - 1)V_{k0} \cdot(e^{-(W_{D0m} - W_{D0k})/\alpha_k} - e^{-(W_{Dm} - W_{D0k})/\alpha_k})]\}, \tag{15}$$

where $q_{1,m}^{\frac{1}{2}} = H_m/H_1$ and $q_{k,n}^{\frac{1}{2}} = H_n/H_k$, H_n being the hardness of crystallites of limiting size λ_n. The rate of workhardening within this range

$$\frac{dH(W_{Dm})}{dW_{Dm}} = H_1\sum_{k=1}^{k=m}\left[q_{1,k}^{\frac{1}{2}}\cdot(q_{k,n}^{\frac{1}{2}} - 1)\cdot\frac{V_{k0}}{\alpha_k}\cdot e^{-(W_D - W_{D0k})/\alpha_k}\right]. \tag{16}$$

If the rate of workhardening is plotted against hardness, a straight line

$$\frac{dH}{dW_D} = C_1 + C_2H \tag{17}$$

is obtained if one term only ($k = 1$) exists in Eq. 15 or, alternately, if α_k is the same for all crystal sizes. The shape of the function $H = f(dH/dW_D)$ thus provides an indication of the character of the process of fragmentation.

It is evident that the only change introduced into Eqs. 13 to 17 by adopting a different assumption concerning the operative workhardening mechanism would be to change the powers of all $q_{k.m}$- and $q_{k.n}$-ratios in those equations from $\frac{2}{3}$ to $\frac{1}{3}$, as required by Eq. 8 which would replace Eq. 7, on the basis of which the workhardening law (15) has been derived. This change is a change in constants only and does not affect the form of the general workhardening function.

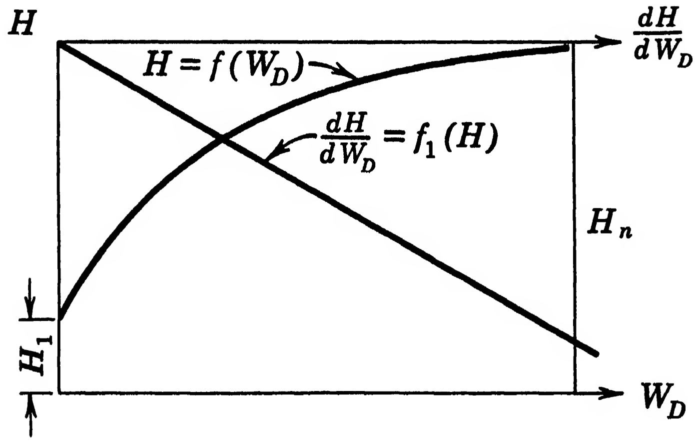

FIG. 2. Workhardening function for fragmentation of single crystal size.

IV. INTERPRETATION OF EXPERIMENTS

In applying Eq. 15 to the interpretation of experimental results under general conditions of stress, the proper measure of H and of W_D must be introduced. The elastic distortional energy is proportional to the second invariant of the deviator of stress I_{02s} or of elastic strain I_{02e}. There is, however, no simple measure of W_D in the case of a general state of stress. In the case of homogeneous stress, W_D can be directly computed from the area under the stress-strain diagram. In first approximation, however, W_D for a general state of stress may be assumed to be proportional to the second invariant of the deviator of plastic strain. Thus, the general workhardening function $\Phi_0 = H = f(W_D)$ for a general state of stress has the invariant form:

$$I_{02e} = F(I_{02p}), \tag{18}$$

where I_{02e} and I_{02p} denote the second invariants of the deviator of elastic and of plastic strain respectively.

In the limiting case of an aggregate with a uniform grain size Eq. 13 (with $V_{10} = 1$) governs the entire range of deformation. Hence

$$H = H_n - (H_n - H_1) \cdot e^{-W_D/\alpha}, \tag{19}$$

and

$$\frac{dH}{dW_D} = \frac{1}{\alpha}(H_n - H). \tag{20}$$

Relations (19) and (20) are represented in Fig. 2. The slope $1/\alpha$ of the relation (20) is constant over the entire deformation range. This constancy is an indication that only one crystal size is being broken up.

Equation 19 can be written in terms of stresses if W_D is considered a function of the plastic strain $\ln p$ alone

$$\left(\frac{s}{s_\infty}\right)^2 = 1 - \left[1 - \left(\frac{s_1}{s_\infty}\right)^2\right] \cdot e^{-(\ln p)^2/\alpha}, \tag{21}$$

where s_∞ and s_1, respectively, denote the stresses for $p = \infty$ and $p = 0$.

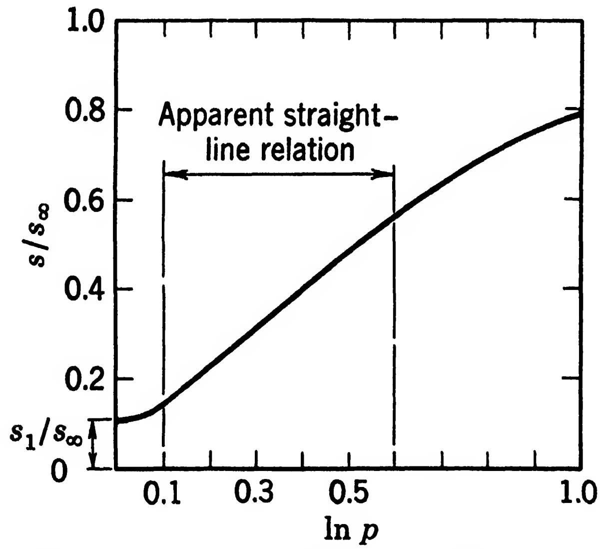

FIG. 3. Stress-strain diagram for workhardening process involving break-up of one crystal size (yield stress $S_1 = 0.1S$).

Hence

$$s = s_\infty \sqrt{1 - \left[1 - \left(\frac{s_1}{s_\infty}\right)^2\right] \cdot e^{-(\ln p)^2/\alpha}}. \tag{22}$$

The form of this relation for assumed values $\alpha = 1$ and $\frac{s_1}{s_\infty} = 0.1$ is represented in Fig. 3. The diagram shows that under the assumed conditions and within the range of the logarithmic plastic strains $0.1 < \ln p < 0.7$ the stress-strain diagram is very nearly a straight line;

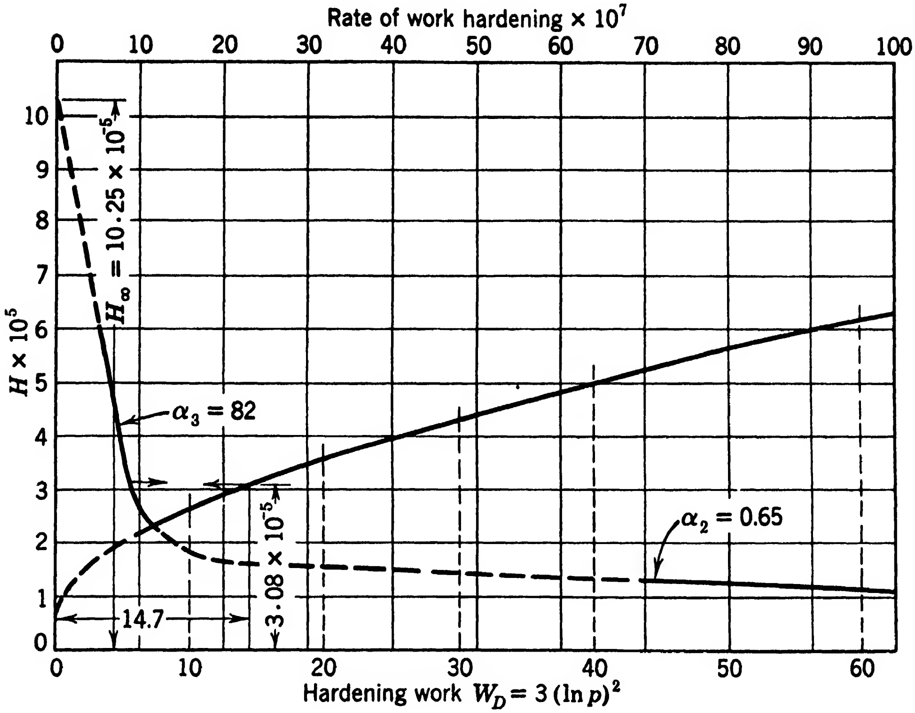

FIG. 4. Workhardening curve of mild steel (8).

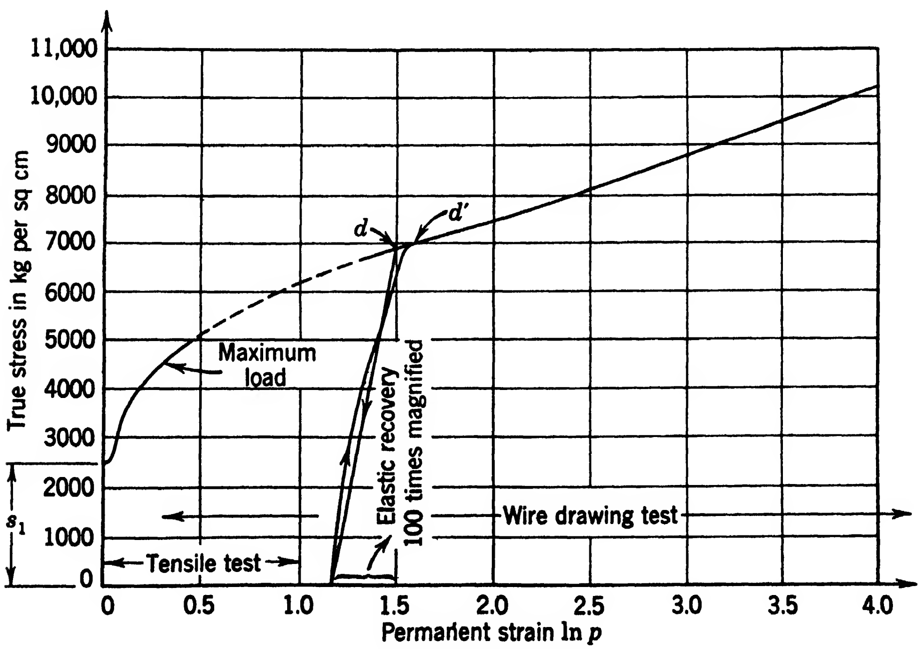

FIG. 5. Workhardening curve for mild steel established from tension test and successive drawing through dies (8).

however, it starts to curve downwards at higher values of strain. The straight-line true stress-strain relation which has been observed in metals within a certain range of strain is, therefore, a transient phenomenon and has no physical meaning. Since in tension tests fracture starts mostly before the curved part of the stress-strain diagram is reached, the apparent straight-line relation has frequently been considered a characteristic feature of deformational behavior; there is, however, no justification for this assumption.

Figure 4 reproduces the function $H = f(W_D)$ and $dH/dW_D = f(H)$ obtained in terms of (e^2) and $(\ln p)^2$ by a combined tension test and wire-drawing experiment on mild steel (Fig. 5) in which very high strain values were reached (6). From the interpretation of the $dH/dW_D = f(H)$ diagram it is evident that the workhardening process consists of three distinct, successive stages, in the course of each of which fragmentation of but one order of magnitude of crystal sizes takes place; only during the comparatively short, curved transition stages does the fragmentation process involve two orders of magnitude of crystals. It is important to note that a detailed analysis of the experimental curve has shown (6) that the true stress $s = \dfrac{P_{max}}{A}$, at which necking starts in the tension test, is attained at the stage of the deformation when the break-up of the large crystal size is practically completed. Since by computing the maximum load in the tension test P_{max} from the condition

$$dP = d(s \cdot A) = A ds + s dA = 0 \qquad \text{and} \qquad \frac{ds}{s} = -\frac{dA}{A} \tag{23}$$

it can be easily verified that necking starts under maximum load when the rate of hardening ds/s is no longer able to compensate for the decrease in cross-section $-dA/A$, this coincidence is to be expected. The discontinuous decrease in the rate dH/H (and therefore of ds/s) after the largest crystal size has been broken up, must necessarily produce instability of deformation, that is, necking under decreasing load. Thus the maximum load P_{max} observed in conventional tension tests and used as "ultimate tensile strength" $s_{max} = P_{max}/A_0$, where A_0 denotes the initial cross-section of the specimen, defines neither a strength nor an "ultimate" limit. It simply designates a transition point of the workhardening rate.

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NOTES FROM THE NATIONAL BUREAU OF STANDARDS *

ELECTRONIC INSTRUMENT TESTS GEIGER COUNTERS

An electronic gating instrument has been designed and constructed at the National Bureau of Standards for the accurate determination of the deadtime and recovery characteristics of Geiger-Müller counters. The deadtime and recovery characteristics are important not only because the errors of counting depend on them, but also because they are intimately associated with the electrical discharge process.

When incoming radiation causes an ionizing event in a Geiger counter, negative electrons are attracted to the positive wire, the electron stream producing a negative voltage pulse across the terminals of of the counter. The highly mobile electrons reach the positive wire of a Geiger counter within a fraction of a microsecond after ionization has occurred. At the same time, the heavy positive ions moving toward the negative outer cylinder remain in the vicinity of the wire long after the electrons have reached it. If a particle enters the counter while the ions are still close to the collecting wire, Geiger action cannot occur.

The deadtime is defined as the time interval after a pulse has occurred during which the counter is insensitive to further ionizing events. Physically it is the time required for the positive ions to travel far enough from the wire to let Geiger action resume. Similarly, the recovery time is defined as the time interval that must elapse after a pulse has occurred before a full-sized pulse can again occur; physically it is the time required for the positive ions to reach the counter wall. Both the deadtime and the recovery characteristics of a Geiger counter, as well as the input sensitivity of the associated circuits, are involved in the "resolving time," defined as the minimum time interval by which two pulses must be separated to be detected as separate pulses by the counter and its accessories. Since the counter is dead for a short period after each pulse, some disintegrations are not detected, resulting in a discrepancy between the number of particles entering the counter and the number of counts recorded.

With the Bureau's electronic gating instrument it is possible to study the pulses that occur in the time interval between the end of the deadtime of a Geiger counter and the end of a predetermined gating period. When the gating instrument is in operation, random pulses from a Geiger counter are fed through the pulse height selector and pulse shaping circuits to give sharp negative pulses of uniform height.

* Communicated by the Director.

These negative pulses follow two paths: one through an amplifier to a heavily biased stage and the other through a delay circuit to a gating multivibrator. An initiating pulse entering the system finds Channel 1 blocked by the heavy bias, but by way of Channel 2 it trips the multivibrator and opens Channel 1 for pulses to pass. Because of the delay in Channel 2, Channel 1 is not opened until the initiating pulse has died away. The delay used is 15 microseconds. Subsequent pulses find Channel 1 open, go through it, and are counted. After a time determined by the circuit constants of the multivibrator, Channel 1 is blocked off again and the whole process is repeated. In effect, the system selects pairs of pulses with a time separation less than a selected value determined by the setting of the controls on the multivibrator, and one count is recorded for each such pair received.

EFFECT OF BORON ON THE STRUCTURE AND PROPERTIES OF CAST IRON

War-induced scarcities of elements such as chromium and manganese called attention to the possible use of boron to increase the hardness and wear resistance of cast iron. However, specific data regarding the effect of boron on the structure and properties of cast irons have been somewhat limited. At the National Bureau of Standards' experimental foundry, an extensive study was recently made of the effect of boron in plain gray cast irons. Useful information on solidification characteristics, strength, hardness, microstructure, and chilling properties was obtained. These data should be of value to those industries which produce cast iron for specialized uses requiring definite characteristics.

Cast iron consists essentially of graphite particles dispersed throughout a steel matrix. Ordinarily the fractured surface of the metal is gray, but when cast iron is cooled rapidly from the molten state, it chills, forming white iron. The core of such castings consists of soft gray iron while the outside portion is the more durable white iron. Because of its hardness and resistance to wear, white cast iron is used in such articles as street-car wheels and rolls for rolling mills. The brittleness of the iron chip and the lubricating properties of the graphite make gray iron easy to machine. The graphite particles also act as cushions against thermal shock and thus prevent crazing. For this reason, gray iron is the only material that has been successfully used on a large scale for brake shoes or brake drum linings, which must absorb tremendous energy, convert it to heat, and still not seize the wheel or axle.

In the Bureau's investigation, plain gray cast irons of nine different stocks were studied. The irons were melted in high-frequency induction furnaces, and boron in the form of ferro-boron was added to the molten metal. When examined under a microscope, the structure of irons with a relatively large boron content was found to consist essen-

tially of a matrix of cementite containing islands of pearlite as well as other structureless islands.

Transverse strength was determined by subjecting bars to a breaking test. A disk was cut from each broken transverse test bar and used for Brinell hardness determinations and for metallographic examination. Fractures could be classified as gray or white-network. For bars with gray fractures, an increase in boron content was accompanied by a small increase in the transverse strength, Brinell hardness, and relative modulus of elasticity of the sample. However, the appearance of a white network or white specks on the fractured surfaces of bars containing larger amounts of boron usually was accompanied by a drop in the transverse strength and deflection, and an increase in the Brinell hardness and relative modulus of elasticity.

An important property of cast iron is its ability to form white iron, when chilled. Depth of chill may be controlled by varying the cooling rate or by adjusting the composition of the iron. In the present study the whiteness of the fractured surface of plate and wedge specimens was taken as a criterion of the iron's susceptibility to chill. For all castings there was a pronounced increase in depth of chill with an increase in the boron content. This was found to be accompanied by a greater development of the white network in the gray portions of the castings.

In general, the results of the investigation show that boron increases the depth of chill of cast iron as well as its hardness. Boron was also found to decrease the size and quantity of graphite particles, to increase the free carbide content, and to produce an undercooling effect which becomes more pronounced with greater percentages of boron.

NBS LIQUID OXYGEN CONVERTER

A fully automatic liquid oxygen converter having significant advantages for use in high altitude flight has been developed at the National Bureau of Standards. Its features include compactness, greatly reduced weight, sturdiness, simplicity of design and operation, rapid attainment of operating pressure, economy of oxygen, and delivery of warm gas.

The NBS Liquid Oxygen Converter consists of a standard 25 liter metal Dewar flask modified by the addition of a bottom drain, together with two coils, one for build-up of pressure and one for warming the gas as it is delivered. Four valves, two on the flask, one in the build-up tube, and the other in the delivery line, prevent excessive rise in pressure.

When oxygen gas pressure is to be built up, liquid is directed through the bottom drain and a liquid trap into the build-up coil, in which it is evaporated and warmed by the atmosphere. The warm gas rises through the coil by thermal convection and is then returned to the top

of the flask, where it mixes with the gas already present above the liquid. Gas pressure rises quickly as this cycle continues. After pressure has been established, oxygen may be withdrawn by forcing liquid through the bottom drain into the withdrawal coil, where it is evaporated and warmed by the atmosphere as it passes to the outlet mechanism.

The total converter unit is 25 in. high and 18 in. in diameter. It weighs 60 lb. and holds 62 lb. of oxygen, enough for ten men for 10 hr. To supply this quantity of oxygen from gas cylinders at 1800 psi. would require 21 tanks of 514 cu. in. each, which weigh a total of 350 lb. when empty. For flows up to 150 liters per minute (5 cu. ft. per minute), gas is delivered within 5° C. of atmospheric temperatures. Greater flow rates may be obtained, but the delivered gas is colder. Pressure may be built up, when the container is full, from 0 to 65 psi. in 10 sec.

The converter has successfully withstood tests under conditions of severe vibration (3000 cycles per minute), rapid acceleration (nine times the acceleration of gravity), extreme cold (−40° C.), and severe heat (75° C.). Performance at tilts up to 45 deg. was not materially different from that observed in the erect position. Converters built on the NBS design by commercial manufacturers have operated satisfactorily in flight tests at 40,000 ft.

In reliability and safety the new converter compares favorably with high-pressure gaseous oxygen systems. Wide variation is possible in container capacity, and in delivery flow, temperature, and pressure specifications. Thus, although designed specifically for aviation use, the device may be adapted with minor modifications to industrial and medical applications.

ELECTRON-OPTICAL SHADOW METHOD

As the result of a series of electron-microscope experiments in the Electron Physics Laboratory of the National Bureau of Standards, an electron-optical shadow technique has been developed which provides a valuable tool for the quantitative study of electrostatic and magnetic fields of extremely small dimensions. The new development should also provide a powerful means for broadening present knowledge concerning space-charge fields, waveguide problems, and the basic magnetic properties of metals. Though similar in some respects to the electron-optical Schlieren method previously developed at the Bureau, the shadow method is much better adapted to precise determinations of field intensity.

In practice, the object to be studied, a recording wire for example, is placed between an electron source and a system of electron lenses. The lens system focuses the electron beam to form an image of the wire on a fluorescent screen. By placing a wire mesh of known gage just

beyond the back focus of the lens system, a shadow image of the mesh is superimposed on the image of the wire. The portions of the shadow network adjacent on the screen to magnetized regions of the recording wire are then found to show considerable distortion.

A complete theoretical analysis of this effect at the National Bureau of Standards has shown that the distortion of the shadow image is due to the deflection of the electron beam by the field of the recording wire at each magnetized region. The displacement and change in size of the shadow image depend on the strength of the field of the magnetized wire. Although it is possible to compute field intensity from the intensity distribution of the pattern obtained by the Schlieren method, the shadow method is of far greater utility for quantitative work since the image displacement and magnification can be measured much more accurately than can the intensity distribution across the Schlieren pattern on a photographic plate.

Perhaps the greatest value of the electron-optical shadow method lies in its utility for exploring complex electric and magnetic fields of extremely small dimensions or in which a probe of greater size than the electron would disturb the field under study. The shadow technique now provides data for accurate calculation of the absolute value of the intensity in the neighborhood of a specimen of any size or shape without altering or disturbing the field. The method is thus well adapted to investigation of the fundamental nature of ferromagnetism.

The Bureau is also applying the principle of the shadow technique to the study of spherical aberration in electron lenses. When a fine wire mesh is placed in the focal region of a lens having spherical aberration, the shadow image of the network is enlarged. The resultant pattern may be interpreted to give information of value in correcting the lens.

NBS METHOD FOR MICROSECTIONING

An improved and relatively rapid method for cutting extremely thin sections of biological tissue for study with either the light or electron microscope has been developed by the National Bureau of Standards. The application of the electron microscope to many biological problems has been seriously hampered by the lack of such a method, because of the very slight penetrating power of the electron beam in commercial instruments and the relatively great depth of field involved. Yet the techniques that have been available for preparing thin sections are quite elaborate and difficult, requiring expensive equipment and producing few usable sections.

An important feature of the Bureau's method is an easily made, inexpensive device which holds the embedded specimen and at the same time advances it gradually and uniformly toward the knife of a micro-

tome. The specimen is embedded in polybutyl methacrylate to produce an optically clear medium having highly desirable cutting properties. The material is fed to the microtome in extremely small increments by means of the thermal expansion of a metal specimen holder. The specimen holder is cooled by carbon dioxide gas and thus contracts. As the holder begins to warm and expand, it moves toward the cutting edge of the microtome. The length of time between cuts and the control of the carbon dioxide determine the thickness of the microsection. Thus, sections of tissue having undistorted structure and uniform thickness of fractions of a micron may be cut, one at a time.

Although the Bureau's new method for obtaining very thin sections has given excellent results, it possesses certain limitations. The greatest chance for failure appears to lie in the polymerization of the embedding mass. However, use of low-temperature catalysts will minimize difficulties at this stage. Efforts are now being made at the Bureau to apply the new sectioning method to the study of the structure of natural and artificial fibers found in leather, paper and textiles.

EFFECT OF SIMULATED SERVICE CONDITIONS ON PLASTICS

The use of laminated plastics in aircraft has increased during the past few years, especially in accessories and semi-structural parts, such as partitions, linings, propellers, and wing flaps. There has accordingly developed a need for more information regarding the action of the weather, temperature, and humidity in order to evaluate these plastics for aircraft applications and to prepare specifications.

To provide the necessary data, the National Bureau of Standards under the sponsorship of the National Advisory Committee for Aeronautics undertook an investigation to determine the effects of simulated service conditions on the weight, dimensions, and flexural properties of phenolic and unsaturated-polyester plastics. The test materials were commercial products and included nine laminated plastics which are the types commonly employed in aircraft.

For accelerated weather testing the specimens were subjected to cycles of ultraviolet light and fog while the accelerated service tests consisted of exposure to various temperatures and relative humidities. Three sets of specimens were exposed for one year to outdoor weathering. Changes in weight, dimensions, and flexural properties were the criteria used in analyzing the data gained. In most of the tests the changes in weight and dimensions were negative, any positive changes being in thickness. In several cases increases in flexural strength were noted and attributed to further cure of the resins.

THE FRANKLIN INSTITUTE

MEDAL DAY PROCEEDINGS

The annual reception, dinner, and presentation of awards, known as "Medal Day," took place at The Franklin Institute on Wednesday, October 19, 1949. The reception began at 6:30 P.M. and dinner was served at 7:20 P.M. Mr. Richard T. Nalle, President of the Institute, presided at the dinner and presented the subsequent awards.

Reception

The reception preceding the dinner was given by the Hostess Committee of The Franklin Institute, and was attended by 294 persons. Immediately following the reception and just before dinner was served in Franklin Hall, Mr. Nalle requested the guests to face the flag above the Franklin statue while Mr. Guy Marriner, director of music at The Institute, played the National Anthem.

Remarks of the President

MR. NALLE: "Tonight we honor a group of individuals who, by dedicating their talents and energies to the investigation of scientific problems, have furthered the value of science in our daily lives.

"The curiosity which prompts such investigation, and the practical channelling of that curiosity, is assuredly in line with Benjamin Franklin's character. That early American gentleman had the most profound respect for every phase of science, but most especially for its practical application to living. He would certainly approve of the work of the scientists gathered here.

"Therefore, may I propose, ladies and gentlemen, a toast on this Medal Day to the living memory of Benjamin Franklin."

* * *

MR. NALLE (*continued*): "On behalf of the Board of Managers and Officers of The Franklin Institute, I welcome all of you here tonight where we have gathered to honor thirteen men for their achievements in the world of Science and the Arts.

"Our special thanks are due the Hostess Committee. They have been responsible for the flowers and have given generous assistance to the reception."

Stated Monthly Meeting

MR. NALLE (*continued*): "Tonight is also the regular stated October meeting of the Institute. The minutes of the last stated meeting in May have been published in the June issue of the JOURNAL OF THE

FRANKLIN INSTITUTE and, if there are no corrections or additions proposed at this time, they are declared approved as printed.

"It is customary to elect recipients of the Franklin Medal to Honorary Membership in The Franklin Institute. Therefore we would like to hear a motion according this privilege to Dr. The Svedberg, of Upsala, Sweden, who is the 1949 Franklin Medalist."

A motion was made by Dr. Allen and duly seconded.

MR. NALLE (*continued*): "It has been moved and seconded that Dr. Svedberg be elected an honorary member of The Franklin Institute. Will all members of The Franklin Institute present, who are in favor signify by saying 'Aye'? Contrary minded—.

"The motion is carried and I declare Dr. Svedberg elected to Honorary Membership."

* * *

MR. NALLE (*continued*): "Among us tonight are twenty-three former recipients of Medals given by The Franklin Institute. As I mention the names, cities and Awards, will each of you please rise and remain standing until the full roll of former Medalists is completed? To save time, I suggest that our applause be withheld until the names of all these gentlemen have been called.

"These Medalists are:

FORMER MEDALISTS ATTENDING MEDAL DAY—1949.

Year	Medalist	City	Medal
1921	Adams, Dr. L. H.	Washington, D. C.	Longstreth
1905	Alteneder, Mr. Theodore G.	Philadelphia,. Pa.	Longstreth
1948	Arone, Mr. Nicholas F.	Philadelphia, Pa.	Longstreth
1930	Bailey, Mr. Ervin G.	New York, N. Y.	Longstreth
1947	Berman, Mr. Samuel	New York, N. Y.	Longstreth
1947	Billner, Mr. Karl P.	Philadelphia, Pa.	Brown
1904	Clamer, Dr. G. H.	Philadelphia, Pa.	Cresson
1947	Davidson, Mr. Kenneth S. M.	Hoboken, N. J.	Wetherill
1946	Dodge, Mr. Ronald D.	Poughkeepsie, N. Y.	Certificate of Merit
1937	Eksergian, Dr. Rupen	Philadelphia, Pa.	Henderson
1945	Eksergian, Dr. Rupen	Philadelphia, Pa.	Levy
1936	Hall, Mr. Peter P. G.	Philadelphia, Pa.	Longstreth
1943	Heyl, Mr. Paul Renno	Washington, D. C.	Potts
1947	Kent, Mr. Robert H.	Aberdeen, Maryland	Potts
1948	Lovell, Dr. Clarence A.	Summit, N. J.	Potts
1924	McBride, Mr. Thomas C.	Philadelphia, Pa.	Longstreth
1925	Nachod, Mr. Carl P.	Birmingham, N. J.	Longstreth
1923	Parks, Mr. Harry S.	Pottstown, Pa.	Longstreth
1948	Rajchman, Dr. Jan A.	Princeton, N. J.	Levy
1946	Southworth, Dr. George	New York, N. Y.	Levy
1947	Southworth, Dr. George	New York, N. Y.	Ballantine
1932	Wenner, Dr. Frank	Washington, D. C.	Wetherill
1939	Wilford, Mr. E. Burke	Philadelphia, Pa.	Certificate of Merit
1941	Wilson, Dr. Benjamin J.	Oreland, Pa.	Longstreth
1915	Young, Brig. Gen. Charles D.	Chester County, Pa.	Longstreth
1948	Young, Brig. Gen. Charles D.	Chester County, Pa.	Henderson

MR. NALLE (*continued*): "Thank you, gentlemen. Now, the next Order of Business is a report from the Executive Vice President and Secretary of the Institute, Dr. Henry Butler Allen. Dr. Allen, will you please make your report?"

REPORT OF THE SECRETARY

"The Institute in 1949"

DR. ALLEN: "Mr. President, Honored Guests, Members and Friends of the Institute. The year 1949 has seen the passing of three beloved members of our Board of Managers, who for many years have been with us on Medal Day: Mr. Malcolm Lloyd, Jr.; Mr. James S. Rogers; and Mr. Philip C. Staples, past president of the Institute.

"You will have seen at your tables this evening a simple little folder titled 'This Is The Franklin Institute.' Most of you know that the Institute's activities embrace the Benjamin Franklin Memorial, the JOURNAL OF THE FRANKLIN INSTITUTE, Lecture and Meeting series, a technical Library, the Science Museum and Planetarium, the Bartol Foundation, the Industrial Research Laboratories; and tonight you are clearly made aware of our Medal Award activity.

"But even though we are this year marking our 125th anniversary, there are quite a few other people around who still are not aware of this rather wide range of activities, most of which have existed since we were founded in 1824, 'For the Promotion of the Mechanic Arts'—as the physical sciences and engineering were then known. Hence this little folder which is being widely distributed.

"This year has seen some growth despite uncomfortably restricted means. We have had some notable gifts, such as Benjamin Franklin's 1757 holograph Will, given us by Mary Curtis Zimbalist in memory of her father Cyrus H. K. Curtis, a past benefactor. Through the aid of industry we have markedly improved the electrical section in our Museum. And a real start is being made toward a better exhibit of the science behind one branch of electrical communications through an outstanding exhibit of telephony now being built for us by the Bell Telephone Company. An intriguing illusions exhibit has been installed where you can go into a room in which your normal ideas about gravity seem to be proven all wrong. A visitors' lounge now exists where one can give his or her museum feet a rest while he or she smokes and sees a televised football game.

"New knowledge has been gained about cosmic rays at our Bartol Laboratories where a world's altitude record has been made by one of their cosmic ray captive balloons. Through their separate trust funds there is under construction a building near Swarthmore to house a new large Van de Graaff machine for studying the atom.

"The industrial Laboratories for Research and Development in which 230 people are employed have been getting useful results, in-

cluding, for example, aid for industry through a sonic testing device for grading grinding wheels; and a reading device for aiding the many thousands of semi-blind people, under a grant from the Kellogg Foundation.

"Our Library, besides keeping up to date through current technical books and periodicals has put on display a noteworthy collection of Franklin imprints given us by Mr. Samuel H. McVitty.

"Our Lecture series was highlighted by the first talk in the newly created Edward G. Budd Lectureship, endowed by his family in memory of that outstanding Philadelphia manufacturer, a former member of our Board, and one who always claimed that he got his technical start in our old Franklin Institute schools. And our 125th Anniversary year, together with the 200th Anniversary of Franklin's classic electrical experiments (which today would be called 'research') were celebrated by a day and evening special meeting. At that time we devoted the afternoon to the Franklin Anniversary. A talk was given by Dr. Conklin, President of Franklin's own American Philosophical Society, another by our past Medalist Robert A. Millikan, on Franklin's work in electricity, and one by Dr. C. Guy Suits, Director of Research of the General Electric Company, on electrical research today. In the evening the Institute's anniversary was well recognized by inimitable remarks from our own Senator George Wharton Pepper, and through a delightful story of the Institute from its founding by a great grandson of its first Board Chairman, our own present fellow Board member, James H. Robins.

"And last but not least, we are back to peace times. Our Board is actively working to increase the numbers in our generous group now numbering about 225 people known as Friends of Franklin who give us life blood through current gifts of money, and to encourage larger gifts through bequests.

"Since last Medal Day three new members have joined our Board: Mr. Francis J. Chesterman, recently President of the Bell Telephone Company of Penna.; Mr. E. Paul du Pont, Engineer and Manufacturer of Montchanin, Del.; and Mr. S. W. Rolph, Executive Vice President of the Electric Storage Battery Company of Philadelphia.

"I have touched upon some of the highlights of our year 1949. We hope you can all visit us often. We want our Medalists to feel that this is their home when in Philadelphia. The Franklin Institute is more than just marble halls—it is, with its 5400 members all over the world, a living creative influence to thousands and thousands of young and old; worthy of your friendship and support."

MR. NALLE: "Thank you, Dr. Allen. And now, will the Sponsors for the 1949 Medal recipients please take their places?"

* * *

MR. NALLE (*continued*): "Most of our guests are aware that we celebrated our 125th year last spring on April 20th. Through that

century and a quarter, many amazing discoveries and inventions have been added to the world's accumulated knowledge. The Institute has rendered a public service by acting not only as custodian in our library and museum of the physical record and evidence of such scientific achievements, but by recognizing and rewarding outstanding individuals who have brought new things to the scientific world. Since 1835 we have been making awards upon the recommendations of our carefully chosen Committee on Science and the Arts.

"These awards have been established through the generosity of far-sighted men who wished to insure recognition over the years of the accomplishment of worthy scientific effort. The Science and Arts Committee carries the responsibility for choosing the recipients in accordance with the terms which accompany each award. Their time is devoted throughout the year to investigating outstanding developments in science in order to determine which cases specially merit recognition. Because of the care and integrity exercised by the Committee members for more than a hundred years, our awards have become internationally known and are highly regarded in all scientific fields.

"Each Medalist, when he comes to the Institute to receive his award, is sponsored by the Chairman of the Sub-Committee from the Science and Arts Committee which had recommended him. This Sponsor gives a brief report about the candidate's work ending with the citation.

"Tonight it is a matter of great regret that three of our Medalists are unable to be here. Dr. The Svedberg has just taken on the duties of Director of the newly created Gustaf Werner Institute for Nuclear Chemistry and therefore could not leave his country this month.

"Dr. Hume-Rothery also is absent because his duties at Oxford University prevent his leaving England just now.

"Mr. Millspaugh, upon arrival in Philadelphia, was, unfortunately, taken ill and is in the Pennsylvania Hospital.

"An invitation was extended to His Excellency, Erik Boheman, Swedish Ambassador to the United States, to receive the Franklin Medal and accompanying Report and Certificates for Dr. Svedberg. Greatly regretting his inability to be present, he delegated Mr. Ingemar Hägglöf, Counselor of the Embassy, to represent him.

"Sir Oliver Franks, British Ambassador to the United States, having likewise been invited to receive the Hume-Rothery award, being unable, unfortunately, to be present, duly designated Dr. William Angus Macfarlane, Scientific Attaché of the Embassy, to act in his behalf.

"The awards to the first two Medalists will be duly transmitted to them after presentation here tonight. That to Mr. Millspaugh will be taken to him as soon as circumstances permit."

PRESENTATION OF THE LONGSTRETH MEDALS

(To Arthur M. Young, Counsulting Engineer, New York City, N. Y.; William H. Millspaugh, President and Director, Centrifugal Steel, Inc., Sandusky, Ohio.)

Arthur M. Young

William H. Millspaugh

MR. NALLE: "I recognize Mr. Ralph H. McClarren, the Sponsor of Mr. Arthur M. Young."

MR. MCCLARREN: "Mr. President, I present Arthur M. Young for an award.

"Since 1927, he has devoted most of his time to the solution of problems pertaining to vertical flight. The well known Bell Helicopter is one major result of his work. Incorporated in the Bell Helicopter is a special mechanism which provides stability in flight.

"Like the airplane, the helicopter will be unstable in flight unless specific means are provided for stabilizing it. The ability of a pilot to keep his helicopter on an even keel, especially when hovering in gusty air, depends upon the degree of stability provided.

"Mr. Young has made a contribution toward improving this stability and has applied it primarily to the single rotor helicopter.

"In developing and improving his ideas, he made use of small scale flying models. This led him into designing, building and flying remote control models, the application of which proved unique and most valuable in advancing his theoretical and practical work.

"I present Arthur M. Young, of Paoli, Pennsylvania, as a candidate for an Edward Longstreth Medal, 'In recognition of his successful application of flying scale models to test unproven ideas in helicopter flight.'"

MR. NALLE: "Mr. Young, by virtue of the power vested in me as President of The Franklin Institute of the State of Pennsylvania, I present to you this Edward Longstreth Medal and the Certificate and Report which accompany it.

"I now recognize the next Sponsor, Dr. Benjamin J. Wilson,"

DR. WILSON: "Mr. President, I present, *in absentia*, William Hulse Millspaugh for an award.

"He has been largely responsible for improvements in paper-making machinery. By devising and successfully applying a method for removing water from the paper pulp by suction, instead of pressing between rolls as was done formerly, he secured, not only an increase in the rate of production, but an improvement in quality as well.

"He also pioneered in the practical application of the centrifugal casting of rolls, and succeeded in making rolls several times as long as was possible to secure by use of the old method of vertical sand-core casting. This permitted an increase in the paper width and accordingly, a further increase in the rate of production of several fold.

"I present, *in absentia*, William Hulse Millspaugh, of Sandusky, Ohio, as a candidate for an Edward Longstreth Medal, 'In consideration of his outstanding contributions to the art of paper making and to his introduction and promotion of the art of the centrifugal casting of rolls for Fourdrinier machines and other purposes.'"

MR. NALLE: "It is my privilege, by the same authority, to present also, to Mr. Millspaugh, *in absentia*, an Edward Longstreth Medal, and the Certificate and Report which accompany it."

PRESENTATION OF THE POTTS MEDALS

(To John William Mauchly, President, and J. Presper Eckert, Jr., Vice President, Eckert-Mauchly Computer Corporation, Philadelphia, Pa.; and Clinton Richards Hanna, Associate Director of Research, Westinghouse Electric Corporation, Pittsburgh, Pa.)

MR. NALLE: "I now call on Dr. Richard M. Sutton, Sponsor of Dr. Mauchly and Mr. Eckert."

DR. SUTTON: "Mr. President, I present John William Mauchly and J. Presper Eckert, Jr. for awards.

"These gentlemen are known for their distinguished work on the ENIAC, or the Electronic Numerical Integrator and Computer, a remarkable electronic calculating machine, conceived with great ingenuity and built on a bold scale at the University of Pennsylvania. Their work represents a significant combination of shrewd electronic design and sound engineering execution to create a computing machine of almost fabulous speed and accuracy.

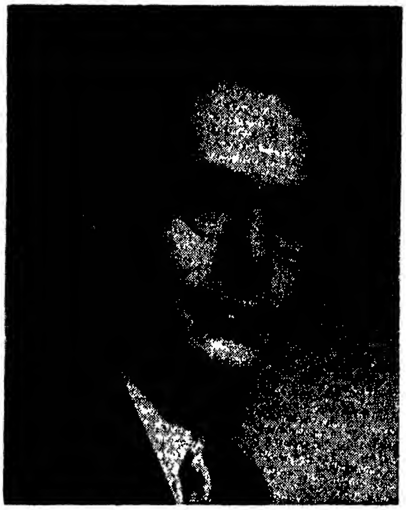
"The ENIAC is now installed in the service of the Ordnance Department at the Aberdeen Proving Grounds where it daily performs

prodigious computing tasks. Capable of doing as many as 5000 additions of two ten-digit numbers in one second, its speed of handling numbers swiftly and accurately outstrips the imagination.

"I present John William Mauchly and J. Presper Eckert, Jr., both of Philadelphia, to each of whom has been awarded a Howard N. Potts Medal, 'In recognition of their design and construction of the ENIAC, the first large-scale, general purpose, digital electronic computer, a machine of great accuracy which makes feasible the application of mathematics to problems which hitherto had either to be ignored or else solved in a much more laborious way.'"

MR. NALLE: "By virtue of my power as president, I present to you, Dr. Mauchly, this Howard N. Potts Medal and the Certificate and Report which accompany it.

"Mr. Eckert, by the same authority, I hereby present to you also a Howard N. Potts Medal and the Certificate and Report which accompany it.



John William Mauchly

J. Presper Eckert, Jr.

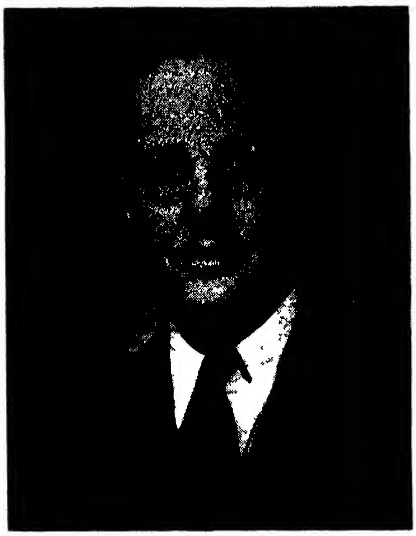
"I recognize Dr. Benjamin J. Wilson, Sponsor of Mr. Hanna."

DR. WILSON: "Mr. President, I present Clinton Richards Hanna for an award.

"He has been largely responsible for inventing and developing a Tank Gun Stabilizer. Before the development of this device, it was customary in tank engagements for alternate tanks to halt in order to cover the advance of the rest of the column. While the halted tanks could aim and shoot their guns effectively, they became, when stationary, 'sitting ducks' for the enemy artillery and were easily put out of commission. It was necessary to use this method, however, because of the difficulty of aiming and shooting from a pitching and rolling tank in motion.

"The development of a gun stabilizer, however, changed this condition. This device keeps the gun aligned on the target despite the pitch and roll of the tank. In tests, the stabilizer enabled a gunner to obtain better than 70 per cent hits, over a variable range up to 1200 yards while traveling at approximately 15 miles per hour over cross-country terrain. Under similar conditions, without stabilizer, fewer than 1 per cent hits were possible, even with experienced gunners. A further advantage was that approximately three times as many sighted shots could be made in a given time, with stabilization.

"I present Clinton Richards Hanna, of Pittsburgh, Pennsylvania, as a candidate for a Howard N. Potts Medal, 'In recognition of his initia-



Clinton Richards Hanna

tive in the conception and development of the Tank Gun Stabilizer, the value of this device in attaining accuracy of fire while the tank is in motion over rough terrain and in securing a greater number of aimed shots than was formerly possible.'"

MR. NALLE: "By the same authority, I award you a Howard N. Potts Medal and Certificate and Report which accompany it."

PRESENTATION OF THE LEVY MEDAL

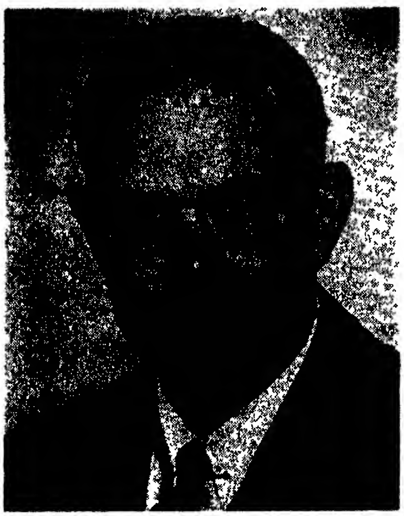
(To Alan Stewart FitzGerald, Consulting Engineer, San Francisco, Calif.)

MR. NALLE: "Dr. Boehne, the Chair recognizes you as the Sponsor for the Levy Medal recipient."

DR. BOEHNE: "Mr. President, I present Alan Stewart FitzGerald for an award.

"Magnetic amplifiers are suitable for a number of important applications for which electron tubes have hitherto been employed. Magnetic amplifiers, so applied, are superior to emission devices because of their greater reliability, longer life, and freedom from maintenance problems. For applications in which a speed of response only slightly less than instantaneous is adequate, magnetic amplifiers play an important role. These include the regulation of such quantities as temperature, voltage, speed, and are particularly useful tools in servo-mechanism techniques.

"The author presented and reviewed circuit arrangements and connections for combining a number of saturating reactors to form multistage magnetic amplifiers. This was followed by a procedure for computing the performance of a saturating reactor when used as a magnetic amplifier, so that the relation between input and output could be



Alan Stewart FitzGerald

quantitatively expressed and indicated in the form of curves. The work outlines the principal elements in the design of magnetic amplifiers and concludes with a description of the design and construction of magnetic amplifiers of various types in which the performance characteristics are presented in curve and tabular form.

"I present Alan Stewart FitzGerald, of Los Angeles, California, as a candidate for the Louis Edward Levy Medal, 'In consideration of his papers on the subject of Magnetic Amplifiers, appearing in the October, November, and December 1947 issues of the JOURNAL OF THE FRANKLIN INSTITUTE.'"

MR. NALLE: "Mr. FitzGerald, by the same authority, I present to you the Louis E. Levy Medal, and the Certificate and Report which accompany it."

PRESENTATION OF THE HENDERSON MEDAL

(To John V. B. Duer (Retired), Pennsylvania Railroad, Fairfield, Conn.)

MR. NALLE: "I now call upon Dr. Rupen Eksergian."

DR. EKSERGIAN: "Mr. President, I present John Van Buren Duer for an award.

"The Pennsylvania Railroad electrification is the world's outstanding electrification, when measured by its power capacity and density of loading. The average power load is in the order of 150,000 KW. and the load factor around 70 per cent. Power is transmitted at 132,000 volts with a trolley voltage at 11,000 volts. The most powerful locomotives per unit weight are the famous G-G-1, developing a peak power at 8100 h.p. and a continuous rating at 4600 h.p.

John V. B. Duer

"While the electrification represents the culmination of a large background from previous electrifications, it is on the other hand unique in many electrical, mechanical and operational characteristics. It carries the densest railway traffic in the world.

"The over-all engineering and construction represents the efforts of a large group of supply companies and consulting engineers who contributed on electrical equipment, motive power and line construction.

"The final responsibility for the coordination of this work, however, rested with John Duer, who, as electrical engineer and later as Chief Electrical Engineer of the Pennsylvania Railroad, showed remarkable insight and engineering judgment in the coordination of the work and its ultimate accomplishment to actual operation.

"His later responsibilities as Senior Officer on over-all Railway Engineering matters covered research developments and studies on steam locomotive and car equipment, on traffic surveys, including special problems, such as the development of a steam turbine locomotive, a high speed freight car truck, the bonding of rails, and colloidal coal developments. His efforts on standardization, both in electrical and mechanical equipment are particularly noteworthy.

"I present John Van Buren Duer, of Fairfield, Connecticut, as a candidate for the George R. Henderson Medal, 'In consideration of his judgment and guidance in the coordination of engineering matters, including full technical responsibility in the electrification of the Pennsylvania Railroad, and in recognition of his contributions and resourcefulness in research developments in over-all railroad problems.'"

MR. NALLE: "Mr. Duer, by the same authority, I present to you the George R. Henderson Medal and the Certificate and Report which accompany it."

PRESENTATION OF THE WETHERILL MEDALS

(To Harlan D. Fowler, Consulting Engineer, Whittier, Calif., Thomas L. Fawick, President, Fawick Airflex Company, Inc., Cleveland, Ohio; and Edgar Collins Bain, Vice President, Carnegie-Illinois Steel Corporation, Pittsburgh, Pa.)

MR. NALLE: "Mr. Fowler's Sponsor, Mr. LePage, who is also Chairman of the Committee on Science and the Arts, is now recognized."

MR. LEPAGE: "Mr. President, I present Harlan D. Fowler for an award.

"He has made an important contribution to the art of airplane design which has resulted in increasing the safety of flying and broadening the range of usefulness of the airplane, both in civilian service and as a weapon of warfare.

"The reconciliation of high top speed with safe and practical landing speed is, in aircraft design, one of the greatest single problems. Obviously, the minimum speed at which an airplane can sustain its weight is a function of its wing area. A plane with a large wing will, generally speaking, have a lower and, therefore, safer and more practical landing speed than a plane with a comparatively small wing. But such a machine will be handicapped in flight by too large and cumbersome a wing structure.

"The problems of structurally changing the wing area in flight are such as to render this obvious solution of the problem impractical.

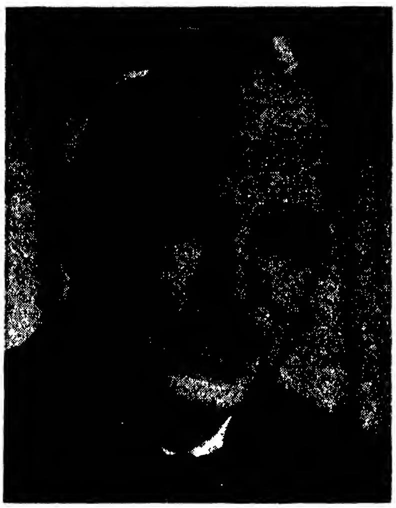
"Our Medalist has succeeded in developing a novel wing-flap device which, when incorporated in the wing structure of an airplane, constitutes the aerodynamic equivalent of variable wing area. His wing-flap principles have been adopted by several leading aircraft manufacturers

and those airplanes which incorporate these principles show remarkable speed range, from landing to top speed, and thereby possess performance characteristics which broaden their utility as well as their safety.

"I present Harlan D. Fowler, of Whittier, Calif., as a candidate for a John Price Wetherill Medal, 'In recognition of his many years of consistent endeavor directed toward a successful solution of the problem of variable lift airplane wings, culminating in the development of the Fowler Flap as an ingenious combination of variable wing area, variable wing camber and adjustable slot.'"

MR. NALLE: "By the same authority, I present to you a John Price Wetherill Medal, and the Certificate and Report which accompany it.

"Mr. Rogers, the Chair recognizes you as the Sponsor for Mr. Fawick."



Harlan D. Fowler

Thomas L. Fawick

MR. ROGERS: "Mr. President, I present Thomas L. Fawick for an award.

"The Fawick Airflex Clutch consists primarily of a pneumatic reinforced rubber bag, usually of flattened shape, mounted in an annular space between a driving and a driven unit, the bag being vulcanized to one of the units and brought in contact with the other unit by admitting air pressure to the bag during operation. The device may be used as a clutch, brake, or combination of both. As the medium for transmitting torque is flexible, shaft vibrations and operating shocks are reduced, slight misalignment of shafts corrected, uniform loading secured, and simplicity of design obtained by the use of a single moving element.

"I present Thomas L. Fawick, of Cleveland, Ohio, as a candidate for a John Price Wetherill Medal, 'In consideration of the adaptation

of the well-known qualities of rubber for expansion, flexibiliy and durability, to its use in the form of an inflatable tire or bag between two rotating members to act as a clutch, and its use between a rotating member and a stationary member to act as a brake, and the excellent design and construction of the details developed, resulting in a practical device, widely and successfully used in industry.'"

MR. NALLE: "Mr. Fawick, by the same authority, I present to you a John Price Wetherill Medal, and the Certificate and Report which accompany it.

"Mr. Francis Foley, will you please present your candidate?"

MR. FOLEY: "Mr. President, I present Edgar Collins Bain for an award.

"The hardening and tempering of steel is one of the world's oldest arts but, through the efforts of men such as Dr. Bain, it has evolved

Edgar Collins Bain

into one of our newest sciences. For hundreds of years, men have hardened steel by the well known process of heating and then rapidly quenching. It has been determined during the past three quarters of a century that this hardening operation involved a change in the crystalline form of the steel, and that hardening resulted from the lowering by rapid cooling of the temperature at which this change took place. It has remained, however, for Dr. Bain to show that the structure and, therefore, the hardness and mechanical properties of steel could be controlled by interrupting the cooling of the metal at predetermined temperature levels during quenching, maintaining it at such temperatures for a predetermined time and then permitting it to cool to room temperature. This has become a standard method of studying the mechanism of transformation of steels and other alloys as

well. Applied commercially, this process is known as 'austempering.' A new micro-structure is developed, now recognized throughout the metallurgical profession as 'bainite.'

"I present Edgar Collins Bain, of Pittsburgh, Pa., as a candidate for a John Price Wetherill Medal, 'In recognition of his pioneer investigations into the structure of metallic alloys and particularly for his original work in the application of the isothermal method of studying the rates of transformation of austenite.'"

MR. NALLE: "Dr. Bain, by the same authority, I present to you a John Price Wetherill Medal, and the Certificate and Report which accompany it."

PRESENTATION OF THE CLAMER MEDAL

(To William Hume-Rothery, Royal Society Warren Research Lecturer, Metallurgical Chemistry, University of Oxford, England.)

MR. NALLE: "Mr. William B. Coleman is now recognized as the Sponsor of the Francis J. Clamer Medalist. Dr. W. A. Macfarlane, Scientific Attaché to the British Embassy in Washington, has come here this evening as proxy for Dr. Hume-Rothery."

William Hume-Rothery

MR. COLEMAN: "Mr. President, I present William Hume-Rothery for an award.

"When metallic elements are melted together they form alloys having characteristic crystalline forms which largely determine their commercial usefulness. What type of crystalline form results has, for

centuries, been attributed purely to chance. There have been no laws to guide the metallurgist, who has had to depend entirely on past experience or on his own cut and try experimentation. Dr. Hume-Rothery has discovered a rule which relates the ratio of electrons to atoms in many of the solid solution alloys of many alloy systems with the various types of crystal structure which are found in them. The successful application of his rule to the crystal forms in alloys has resulted in the use of the designation, 'Hume-Rothery Compounds,' in referring to them.

"I present William Hume-Rothery, of Oxford, England, as a candidate for the Francis J. Clamer Medal, 'In recognition of his brilliant work in scientifically determining and interpreting the structure and behavior of metallic equilibrium systems.'"

MR. NALLE: "Dr. Macfarlane, by the same authority, I take pleasure in presenting to Dr. Hume-Rothery, through you, the Francis J. Clamer Medal, and the Certificate and Report which accompany it."

PRESENTATION OF THE BALLANTINE MEDAL

(To Sergei A. Schelkunoff, Research Mathematician, Bell Telephone Laboratories, Murray Hill, N. J.)

MR. NALLE: "Dr. Albert F. Murray is Sponsor for our Ballantine Medalist. The Chair recognizes Dr. Murray."

DR. MURRAY: "Mr. President, I present Sergei A. Schelkunoff for an award.

"The ever-broadening field of radio engineering owed much of its progress into the newer, little-explored high frequency and microwave regions to our Medalist, a mathematical theorist.

"During the past twenty years that he has been at the Bell Telephone Laboratories, his work has pointed the way to many important communication developments. One of these, of interest to those who observe network television programs, concerns wave propagation in the coaxial cable.

"Dr. Schelkunoff did early theoretical work on the propagation of microwaves in wave guides. These guides later were used extensively in Radar installations during World War II. He made the interesting discovery of the so-called $TE_{0,1}$ mode of propagation, in which the attenuation of the guide decreases as the transmitted frequency increases. This discovery may have a bearing on long-distance guided communication in the future.

"Here are a few of the many theoretical problems on which our Medalist has worked: Antennas, both broadband and highly-directive as well as metal lens antennas; radiating horns; the study of wave propagation in the upper atmosphere.

"Dr. Schelkunoff, during World War II, acted as consultant on Wave Propagation at the U. S. Naval Station at San Diego. To his credit is an unusually long and impressive list of thirty-one publications. He is the author of three books.

"I present Sergei A. Schelkunoff, of New York, New York, as a candidate for the Stuart Ballantine Medal, 'In consideration of his out-

Sergei A. Schelkunoff

standing contributions to the extension of electromagnetic wave theory, particularly his mathematically-based concepts so helpful to the radio engineer.'"

MR. NALLE: "By the same authority, I present to you, Dr. Schelkunoff, the Stuart Ballantine Medal, and the Certificate and Report which accompany it."

PRESENTATION OF THE FRANKLIN MEDAL

(To The Svedberg, Director, The Gustaf Werner Institute for Nuclear Chemistry, Upsala, Sweden.)

MR. NALLE: "The final Sponsor of the evening, Mr. C. H. Masland, II, is now recognized, to present the candidate for the Franklin Medal. Mr. Ingemar Hägglöf, Counselor of the Swedish Embassy in Washington, has come here this evening as proxy for Dr. Svedberg."

MR. MASLAND: "Mr. President, I have the honor to present as a candidate for the Franklin Medal The Svedberg, for many years Professor of Physical Chemistry at Upsala University and, since the first of September of this year, Director of the Gustaf Werner Institute for Nuclear Chemistry at Upsala, Sweden.

"Ever since the invention of the wheel, of the inclined plane and of the lever, the invention and development of tools have been among the most important of man's activities. Professor Svedberg has invented a new and very useful tool. He has developed it so that it performs the functions for which it was created and he has taught others to use it.

"Under the influence of the earth's gravitational field, particles suspended in a fluid settle. Particles of different masses settle at different rates. By observing this rate of sedimentation, it is possible to draw conclusions regarding the mass of the particles.

"As the size of particles becomes smaller, the rate of sedimentation becomes slower until, when we reach particles of the size of large molecules, the rate of sedimentation is so slow under the influence of the

The Svedberg

earth's gravitational field that other factors interfere with the observation of that rate of sedimentation. In order to secure more rapid sedimentation of particles of molecular size, Professor Svedberg turned to the centrifuge. We have centrifuges and we have super-centrifuges and Professor Svedberg has given us the ultracentrifuge. With these devices, Professor Svedberg has been able to develop centrifugal field intensities 900,000 times that of the earth's gravitational field and centrifugal field intensities 100,000 to 200,000 times that of the earth's gravitational field are in regular use for the purpose of determining molecular weights of molecules from the largest to the smallest, from the large protein molecules down to molecules as small as that of lithium chloride. With this device, Professor Svedberg has determined the weight average molecular weights of the very large molecules, particularly proteins in which scientists are so much interested these days.

This method is the only method in existence for determining such molecular weights.

"Mr. President, 'In consideration of his development of the ultracentrifuge, of his development of methods for use of the ultracentrifuge in determining molecular weights and the distribution of molecular weights, of high molecular weight proteins, and in consideration of his inspiring leadership of others in the field of colloid chemistry,' I take great pleasure in presenting The Svedberg."

MR. HÄGGLÖF: "Mr. President, Professor Svedberg has asked me to say how very much he regrets not being able to be here to-night. On his behalf, I wish to express his deep gratitude for the high distinction that the Franklin Institute has bestowed upon him.

"Professor Svedberg has lived and worked in the United States. Throughout his life he has maintained close contact with American scientists. He has in these ways acquired a profound knowledge of, and great admiration for, American science, scientists and scientific institutions. To have been honored by the distinguished Franklin Institute and to have received the Franklin Medal, the highest award for physical sciences in the United States, is therefore to him a very great satisfaction indeed.

"On behalf of Professor Svedberg and on behalf of Sweden, his country and mine, I wish to thank you most sincerely."

MR. NALLE: "Mr. Hägglöf, it is my pleasant privilege to present to Dr. The Svedberg, through you, this Franklin Medal with its accompanying Certificate and Report, together with a Certificate of Honorary Membership in The Franklin Institute."

"Dr. Allen, will you, as Secretary of The Institute, please assist Dr. Macfarlane and Mr. Hägglöf in any arrangement necessary to send these awards to their recipients in England and Sweden?"

DR. ALLEN: "Yes indeed, Mr. Nalle."

MR. NALLE: "It is now my pleasure to present Dr. J. Burton Nichols, of the duPont Experimental Station, who, as a personal friend and former student and colleague of Dr. Svedberg, will read to us a paper entitled "Giant Molecules in Solution," specially prepared for this evening's event by Dr. Svedberg.*

Mr. Nalle, after Dr. Nichols had read this paper, conveyed to him the thanks of the Institute, and dismissed the assembly with a cordial word and declared the meeting adjourned.

* See page 469 for "Giant Molecules in Solution."

MINUTES OF STATED MEETING OF THE FRANKLIN INSTITUTE

Wednesday, November 16, 1949

The November Stated Monthly Meeting of The Franklin Institute was held November 16, 1949 at 8:15 P.M. together with the Joint Lecture of the Philadelphia Section of the Illuminating Engineering Society. The meeting took place in the Lecture Hall with a capacity house in attendance.

President Nalle opened the short business session of the Stated meeting by announcing that the minutes of the October meeting appeared in the November issue of the JOURNAL OF THE FRANKLIN INSTITUTE and if there were no corrections they would stand approved as printed. There was no contrary motion. The President then called for a report from the Secretary.

In his report the Secretary announced the following additions to the membership for the month of October:

Active	42
Associate	45
Student	24

The total membership recorded as of the 30th of October was 5384.

Two announcements of interest to the membership were made by the Secretary. The first concerned the Christmas Show in the Planetarium which is scheduled to open to the public on November 29. All were urged to come and bring their friends to see "The Star of Bethlehem." The second announcement concerned the joint meeting of the Americal Society for Testing Materials and The Franklin Institute which is to be held on December 14, 1949. This will be a symposium on the subject "Research for Management" and many excellent speakers will be heard on this topic.

President Nalle then welcomed the officers and the members of the Philadelphia Section of the Illuminating Engineering Society to their first Joint Lecture Meeting at The Franklin Institute. He introduced Mr. Fred Pyle their Chairman, who in turn introduced Mr. Alston Rodgers, the speaker of the evening.

Mr. Rodgers, Senior Lighting Engineer of General Electric Lighting Institute in Cleveland, Ohio, chose for his topic "Light Sorcery." He presented an entertaining story of the science and research which made possible the lamps of today. He demonstrated a series of spectacular experiments with infra-red, ultra-violet and high-frequency radiations, and bewitched the audience with the mysterious "Ukelitely," which uses a glow lamp as a basis for a musical instrument.

HENRY B. ALLEN
Secretary

LIBRARY

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. The average cost for a print 9 × 14 inches is thirty-five cents.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A.M. until 5 P.M.; Wednesdays and Thursdays from 2 P.M. until 10 P.M.

RECENT ADDITIONS

ARCHITECTURE AND BUILDING

FRANKL, LEE. Home Repairs Made Easy. 1949.

ASTRONOMY

MAYALL, R. NEWTON AND MAYALL, MARGARET. Skyshooting; Hunting the Stars. 1949.

BIOGRAPHY

AMERICAN DENTAL ASSOCIATION. Horace Wells; Dentist, Father of Surgical Anesthesia. 1948.

CHEMISTRY AND CHEMICAL TECHNOLOGY

Colloid Science. Volume 2. 1949.
FIERZ-DAVID, HANS EDWARD. Die Entwicklungsgeschichte der Chemie. 1945.
GAADE, W. Begenselen der Organisch-chemische Nomenclatuur. 1948.

DICTIONARIES

Blakiston's New Gould Medical Dictionary. 1949.

ELECTRICITY AND ELECTRICAL ENGINEERING

HINDE, D. W. AND HINDE, M. Electric and Diesel Electric Locomotives. 1948.

ELECTRONICS

LYTEL, ALLAN. TV Picture Projection and Enlargement. 1949.
GROB, BERNARD. Basic Television Principles and Servicing. 1949.

ENGINEERING

WILLIAMS, CLIFFORD DAVID AND HARRIS, ERNEST C. Structural Design in Metals. 1949.

MANUFACTURE

Steel Plates and Their Fabrication. 1947.

MATHEMATICS

BOCHNER, SALOMON AND CHANDRASEKHARAN, K. Fourier Transforms. 1949.
GOLDMAN, STANFORD. Transformation Calculus and Electrical Transients. 1949.
HOPF, L. Introduction to the Differential Equations of Physics. 1948.
JEFFREYS, HAROLD. Theory of Probability. Second Edition. 1949.
JOHNSON, LEE H. The Slide Rule. 1949.
SEMPLE, J. G. AND ROTH, L. Introduction to Algebraic Geometry. 1949.

MECHANICAL ENGINEERING

AMERICAN SOCIETY OF TOOL ENGINEERS. Tool Engineers Handbook. 1949.
GIRVIN, HARVEY F. Applied Mechanics. Second Edition. 1949.

METALLURGY

SCHULZE, R. BURT. Aluminum and Magnesium Design. 1949.

PHYSICS

CHALMERS, JOHN ALAN. Atmospheric Electricity. 1949.
JAKOB, MAX. Heat Transfer. Volume 1. 1949.
ROSSI, BRUNO B. AND STAUB, HANS HEINRICH. Ionization Chambers and Counters. 1949.
RUSHBOOKE, G. S. Introduction to Statistical Mechanics. 1949.

RADIO

FEDERAL TELEPHONE AND RADIO CORP. Reference Data for Radio Engineers. Third Edition. 1949.

SANITARY ENGINEERING

HARDENBERGH, WILLIAM A. Water Supply and Purification. Second Edition. 1949.

STEAM ENGINEERING

POTTER, PHILLIP J. Steam Power Plants. 1949.

WOOD

CORKHILL, THOMAS. A Glossary of Wood. 1948.

MUSEUM

Astronomy being the oldest of the sciences, it is inevitable that shreds of very ancient beliefs should still cling to its fringes in spite of all that astronomers can do to dislodge them. However, education has done much to free the human mind from the tyranny of superstitious faith in celestial omens and portents. Simple natural phenomena, such as a solar eclipse or the more rare appearance of a comet, could strike terror into a whole population. Our thinking has progressed beyond this state of ignorant superstition but fear of the future plays such an important role in people's lives that a conspicuous blot upon the intelligence of our contemporary thought is the widespread belief in astrology.

On the other hand, certain celestial phenomena, like the rainbow, have always been regarded as favorable omens, although their credibility was no greater than that of the evil omens, since all depended upon quite simple natural causes. The one celestial phenomenon which has retained its grip upon the imagination of multitudes is the Star of Bethlehem. Without denying that the appearance of this star might have had a supernatural origin, astronomers have submitted the question to a careful examination with a view to ascertaining whether or not there is a rational explanation for the appearance of some unusual celestial object at the time of the Nativity. Their investigations have brought to light some exceedingly interesting information relating to the errors which have crept into our accounting of time, and to the odd combinations in proximity which result from the planetary motions.

Each year during the month of December it is customary to devote the Planetarium demonstration to describing the various celestial phenomena which could have had an especial significance to the minds of those who saw the Star of Bethlehem. The extraordinary flexibility of the planetarium instrument enables the demonstrator to reproduce the skies as they appeared at the time in Palestine. This leads to an account of a rare conjunction of planets which occurred in the year 7 B.C. and to reasons why the possible date of the Saviour's birth should be post-dated to that year. The whole makes a story of absorbing interest.

The scientific aspect of the heavens at about the time of the Nativity having been adequately treated, the devotional aspect is then emphasized. Appropriate music and the ingenious applications of light and color are used to tell the story of the Nativity.

This annual demonstration has become a well established tradition of Philadelphia's Christmas week, and is attended by thousands of people who find this a most acceptable opportunity for recapturing the spirit of the season. Those who witness the demonstration for the first time almost invariably remark upon the impressive nature of the effort to combine the wisdom of today with the simple faith of the past in order to tell a complete story of an event, the spiritual significance of which has been and will continue to have an enduring influence.

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in the JOURNAL within the next few months:

COULSON, THOMAS: The Franklin Institute from 1824 to 1949.

FANO, ROBERT M.: Theoretical Limitations on the Broadband Matching of Arbitrary Impedances.

FANO, ROBERT M.: A Note on the Solution of Certain Approximation Problems in Network Synthesis.
OUGHTON, CHARLES D.: Xerography.
MINAMIOZI, K. AND H. OKUBO: A Note on the Notch Effect of Metals.
SUSSKIND, CHARLES: Henry Cavendish, Electrician.
BOYAJIAN, A.: Origin and Meaning of Circular and Hyperbolic Functions in Electrical Engineering.
SILBERSTEIN, LUDVIG: Developable and Developed Silver Halide Grains.
ROOP, WENDELL P.: The Part of Octahedral Theory in the Study of the Plasticity of Metals.
HERSHBERGER, W. D. AND L. E. NORTON: Servo Theory Applied to Frequency Stabilization with Spectral Lines.
LEITNER, A. AND R. D. SPENCE: The Oblate Spheroidal Wave Functions.

MEMBERSHIP

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS, NOVEMBER 16, 1949

ACTIVE FAMILY

Alfred J. Cooke
Maurice J. Cooper
Allen A. Gale
Albert Greenberg, Sr.
Clyde H. Stephens

ACTIVE

Frederick H. Allen, M.D.
Alfred Arden
Royden Astley, M.D.
Ardrey M. Bounds
Walter E. Champlin
Frank Wade Conway
Raymond B. Crean
Louis Davis
William L. Davis
Bernard R. Horn
Joseph La Rocca, Jr.
William L. Lyon
Robert E. Mack
William F. Maxfield
Leo P. McManus
Walter Mulligan
W. A. Myers
J. W. Noll
Arnold Raines
Robert F. Robbins
Edward Semel
Lewis S. Somers
George B. Thomas

ACTIVE NON-RESIDENT

Richard Kroth
Robert McKinley, Jr.
Francis J. O'Brien
David E. Pearsall
Paul Wengraf

NECROLOGY

Harry W. Ellis '36
Charles L. Gabriel '45
Jonathan Godfrey '40
Sigmund S. Greenbaum, M.D. '48
George Walter Jarrett '46
W. J. Jeffries '42
Jack Kapp '48
Joseph A. Martocello '21
James H. McKee, M.D. '48
F. M. Murphy '43
John H. Packard '89
Ralph Pemberton, M.D. '24
Frank H. Stewart '20
Walter S. Thomson '36

BARTOL RESEARCH FOUNDATION

Deuteron Bombardment of N^{14} and C^{14}.*—E. L. HUDSPETH AND C. P. SWANN. Further bombardments of N^{14} have been made with 1.05 Mev deuterons. The arrangement was identical with that previously reported, except that the gas cell was covered with nickel rather than aluminum foil. This change was made when we found that the reaction Al(d,n) was producing an objectionable background. Under these conditions, we find that the neutrons from $N^{14}(d,n)$ belong to only one group; a second group observed in this work probably arises from carbon contamination. It is therefore likely that $N^{14}(d,n)$ is a good source of monochromatic neutrons. Our results are in agreement with observations recently made in another laboratory,[1] but differ from earlier results.

The $C^{14}(d,n)$ reaction has also been observed with a target of $BaCO_3$ which was enriched about 40 per cent in C^{14}. The calculated Q-value for the formation of N^{15} in its ground state is 8.02 Mev, and we have observed neutrons corresponding to this case. Other groups, expected from consideration of the known excited levels of N^{15}, are also observed. The $C^{14}(d,n)$ reaction yields more than twice as many neutrons as $C^{12}(d,n)$ for deuterons of 1.26 Mev energy.

* Work in progress.

[1] W. M. GIBSON ANA D. L. LIVESEY, *Proc. Phys. Soc.* (London), Vol. 60, p. 523 (1948).

Abstract of* An Investigation of the Disintegration Schemes of Ru^{103} and Rb^{86}.—C. E. MANDEVILLE AND E. SHAPIRO. The disintegration schemes of the 42-day Ru^{103} and the 19-day Rb^{86} have been studied with the use of absorption and coincidence methods. Ninety-two per cent of the disintegrations of Ru^{103} proceed by way of an inner beta ray group at 0.15 Mev coincident in time with a gamma ray of energy 0.52 Mev, which is (4 ± 1) per cent converted. A harder beta spectrum having a maximum energy of 0.68 Mev leads to the ground state of Rh^{103}.

It is estimated that (12 ± 2) per cent of the beta rays of Rb^{86} are contained in the inner spectrum. Absorption measurements confirm the energy values given by the Indiana group.

* As yet unpublished, but to appear in the *Physical Review.*

Unpublished Notes.—Dr. S. C. SNOWDON.

Ion Source

The possibility of using 10 cm. microwave radiation for the efficient production of protons and deuterons in the Van de Graaff generator is being investigated. Since a small region of the order of a cubic millimeter can be efficiently ionized with microwave energy, it was thought that the gas and power consumption of the source could be kept to a minimum by the use of 10 cm. CW microwave power. At present the CW magnetron supplying 100 watts and the R.F. voltage necessary for the Cockroft-Walton cascade rectifier (60 K.V.) are available. The oil insulated cascade rectifier and the vacuum system for testing have yet to be built.

Spectrometer

A large air-core double magnetic lens spectrometer for measuring beta-ray and gamma-ray energies is under consideration. The necessary copper and motor generator set with amplidyne control are available. A detailed theoretical investigation of the actual trajectory—accurate to the third order in the initial slope—of an electron in an immersion lens has been completed. The coil design should be able to handle quite high currents but, at present, we are keeping the power requirements down to 20 K.W. and expect to be able to focus three Mev beta-particles with high gathering power (1 to 10 per cent of a sphere) and with moderate resolution (1 to 3 per cent).

THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

***Abstract of* A New Fourier Coefficient Harmonic Analyzer.**[1]—S. CHARP.[2] The Fourier coefficient harmonic analyzer is a device for evaluating the Fourier coefficients of periodic waves, or sections of complex waves which are assumed to be periodic outside of the interval within which the wave is to be analyzed. The machine (Fig. 1) can be considered to be a special-purpose differential analyzer, permanently interconnected to solve for the Fourier coefficients. Figure 2, a top front view of the analyzer with the protective cover removed,

[1] *AIEE* Paper 49–163, presented at the *AIEE* Summer General Meeting, Swampscott, Mass., June 22, 1949. *AIEE Proceedings,* Section T9163, Vol. 68 (1949).

[2] Senior Research Engineer, The Franklin Institute Laboratories for Research and Development, Philadelphia 3, Pa. The harmonic analyzer was developed while the author was Research Associate in Electrical Engineering, Moore School of Electrical Engineering, University of Pennsylvania, Philadelphia, Pa.

shows the ball and roller type mechanical integrators which carry out the following integrations for the coefficients:

$$A_0 = \frac{1}{\pi}\int^{2\pi} F(\theta)d\theta, \tag{1}$$

$$A_n = \frac{1}{\pi}\int_0^{2\pi} F(\theta)\cos n\theta d\theta, \tag{2}$$

and

$$B_n = \frac{1}{\pi}\int_0^{2\pi} F(\theta)\sin n\theta d\theta, \tag{3}$$

$$F(\theta) = A_0/2 + \sum_1^{\infty} A_n \cos n\theta + \sum_1^{\infty} B_n \sin n\theta. \tag{4}$$

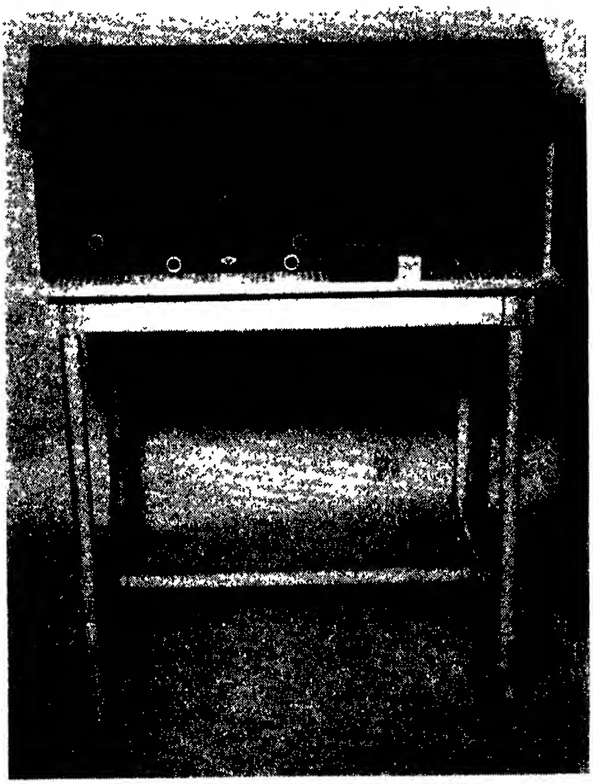


FIG. 1. Over-all view of the harmonic analyzer.

The function, $F(\theta)$, is the wave whose periodical components are desired. In accordance with the theory of the Fourier series, it is also assumed to be given by the series in Eq. 4.

The coefficient A_0 is developed by an integrator whose incremental output is given by

$$dA_0 \propto F(\theta)d\theta. \tag{5}$$

This relationship can be used in Eq. 2 and 3 resulting in:

$$A_n = \frac{1}{\pi}\int_{\theta=0}^{\theta=2\pi} \cos n\theta d[\int F(\theta)d\theta] = \frac{1}{\pi}\int_{\theta=0}^{\theta=2\pi} \cos n\theta dA_0, \tag{6}$$

and

$$B_n = \frac{1}{\pi}\int_{\theta=0}^{\theta=2\pi} \sin n\theta d[\int F(\theta)d\theta] = \frac{1}{\pi}\int_{\theta=0}^{\theta=2\pi} \sin n\theta dA_0 \tag{7}$$

With Eq. 1, these equations are much more suitable for mechanical solution. Since the sine and cosine coefficients of the Fourier series are developed in this machine, the magnitude and phase of each frequency component can be determined by the following:

$$C_n = [A_n^2 + B_n^2]^{\frac{1}{2}}, \tag{8}$$

and

$$\phi_n = \tan^{-1}(B_n/A_n). \tag{9}$$

Four ball and roller integrators are used in the analyzer; three to perform the integrations called for in Eqs. 1, 6, and 7, and one to serve as a variable speed drive to adjust for frequency, obviating the use of change gears. The outputs of the integrators are individually recorded on counters, located at the front upper left hand part of the analyzer These can be read to 0.02 revolution

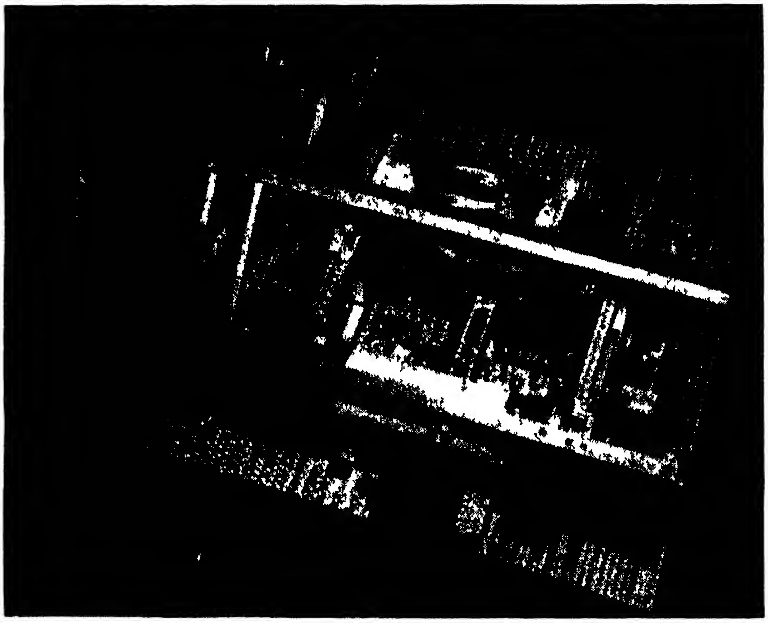


FIG. 2. Top front view of the analyzer with plastic cover removed.

The Fourier coefficients are related to the counter readings as follows:

$$A_0 = \frac{\text{Counter Reading}}{576K} + 2f_{\text{avg}}, \tag{10}$$

$$A_n = \text{Counter Reading}/576K, \tag{11}$$

and

$$B_n = \text{Counter Reading}/576K. \tag{12}$$

Here,

$K = \dfrac{5}{[f_{max} - f_{min}]}$ = conversion factor to change the physical measured data to inches displacement along the plotted curve,

f_{avg} = arithmetic mean of the peak to peak values of the function $F(\theta)$ to be analyzed,

f_{max} = maximum value of the function $F(\theta)$, and

f_{min} = minimum value of the function $F(\theta)$.

The curve to be analyzed is plotted on a piece of paper to a size not greater than 5 × 10 in. It is then wrapped around the plastic cylinder shown in Fig. 2 and inserted in the machine.

When the machine is in operation, the cylinder rotates an amount proportional to the independent variable. The operator tracks the curve by sighting through an eyepiece and turning the tracking handwheel located on the front panel. This operation introduces the function $F(\theta)$ into the integrator which solves for Eq. 1, the output of which is the differential input to the integrators for Eqs. 6 and 7. The sine and cosine integrands of these latter equations are developed by a Scotch-yoke mechanism.

Each time the curve is scanned the Fourier coefficients for that harmonic appear on the counters. The frequency is altered by adjusting the variable speed-drive to the Scotch-yoke.

Since the completion of the machine it has been applied to solving problems in many engineering fields including studies in servo-mechansims, crystal structure periodicities, radar tracking of airplanes, and others.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

Seminar on Pathological Sera.—DR. KAI O. PEDERSEN of the Institute of Physical Chemistry of the University of Upsala, Sweden, addressed the Seminar of the Biochemical Research Foundation on Wednesday afternoon, June first.

For a number of years Dr. Pedersen has utilized the Svedberg ultracentrifuge and the Tiselius electrophoresis apparatus to examine pathological sera furnished by Dr. Jan Waldenström of the University Medical Clinic. Dr. Waldenström has supplemented the ultracentrifugal and electrophoretic analyses by salt fractionation and Kjeldahl analyses, by measurements of specific gravity, refractive index, and relative viscosity at 13° and 37° C., and by the formol gel, cadmium, and alcacid tests. The more recent electrophoretic work has been carried out in collaboration with Dr. Niels Harboe of the University of Copenhagen.

In the ultracentrifuge diagram of normal human serum, the albumin is found in the boundary with a sedimentation constant of 4.5 S. The alpha globulin is divided between the 4.5 S and 20 S boundaries, and the beta globulin among the 4.5 S, 7 S, and X boundaries; the gamma globulin appears almost always to sediment with the 7 S boundary.

Many pathological sera are characterized by hyperglobulinemia. In severe burns and in acute infections accompanied by tissue destruction (acute tuberculosis, pneumonia, cancer with metastases) there is an increase of alpha globulin; the globulin thus formed sediments with the 4.5 S boundary. In certain renal diseases there is an increase of the alpha and beta components in electrophoresis, and of the 4.5 S component in the ultracentrifuge. In most cases where there is an increase in the total plasma protein, however, the augmentation is due to an increase in the gamma globulin, and this is usually correlated with an increase in the 7 S component.

In some cases of myeloma there may be an increase of one of the slow moving beta components; this is correlated either with an increase in the 7 S component or with the appearance of a new component of sedimentation constant 11 S. In a few rare cases of essential macroglobulinemia (Waldenström, *J.*, *Acta Med. Scand.*, **117**, 216 (1944)), a very high concentration of the 20 S component has been found. This is associated with the beta or gamma globulin in electrophoresis. It always gives rise to a sharp and well-defined component both in electrophoresis and in the salting-out procedure of Derrien (*Svensk. Kem. Tid.*, **59**, 139 (1947)). The molecular weight is about one million. Augmentation of this component confers certain abnormal characteristics on the

serum: high relative viscosity with a negative temperature coefficient, leading to a viscosity index, $100 \cdot \eta_{13°}/\eta_{37°}$ (Waldenström, *loc. cit.*), above the normal value of 100; high euglobulin content; and rapid formol gel, cadmium, and alcacid tests.

Dr. Pedersen and Dr. Waldenström in collaboration with Dr. Sonck of Helsingfors have chosen lymphogranuloma venereum, a compara-

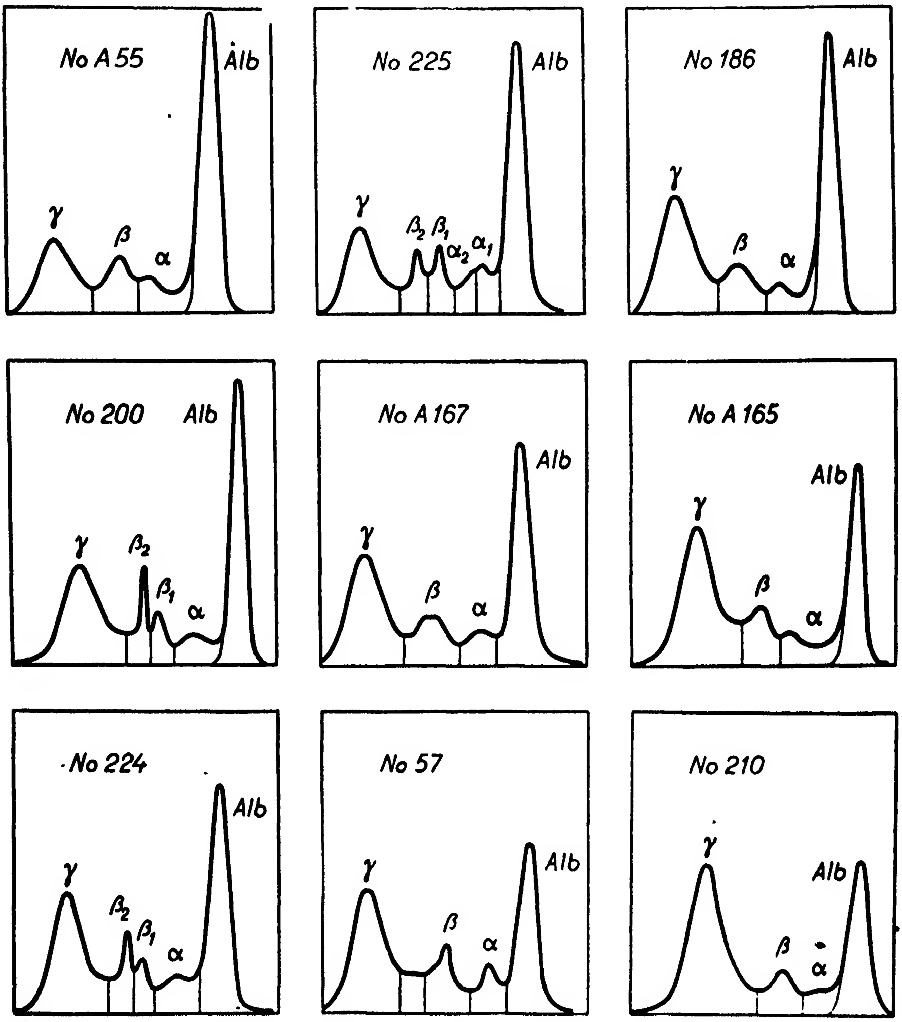


FIG. 1. Electrophoretic diagrams from nine cases of lymphogranuloma venereum with varying hyperglobulinemia.

tively rare disease in Sweden, as a type of chronic virus infection. In the serum of patients with this disease (Fig. 1) the globulin concentrations are high and the albumin concentrations low. The increased globulin is associated with the gamma globulin, and forms a very broad boundary; the sedimentation constant is 7 S. The relative

viscosity of the serum parallels the gamma globulin content and exhibits no pathological temperature coefficient. The formol gel and other tests are rapid. When separated by salt fractionation methods, the gamma globulin from lymphogranuloma venereum is apparently inhomogeneous with respect to both charge and mass.

Successful treatment of the disease by sulfa drugs or other therapeutic measures is accompanied by a drop in the gamma globulin concentration of the serum. The excess gamma globulin thus appears to be a pathological globulin produced in response to the infection, rather than an antibody; serological tests will be required to verify this

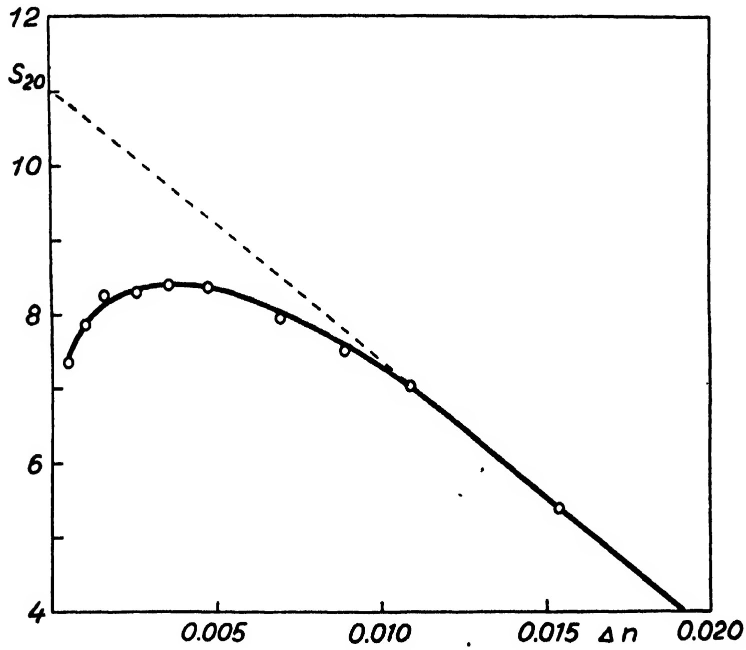


FIG. 2. Sedimentation of the pathological globulin from a case of multiple myeloma. Variation of sedimentation constant with concentration and temperature. Ordinate: sedimentation constant, s_{20}. Abscissa: concentration of globulin, in refractive units. Heavy curve: measurements at 22° C. Dashed curve: measurements at 10°C.

hypothesis. The production of the pathological globulin is apparently the primary change produced in the serum as the result of the disease. The reduced albumin concentration is believed to be a secondary change, for the extent of the reduction appears to vary with, and possibly to be determined by, the concentration of pathological globulin (see Fig. 1).

Occasional instances of hyperglobulinemia with increased gamma globulin such as characterizes lymphogranuloma venereum have been found with no apparent pathological cause. The condition might reasonably be the result of chronic infection by a non-pathogenic virus; it has not, however, responded to treatment with sulfa drugs. Hyper-

globulinemia with increased gamma globulin has also been found in liver cirrhosis.

A different type of pathological globulin is produced in myeloma. One sample of myelomatous serum contained 58 per cent of beta$_2$ globulin; the pathological globulin could be separated as a homogeneous

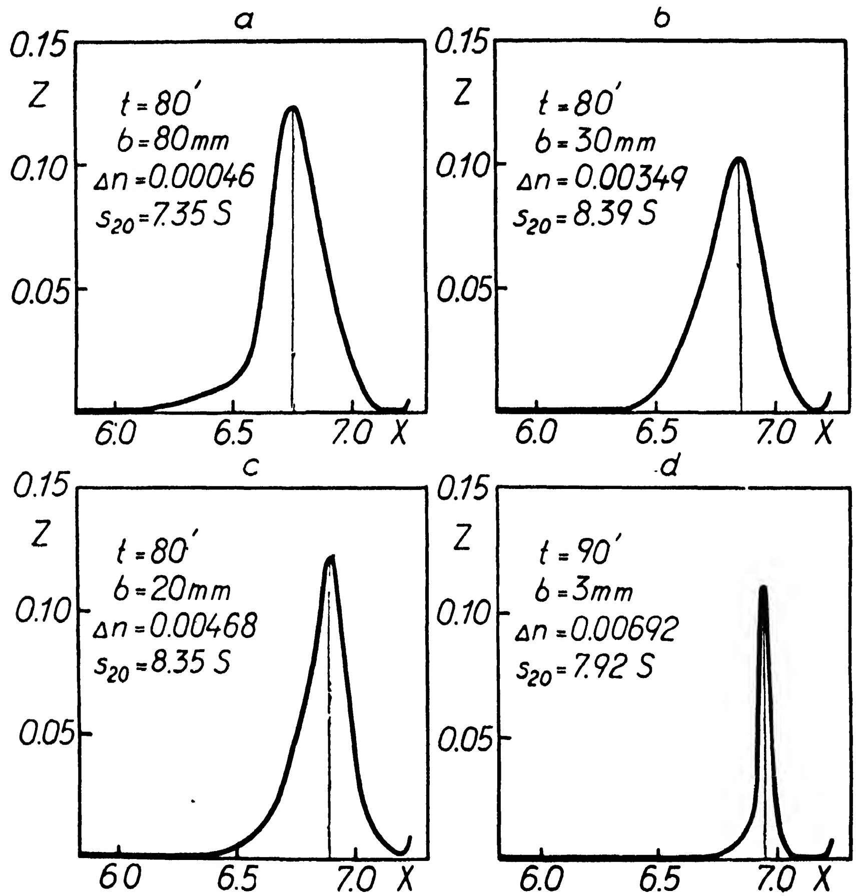


FIG. 3. Sedimentation of the pathological globulin from a case of multiple myeloma Sedimentation boundaries at 22° C. for various concentrations of the globulin. Ordinates: scale line displacements, in mm. Abscissae: distances from center of rotation, in cm. The symbols in each block have the following significance: t = time in minutes from the attainment of full rotor speed; b = distance from the scale to the middle of the cell; Δn = globulin concentration in refractive units; s_{20} = sedimentation constant.

component by salt fractionation methods (Derrien, Y., *loc. cit.*). Other samples of myeloma serum contained much gamma globulin, whereas both alpha and beta globulins were normal. In each instance the pathological component gave a sharp boundary on electrophoresis and a homogeneous component on salt precipitation. The salt precipitation ranges varied, however, with the mobility.

A patient in the early stages of myeloma showed a moderate increase of gamma globulin, with little change in the other components of the serum. Three years later the same patient, in the terminal stages, showed a large increase of gamma globulin.

A sample of myeloma serum from Holland separated spontaneously at 2° C. into two layers, the lower of which proved to be a concentrated solution of the pathological globulin. When separated and purified, the lower layer provided an excellent source of material for investigation of the pathological component. The sedimentation constant was found to vary with concentration as shown in Fig. 2. At 22° C. it had a maximum value of about $s_{20} = 8.5$ S at a refractive increment of 0.007 ($\Delta n = 0.002 \sim 1\%$). The occurrence of a maximum is evidence of an association-dissociation reaction. The change in shape of the sedimentation boundaries with concentration of the globulin at this temperature (Fig. 3) is further indication of this type of reaction. At 10° C. the points followed the dashed line shown above the heavy curve in Fig. 2. If dissociation occurred at all at this temperature, it took place at a concentration below 1 per cent. The value for s_{20} extrapolated to zero concentration was 11 S. On the other hand, at 30° C. the points were situated below the heavy curve in Fig. 2, an indication of the presence of monomers. Dissociation appeared to be complete at this temperature in the concentration range studied. The limiting value for s_{20} was 7 S. Diffusion measurements at 20° C. showed a profound dependence of the diffusion constant on concentration. When the protein was diffused against pure buffer solution, the curves were extremely skew. When diffusion was carried out with two solutions of the same globulin differing only slightly in concentration, the curves were symmetrical, but the diffusion constant varied strongly with concentration.

In liver cirrhosis there is apt to be a large increase in gamma globulin and in the 7 S component, a decrease in the alpha and beta globulins. In Cushing's syndrome the alpha and beta globulins are fairly constant; the decreased total protein content is caused by the decrease in albumin and gamma globulin. A typhoid patient with complete loss of appetite showed an extremely low content of albumin and of total protein, with almost normal values for the alpha and beta globulins. With return of appetite the patient recovered, and recovery was paralleled by return of the albumin content to normal.

It is apparent that the hyperglobulinemia which characterizes many diseases can manifest itself in numerous ways. The results so far obtained have encouraged the investigators in their hope that physicochemical methods may give a clearer insight into protein derangement than has hitherto been possible.

BOOK REVIEWS

MAX PLANCK'S SCIENTIFIC AUTOBIOGRAPHY AND OTHER PAPERS, translated from the German by Frank Gaynor. 192 pages, 14 × 22 cm. New York, The Philosophical Library, 1949. Price, $3.75.

It is hard to believe that almost half a century has passed since Planck made his first disclosure of the law of radiation and its deduction which raised him to the front rank of physicists of all time. He was not a prolific writer but he was certain of an interested audience whenever he chose to write or speak, and this small volume of essays is the pendant which completes his work. The essays deal with problems which, for many decades, occupied the mind of this great thinker, such as:—true and fictitious issues in science, the meaning and limit of exact science, physics and causality, natural science and the religious world. All belong to the period near the end of his life, so that they represent the essence of his teaching.

There is nothing in the volume which ranks with the two papers Planck presented before the Berlin Physical Society in 1900, establishing him as a thinker of the foremost order. Such was not to be expected for it is not given to a single human being to produce two conceptions equalling in importance the conception of the quantum theory.

The autobiographical fragment which gives title to the volume is of especial significance. In the simplest possible terms and with great charm the living mind of a great contemporary is unfolded. It tells how the first papers of the young student were not even read by the famous men at whose feet he sat. Helmholtz and Kirchhoff, under whom he studied, so ignored him that they contributed little to his mental advancement. Clausius did not even answer his letters. Rebuffed by the men who would have found it easiest to appreciate his work, Planck continued in his own lonely way, obeying an inner call, until he arrived at the great problem for which the path he had taken without guidance proved to be the best preparation. Thus he was able to recognize and to formulate, from measurement of radiations, the law which bears and immortalizes his name. Acceptance was not achieved without a struggle and the experiences through which he passed led him to the sad reflection that "a new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it."

This sketch of the mental development of a man who looked upon the search for the absolute as the noblest and most worth while task of the scientist is a document of touching simplicity. Everyone interested in the evolution of a great physical principle will benefit from its reading, although the story is carried only up to the stage where Bohr laid the foundation for a reasonable unification of the quantum theory with classical theory, and Schrödinger created wave mechanics and thereby the dualism between wave and particle.

Planck's long life was not all serenity. The controversy over his first original ideas, waged by Boltzmann, the atomist, against Ernst Mach, Ostwald, and the Energetics engendered no small amount of heat and irritation, but it is graciously overlooked in this retrospective view. Nor will any allusion be found to the bitter personal tragedies of his life, the death in action of his soldier son, and the stunning loss of his second son in the reign of terror which followed upon the attempt on Hitler's life in 1945. These were terrible blows to a man whose private life would otherwise have been as beautiful and contented as any man could wish for. The aloofness of his mind from personal sorrows is reflected in the omission to all allusions which would have caused Max Planck's recollections to be tinged with bitterness and disappointment. His admirers and his biographers will find this all too brief outline of his intellectual development a valuable work, not on any account to be overlooked.

T. C.

BASIC ELECTRONICS, by Royce G. Kloeffler and Maurice W. Horrell. 435 pages, 15 × 23 cm., drawings and photographs. New York, John Wiley & Sons, Inc., 1949. Price, $5.00.

"Basic Electronics" is less basic than it is comprehensive. The authors have attempted to introduce the entire field of electronics emphasizing the multitude of operations that can be

performed within this field, and they have done this very skilfully. Examples of every type of electron tube from the proximity fuse sub-miniature to the cavity magnetron used in pulse radar are discussed. Electronic apparatus ranging from the simple photocell exposure meter to the electron microscope and the G. E. betatron is described.

One of the most important points in justification of this book is its up-to-dateness, for it includes such recent developments as the Bell Telephone Laboratories' transistor and R. C. A.'s triode mechano-electronic transducer. It also includes the large variety of devices such as germanium and silicon crystals developed for radar and other purposes during the War.

Nearly every page of the volume contains some sort of illustration. In most cases these are drawings prepared to illustrate a teaching point discussed in the text; there are few vague apparatus photographs so often found in this type of non-mathematical treatment. Although the book is primarily descriptive, the use of these well-drawn, line illustrations and a simple choice of words convey a large amount of information.

The authors state in the preface that they intend this as a first course in electronics. The reviewer feels that the book provides material adequate for a first course in electronics for the liberal education of electrical power majors, general engineering students and those who do not intend to specialize in electronics. However, the rigorous preparation required for professional performance in electronics and communications indicates that a more thorough grounding in the physics of fundamental electrical phenomena would be a more profitable starting point than this general survey. The volume is useful for teaching purposes but has little value as a reference.

C. W. HARGENS

CHYMIA, ANNUAL STUDIES IN THE HISTORY OF CHEMISTRY, Tenney L. Davis, Editor in Chief. Volume 2, 143 pages, illustrations, 15 × 24 cm. Philadelphia, University of Pennsylvania Press, 1949. Price, $4.00.

It is a pleasure to record the appearance of the second volume of this interesting series. However, one regrets the untimely death of Mr. Davis, for as editor of these first two volumes he has builded well. In the present volume he has succeeded admirably in emphasizing the international character of the work, for more than half of the twelve articles represent the work of residents of five foreign countries: Canada, France, Germany, India, and the Netherlands. Such variety of nationality in authorship almost automatically insures the variety of subject matter which is a desirable feature of such a volume.

The lead article is by H. S. van Klooster and treats of the beginning of laboratory instruction in chemistry in the United States. After reviewing the early teaching of chemistry in this country, the author discusses the work of Amos Eaton, the first teacher of practical chemistry in America. The subsequent introduction of laboratory teaching into other colleges than that of Rensselaer Polytechnic Institute is sketched briefly.

Wilhelm Prandtl offers an interesting account of the chemical laboratory at the Bayerische Akademie der Wissenschaften in Munich. In the number and caliber of the teachers and industrial chemists who have been trained there, it holds an outstanding position. The article is well illustrated with portraits of the laboratory's leaders and of its buildings, unfortunately destroyed in the last war.

Individual studies of several figures in chemical history are given by W. Miles on Sir Kenelm Digby; by D. Reilly on William J. MacNeven, an Irish-American chemist, 1763–1841; and by R. J. Forbes on Sir Isaac Newton and his interest in chemistry.

Personal reminiscence is contributed by Edmund P. Hillpern who writes of some personal qualities of Wilhelm Ostwald. As a former assistant of Ostwald, Hillpern is able to furnish several interesting sidelights on the former's beliefs and habits.

R. Hooykaas discusses the experimental origin of chemical atomic and molecular theory before Boyle, and Mr. Davis, the editor, has contributed a well documented paper entitled "Pulvis fulminans" in which he discusses the various references to fulminating power. Other articles treat of some seventeenth century chemists and alchemists of Lorraine, the discovery of catalysis, by Désormes and Clément, the history of ambergris in India, and burning glasses.

With its wide variety of articles and well chosen illustrations attractively presented,

Chymia should offer something of interest to every student of chemistry. To facilitate its use the editors have added a name index of the first two volumes, further emphasizing its permanent reference value.

G. E. PETTENGILL

THE SPECTROSCOPIC PROPERTIES OF URANIUM COMPOUNDS, by G. H. Dieke and A. B. F. Duncan. Volume 2, Division III, National Nuclear Energy Series, 290 pages, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Co., 1949. Price, $2.75.

This book describes in a rather complete manner the spectra of a large number of uranium compounds. It is one of a series, whose purpose is to record results of research done under the Manhattan Project and the Atomic Energy Commission.

The descriptive part of the book deals mainly with experimental work which has taken place at several different laboratories. The spectroscopic work was done at Columbia and The Johns Hopkins Universities. The investigation of crystal structure was carried out at Cornell University.

The book is divided into two parts. Part one deals with the spectroscopic properties of uranium compounds, while part two discusses their preparation. This is followed by three appendices and a bibliography.

The first chapter of Part I discusses experimental procedures. The instruments used are described. The preparation of samples and the methods used for low temperature studies are also discussed. The chapters following discuss the crystal structure and the general features of the spectra of uranyl compounds. This is followed by a more detailed discussion of their fluorescence and absorption spectra. Other chapter headings are "Intensity Measurements," "Miscellaneous Observations" and "Miscellaneous Uranium Compounds."

The second part begins with the chapters on the synthesis and preparation of uranyl compounds. There is a chapter which discusses the methods used in the study of crystal growth. This part is concluded with a summary of the whole work.

The book contains as part of the appendix about eighty-five pages of tables giving wavelength measurements and other spectral data for many uranium compounds.

An extensive bibliography is included which contains references not only to the spectroscopic and optical properties of uranium compounds but also to other properties such as photochemistry, color, and crystal structure. This bibliography has the references arranged in chronological order beginning with the famous papers of Nichols and Howes which were published in 1919.

This book deals with a highly specialized topic and consequently may not be of direct interest to a large number of scientists. It does, however, contain a great deal of information and should prove very valuable to persons working in the field of spectroscopy, especially those who are concerned with the compounds discussed.

The results of the research performed by a large number of people are recorded in this volume. It therefore represents an important contribution to our total of scientific knowledge.

MARSHALL D. EARLE

CONSTRUCTIVE USES OF ATOMIC ENERGY, edited by S. C. Rothman. 258 pages, plates, 14 × 21 cm. New York, Harper & Brothers, 1949. Price, $3.00.

Mr. Rothman has brought together fourteen articles by scientists in various fields, in an attempt to produce a book which would point up the potentialities of atomic energy for other than military purposes. The articles were, for the most part, written originally in technical journals and will not be well understood by the layman. Furthermore, this method of assembling a book leads to an ultimate product which is not a book as a unit, but is merely fourteen papers bound together.

The book shows clearly that the constructive aspects of atomic energy lie for the most part in research fields. The large industrial applications of atomic power for industry and atomic engines for aircraft are, as of July and August, 1948, applications for the future beset with many difficult problems. As for the uses in medicine, the article by Dr. C. P. Rhoads does a good job of making clear that only in rare cases can diseases be cured with atomic

radiations, and he feels that the important role of radioactive isotopes in medicine lies in the research studies they make possible, rather than in direct therapeutic application.

Mr. Rothman, in his preface, states that the articles included in the book were chosen in an effort to find material which would dispel the common misconception that atomic energy is beyond the comprehension of the layman. I agree with him that this is a common misconception, but I do not feel that this book does much to dispel it.

E. SHAPIRO

FREEZE-DRYING, by Earl W. Flosdorf. 280 pages, illustrations, 15 × 23 cm. New York, Reinhold Publishing Corp., 1949. Price, $5.00.

This is the first book entirely devoted to the subject of freeze-drying, a technique of drying material from the frozen state now being used for the preservation of antibiotics, foods, blood plasma and bacterial and viral cultures. The author has done much of the pioneer work on freeze-drying and is well qualified to present both the theoretical and the practical aspects of freeze-drying on either laboratory or industrial scale.

One portion of this book is devoted to the basic principles of physical chemistry on which the technique depends and which govern the design and operation of equipment. Much of the remainder of the book is devoted to the practical application of freeze-drying in research and industry, with particular attention to the conditions under which drying is carried out, and the equipment used in the commercial processing of medical and food products. A complete bibliography, a list of patents in this field, and an adequate index make this book doubly valuable to anyone who is interested in the preservation of labile materials.

LOUIS DESPAIN SMITH

TRANSFORMATION CALCULUS AND ELECTRICAL TRANSIENTS, by Stanford Goldman. 439 pages, 14 × 22 cm., illustrations and tables. New York, Prentice-Hall, Inc., 1949. Price, $6.25.

Research and development workers in electrical engineering will find this book to be of great value as an aid in the analysis of transient electrical phenomena. Although practically no results are presented that have not appeared in other books, something new has been created here in the form of a consolidation and correlation of divers subjects in the field of applied mathematics as they pertain to transient-analysis problems. In this book one finds such seemingly unrelated mathematical topics as determinants, theory of functions of a complex variable, gamma and error functions, solutions in series, Bessel functions, etc. welded into a single cohesive bundle and clearly presented in direct relationship to transient problems in which they are useful. This is a coordinated, comprehensive treatment that has been needed in this field and which penetrates into an old subject with a modern, expanded viewpoint.

The principal aim of this book is to develop up-to-date methods of the Laplace transformation and its inverse as applied to electrical networks. In accomplishing this aim certain mathematical topics are introduced and used in numerous specific examples. Non-linear circuits are not treated nor is the problem of synthesis considered. Because the mathematical material is also useful in other phases of electrical engineering, the reader will strengthen his general background as well as aquire a familiarization with the tools of transient analysis.

Those individuals to whom the mathematics of this book is relatively unfamiliar will experience great difficulty if they attempt to study this book alone, despite its clarity. It deals largely with the important highlights of the subject rather than a thorough inquiry into each separate topic. This feature is apparently recognized, and partially compensated for, by excessive use of footnotes (over 230 of them!) many of which refer to other texts where a thorough treatment of the subject under discussion may be found. However, one wonders why an author index or bibliography has not been included, particularly because this is in the nature of a "consolidation" book. Perhaps it is because of the abundance of references to the author's previous book, *Frequency Analysis, Modulation and Noise*, which, despite its frequent "plugging," is a very worthwhile volume.

A significant weakness of this book is rooted in Dr. Goldman's attempt to bring moderately advanced material within the reach of college seniors. Certain topics are ridiculously simple

by contrast with the meaty middle and latter portions of the book. For instance, Chapter 1, "Determinants," contains material that can be found in many elementary mathematics books. This exceedingly elementary chapter contains some examples of the use of determinants in the solution of network problems which may be of some value, but the phasor diagrams and analytical geometry applications seem rather inappropriate. Similarly, a nineteen-page appendix "Basic Theory of Transmission Lines in the Steady State" (material that can be found in at least a half-dozen textbooks for undergraduates) seems far too long and unrelated to be justified by the very excellent and basic Chapter 10, "Transients in Transmission Lines." Another example is the author's failure to demonstrate the disadvantages of the classical method of analysis which is certainly a very desirable and logical way of introducing the novice to the more powerful and convenient Laplace transformation method. There are other examples of this divergence in which it seems that the two extremes are separated too far in level.

The net result is that many of these pages are essentially wasted. A senior in a high-level college would be able to cope with the first 128 pages but would tread on dangerous ground in proceeding further without a lengthy digression to several other works for fundamental background which is considered by the reviewer to be insufficient in this book for a first introduction to the subject. On the other hand, a person who is qualified to use this book to full advantage as a good centralized source of judiciously-selected mathematical information will simply ignore much of the elementary material. It would seem much better to have eliminated the latter, replacing it with a more complete treatment of the advanced aspects.

ERNEST FRANK

HEAT TRANSFER, Vol. I, by Max Jakob. 758 pages, illustrations, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1949. Price, $12.00.

This book is a comprehensive product of the author's many years of activity in research and teaching elementary, intermediate and graduate courses on the subject of heat transfer, and serves both as a treatise and as a textbook. Its main purpose is to show the development of ideas which led to the present state of knowledge on the subject. The main contents of this book, including the most important derivations, are presented in such detail that study of the original papers will not be necessary except in very rare cases. The book is international in scope, with particularly thorough coverage of extensive German literature.

Since the interrelation and peculiarities of conduction, convection and radiation become more evident in the parallel treatment, they are not dealt with separately. Accordingly, the first part of the book contains the basic equations of all three brancehs of heat transfer; the second part deals with the most characteristic properties occurring in each of these branches; then the solutions of the equations, with all their consequences, are shown in the conventional separation into special parts for each of the three kinds of heat flow as follows: Heat Conduction in Simple Bodies; Heat Transfer without Change of Phase or Constitution; and Heat Transfer Including Changes of Phase. Each of these parts contains straight solutions, similarity treatments, techniques, graphical methods, experimental methods, etc. The treatment of heat radiation in spaces of simple configuration, and selected fields for application of heat transfer will be presented in Volume II, now in preparation.

Appendices contain sample problems to be solved, a list of the symbols most often needed, conversion factors, bibliography and author index and a subject index.

This book, like the courses of the author, should serve the needs of mechanical, chemical, electrical, civil, and aerodynamic engineers, and physicists alike. The presentation is, therefore, an arbitrary compromise between the highly mathematical procedure in most technical publications and the presentation of empirical facts in others. For the benefit of readers with only a modest knowledge of higher mathematics, the mathematical procedures are explained in the first chapters and parts so that they may become acquainted with such typical features as finding and substituting suitable boundary conditions. The empirical material has been distributed throughout the text and presented wherever the analysis required it for illustration, verification or for showing the deficiencies of the theoretical analysis.

SAMUEL M. BERKOWITZ

TABLES AZÉOTROPIQUES, by Maurice Lecat. Tome premier, Azéotropes binaires orthobares. Deuxieme edition, 406 pages, 16 × 26 cm. Bruxelles, Chez L'Auteur, 1949. Price, 1000 Belgian francs.

In 1918 there appeared M. Lecat's first book on azeotropy which included some elementary considerations on the subject, tables of azeotropes, and a bibliography. Since that time he has had published some ninety works on the subject including both books and periodical articles. The bibliography which appeared in the 1918 volume has since been revised and expanded, and has been issued in two volumes, one in 1932 and the other in 1942.

The present title represents a revision or second edition of the tables of the 1918 work. Those tables were divided into four parts: binary systems with minimum boiling point; binary systems with maximum boiling point; ternary systems and quaternary systems. There were 2200 binary systems included and 250 ternary systems. The present volume is limited to binary systems and includes 6287 azeotropes and 7003 zeotropes, nearly all determined under 760 mm. of pressure. The forthcoming second volume will treat of the ternary systems as well as presenting M. Lecat's discussion of the empirical laws which may be set forth.

The tables are arranged by groups alphabetically from *Acides et Acides-cétones* to *Varia et Varia*, passing through, for example, *Acides et Oxydes*, *Cétones et Nitrodérivés*, and *Esters et Phénols*. The tables include the boiling point of each constituent, their difference, the boiling point of the azeotrope and its deviation, and the percentage of the first constituent in the mixture.

Bibliographical reference to the original sources is made by number to the entries in the two volumes of bibliography previously published, and in the case of M. Lecat's own work, to a list included in the present volume. References later than those included in the bibliography are given in full. An index by author and an index of the constituents of systems have been included.

In the forty odd years in which M. Lecat has been studying azeotropes, there has been a great increase in the knowledge of these mixtures and in their commercial importance. That much of this increase is due to M. Lecat will be recognized by all students of the subject and they will be grateful to him for providing this up-to-date comprehensive table of azeotropic values.

G. E. PETTENGILL

ALTERNATING CURRENT MACHINERY, by L. V. Bewley. 376 pages, illustrations, 16 × 24 cm. New York, The Macmillan Co., 1949. Price, $5.25.

Professor Bewley, in writing this new book on alternating current machinery for junior and senior students in electrical engineering, no doubt kept in mind the order in which most newly graduated engineers encounter the most important types of electrical machinery. The book introduces the general principles of magnetic circuits, followed by transformer theory. The basic design principles pertinent to rotating machines precede detailed treatments of specific machines, again in the approximate order in which these are encountered, namely: polyphase induction motors; single-phase induction motors; alternating current commutator machines; synchronous generators and motors; and last, synchronous converters.

What is different in this book? The author has approached the writing of this text with a fresh and long needed point of view, based upon his many years of industrial engineering and educational experience. For a long time, inadequate books on alternating current machinery have adhered to concepts and methods of analysis which should long ago have been abandoned because of their restricted and narrow approach. Here the author has attempted to unify the underlying theories of electrical machinery by introducing general methods of analysis which hold for many kinds of rotating machines. Several new concepts are introduced in the book which simplify an understanding of machinery theory.

The section devoted to transformers departs from the traditional treatment and assumes: (1) that all coils on the iron structure are independent, with interconnections to be made as desired for the transformer configurations to be studied; and (2) that the presence of a fictitious

short circuited coil can account for core loss. The use of differential equations greatly simplifies the presentation of the material.

Since most alternating current rotating machines have coil structures which have basic designs in common, a realization of the importance of the many empirical and computed design reduction factors is called for. This is essential to a full understanding of synchronous machines.

In an attempt to generalize the machine equations, the author has succeeded in developing all-inclusive basic equations for voltage generation and armature reaction which hold for a-c. or d-c. machines. A unification of concept naturally follows since one kind of machine differs from another only in the presence or absence of specific terms.

Many such examples of essential departures from the traditional treatment of machinery theory appear in the book. To review them in detail with an evaluation of their merit would not be proper in this resumé.

The many worked out examples and problems included in each chapter make the text ideal for a two semester course, to be supplemented by courses in electro-magnetic field analysis applied to magnetic structures. The one fault which this reviewer finds is the almost complete lack of references to the technical literature. This oversight should be corrected in future editions to make the book more desirable as a reference work not only for electrical engineering students but for the practicing engineer who desires a fresh approach in a review of basic a-c. machinery.

S. CHARP

TWO LECTURES. I. PRESENT SITUATION IN THE THEORY OF ELEMENTARY PARTICLES. II. ELECTRON THEORY OF SUPER-CONDUCTIVITY, by W. Heisenberg. 51 pages, 12 × 19 cm. New York, Cambridge University Press, 1949. No price.

The two lectures brought together in this slim book were delivered at the Cavendish Laboratory in December, 1947, as introductory discussions of two interesting, and as yet unresolved, topics in atomic theory. It is always a great pleasure to read Professor Heisenberg's lucid work, and he is perhaps at his best when, as in these lectures, he is directing his attention toward the understanding of physical fundamentals.

In the first lecture on the "Present Situation in the Theory of Elementary Particles" Heisenberg attempts to sketch, in broad outline, some necessary characteristics of a theory capable of describing the elementary particles. He suggests that the present quantum mechanical formalism based on the Correspondence Principle cannot be expected to apply for the relativistic energies at which the various elementary particles inter-convert. He doubts the existence of a primary Hamiltonian for the elementary particles and suggests that the various quantized field equations now employed for the description of particular particles will appear as low energy approximations in a complete theory. He finally describes briefly the S-matrix formalism which he has developed in several papers over the past few years as a possible alternative description of the elementary particles. At present, as Heisenberg admits, the S-matrix formalism is an "empty frame," but he implies that the attempt to discover the basic equations satisfied by the S-matrix is likely to prove more fruitful than the attempt to find a primary Hamiltonian.

In the second lecture Heisenberg describes his and H. Koppe's recent theory of super-conductivity. In this theory Heisenberg assumes that at low temperatures the electrons at the top of the Fermi distribution may, through purely Coulomb interactions, form a condensed phase, which has the character of a low density electron lattice, through the super-conductor. This assumption leads to the existence of current domains similar to the magnetic domains of a ferromagnet. Heisenberg describes the phenomena of super-conductivity in terms of the interactions between the Fermi electron gas, the ionic lattice, and the assumed electron lattice. He shows that the phenomenological equations of F. Landon may be derived from his assumptions.

Of course, all who are interested in the subjects discussed will be eager to read these lectures; Professor Heisenberg requires no commendation from a reviewer.

LEONARD EISENBUD

BOOK NOTES

EXPERIMENTAL GENERAL CHEMISTRY, by J. W. Neckers, T. W. Abbott and K. A. Van Lente. Second edition, 307 pages, diagrams, 20 × 27 cm. New York, Thomas Y. Crowell Company, 1949. Price, $2.25 (paper).

This laboratory manual for a course in general chemistry now appears in a revised edition to incorporate changes which observation of its use has indicated would make it more effective. About sixty per cent of the experiments have undergone at least partial revision. A feature which will commend itself both to the instructor and the student is a table correlating the experiments with seven different college chemistry texts.

ELEMENTARY MODERN PHYSICS, by Gordon F. Hull. 503 pages, illustrations, 14 × 22 cm. New York, Macmillan Company, 1949. Price, $5.25.

Originally published in 1936 under the title "An Elementary Survey of Modern Physics," this volume has been brought up to date. No marked change has been made in the treatment of the subject matter. Primarily a college textbook, it may interest some seeking a not too technical presentation of the ideas of modern physics since 1895.

ENCYCLOPEDIA OF CHEMICAL REACTIONS, compiled and edited by C. A. Jacobson. Volume III, 842 pages, 15 × 24 cm. New York, Reinhold Publishing Corp., 1949. Price, $12.00.

Beyond noting its appearance, there is little need to comment on this third volume of a useful reference work. Nineteen elements from cobalt to iridium, considering the elements alphabetically, are covered in this volume. This is more than in the previous volumes, since few reactions have been recorded in the chemical literature for the rarer elements, several of which are treated in this volume. Holmium and illumium are both represented by only one reaction; on the other hand copper has 610 and cobalt 822. The arrangement and indexes follow the standards set in the previous volumes.

AMERICAN COTTON HANDBOOK, by G. R. Merrill, A. R. Macormac and H. R. Mauersberger. Second revised edition, 943 pages, illustrations, 13 × 19 cm. New York, Textile Book Publishers, Inc., 1949. Price, $9.50.

Now appearing in a second revised edition, this standard work on the various phases of cotton manufacture, first published in 1941, has been brought up to date. The authors, the same as those of the first edition, have all had considerable experience in the textile field.

Several new chapters have been added, although to maintain the book within reasonable limits, some other material has been omitted. One of these new chapters is on cellulose and the cotton fiber, of particular interest in view of the newer developments in cotton finishing, many of which are based on fiber modification. The chapter on physical and chemical testing of fibers, yarns and fabrics has been split into two individual chapters and considerably augmented. Much new material has also been added on bleaching, dyëing and finishing. A chapter on "Nomenclature of Cotton Dyes" contains an alphabetical list of trade names giving the manufacturers and arranged by classes, according to fastness and dyeing properties. A comprehensive bibliography is included.

In addition to the major changes noted, the work shows evidence of many minor changes which result in clarification of the text. Containing much valuable information, this book should appeal to all those who are concerned with the manufacture of cotton goods.

CURRENT TOPICS

Ultra-Modern Shipping Center.—An ultra-modern $2,500,000 Shipping Center which employs many revolutionary new methods of material handling, and which will cut down from days to hours the time required to assemble and ship an order, has been opened at New Brunswick, N. J. by Johnson and Johnson.

The new Shipping Center, located just off New Jersey Highway 27 between Metuchen and New Brunswick, is set on a 58-acre plot, and its 207,000 gross square feet of floor space is sufficient to accommodate four regulation-size football fields.

This new unit is of modern industrial design. It brings into being an entirely new concept of warehousing, materials handling and shipping which is expected to revolutionize present methods in these fields.

An underground drag-line system pulls especially designed trucks through aisles along which some 500 different items are stored. An order assembler precedes these trucks, taking merchandise from piles of good stacked on pallets. An order calling for hundreds of different items can be assembled and shipped within a matter of hours, whereas previously as much as several days time were required.

To help keep this super-speed Shipping Center in high gear, three new inventions will be introduced for the first time:

A four wheel drag-line truck; a hand-operated label applicator; and a motor-driven dockboard are the new inventions designed to better the efficient operation of the new center. No employee will ever have to carry a carton more than four feet, or lift one more than four feet.

The drag-line trucks will follow the shipping employees around the center like pets anxious to carry boxes from the numerous vast stockpiles to the outgoing trucks, or railroad cars at the sidings. Each will carry an average of 900 lb., with a capacity of better than 4000 lb.

New hand-label applicators, twelve times faster than stenciling machines, will also help speed surgical dressings and other J & J products on to customers. They are light weight, hand-operated machines that apply labels, pre-printed with names and addresses of customers, on shipping cases while being loaded on a drag-line truck. Such on-the-spot labeling will minimize errors.

A mechanical dockboard for loading trucks is the third invention. Spanning the gap between building and carrier, the dockboard is electrically operated and can be swung into position in 10 seconds.

The new Shipping Center has two railroad tracks on a siding which leads to the main line of the Pennsylvania Railroad. Fifteen railroad "car spots," or loading and unloading bays, permit this number of cars to be loaded and unloaded simultaneously. Fourteen loading docks, seven for inbound and seven for outbound, will handle truck traffic. A special truck trailer parking lot will permit trailers to remain at the center until ready for loading. The Shipping Center's own truck tractor will haul the trailers to a loading bay and then back to the trailer parking lot loaded, where their own truck tractors

can conveniently remove them. This will eliminate idle time for the truck tractors.

As an order is received it is coded and shipping labels are prepared for insertion in an especially designed hand applicator which the order assembler uses. The assemblers have a sheet on which is listed the number of cartons or cases called for and the stock location numbers from which they are to be taken and placed on the small drag-line trucks. At the end of the dragline, trucks are removed from the line and the order is checked. If broken lots are required, they are assembled adjacent to this point from a series of shelves of loose merchandise. Here the small lots are placed on a belt-line, and wrapped. Then the packages are placed on marked shelves from which they are removed to the previously assembled whole-carton, or whole-case order shipment.

This new method of assembling and shipping an order will increase the capacity of the shipping plant ten times over that of the system previously in use.

The Shipping Center also employs an especially designed locator board at the receiving docks. It is a miniature of the reserve goods area and of its state of merchandise occupancy at any given time. This is supplemented by a series of visible index cards which serve as a perpetual inventory. As a pallet of merchandise is emptied in the order picking area a clip from the empty pallet is placed on the index card as a warning that additional reserve stock must be obtained from the factory.

Fruit "Essence" Now Free of Alcohol Tax.—Consumers should soon be getting food products with finer fruit flavors, say U. S. Department of Agricultural researchers. Changes in the alcohol-tax laws, ordered recently by Congress, now permit manufacture of fruit "essences" without payment of the $9-per-gallon tax formerly imposed. These flavor concentrates must still meet certain requirements, but Department scientists in the Bureau of Agricultural and Industrial Chemistry expect that the new regulations will result in rapid expansion of the fruit-essence industry—and in tastier fruit-flavored foods on your grocer's shelves.

A process for recovering and concentrating the volatile flavor and fragrance constituents of fresh apples and of grape juice has been developed by the Bureau at its Eastern Regional Research Laboratory in Philadelphia. Known technically as volatile fruit concentrates, the essences can be used by food manufacturers to enhance the taste and aroma of fruit products. A number of commercial companies are already set up to make concentrates from apples and other fruits by the Laboratory's method. Their demands for fresh fruit will help to expand markets for this year's tremendous apple crop.

The new alcohol-tax regulations governing production of volatile fruit concentrates were published in the *Federal Register* of September 27, 1949, and became effective on that date. They include revisions of the Internal Revenue Code in accordance with public Law 240, 81st Congress, which exempt fruit concentrates from the tax and various other provisions of the beverage-alcohol laws, provided certain requirements are met.

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References with (N) are Notes from the National Bureau of Standards and references with (C) are Current Topics

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